Algerian People's Democratic Republic

Ministry of Higher Education and Scientific Research

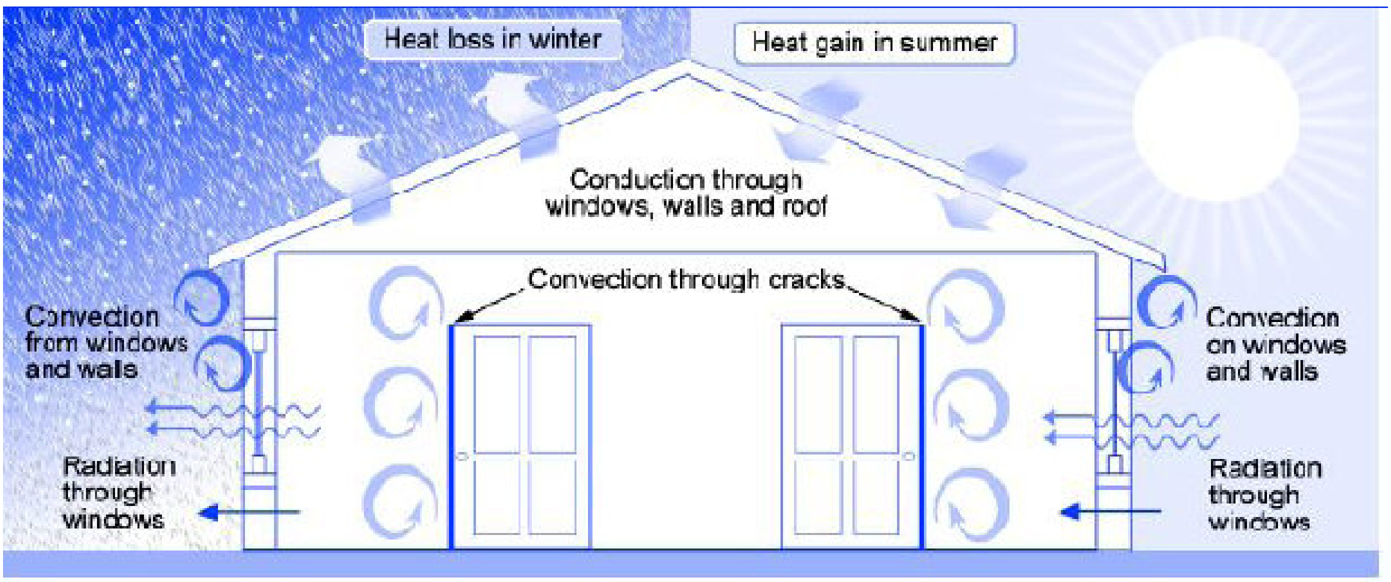
Ziane Achour University – Djelfa

Faculty of Science and Technology

Course Handout

Intended for students of the 2nd year Process Engineering

Heat Transfer



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Abstract

Heat transfer is directly related to numerous problems in many engineering and scientific applications. It is perhaps the most important, as well as the most applied process, in refining, gas processing, and chemical and petrochemical plants.

This course on heat transfer is intended for second-year students in process engineering, as well as anyone who needs to acquire a foundation in this discipline. It conforms to the official curriculum issued by the Ministry of Higher Education and Scientific Research. Through the various chapters, the reader will be introduced to the modes of heat transfer. They will thus be able to identify the mechanisms and calculate the quantities of thermal energy that migrate from one medium to another.

Keywords: Heat transfer, conduction, convection, radiation.

Contents

Chapter 3 Heat Transfer by Convection

Chapter 4 Heat Transfer by Radiation

Preface

This learning material consists of a lecture, tutorials to reinforce acquired knowledge, and review exercises. The lecture portion comprises three main chapters, in accordance with the official curriculum of the Ministry of Higher Education and Scientific Research:

The first chapter begins with an introduction to modes of heat transfer, followed by basic concepts of steady-state heat conduction, including thermal conductivity and fin design.

The second chapter deals with heat convection. Methods for quantifying heat flux under this regime are presented for both types of convection: natural and forced. This section also includes the main correlations used to calculate the convection coefficient for different regimes and geometries.

The final chapter is devoted to radiation, a complex phenomenon with some still poorly understood aspects. The fundamental laws are presented, along with the main methods for calculating thermal power in the most common cases.

Each chapter contains a theoretical part and is enriched with calculation examples that are a direct application of the concepts studied.

Course Scientific Objectives

From a scientific perspective, the objectives of this course align with the interests of studying heat transfer:

To be able to estimate the amount of heat energy flux that passes from one system to another;

- To determine the temperatures of the different systems, which allows us to understand how temperature varies with the material and over time;
- To achieve maximum heat flux at minimum cost;
- To recover heat from certain systems.
- Ensuring the insulation of certain systems to minimize heat loss; • Choosing appropriate materials to transfer heat, prevent heat loss, or select materials that withstand high temperatures.

Course Learning Objectives

This heat transfer course provides students with the fundamental knowledge needed to address heat transfer problems and equips them to:

- Correctly identify a heat transfer problem;
- Analyze situations to:
  - Easily determine the steps to follow and the laws to apply;
  - Find effective solutions and improve the operation of certain systems;
  - Be able to reason and analyze systems from a thermal perspective.

For each heat transfer problem, a series of steps is presented to facilitate the student's task, enabling them to easily arrive at a solution. An example in this context can be found in each Chapter. This course gives the learner the tools to analyze and solve common heat transfer problems as easily as possible using basic techniques.

Chapter 1

General Principles of Heat Transfer

I.1. Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference. Thermodynamics teaches that this energy transfer is defined as heat. The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange will take place under certain specified conditions. The fact that a heat-transfer *rate* is the desired objective of an analysis points out the difference between heat transfer and thermodynamics. Thermodynamics deals with systems in equilibrium; it may be used to predict the amount of energy required to change a system from one equilibrium state to another; it may not be used to predict how fast a change will take place since the system is not in equilibrium during the process. Heat transfer supplements the first and second principles of thermodynamics by providing additional experimental rules that may be used to establish energy-transfer rates. As in the science of thermodynamics, the experimental rules used as a basis of the subject of heat transfer are rather simple and easily expanded to encompass a variety of practical situations.

As an example of the different kinds of problems that are treated by thermodynamics and heat transfer, consider the cooling of a hot steel bar that is placed in a pail of water.

Thermodynamics may be used to predict the final equilibrium temperature of the steel bar–water combination. Thermodynamics will not tell us how long it takes to reach this equilibrium condition or what the temperature of the bar will be after a certain length of time before the equilibrium condition is attained. Heat transfer may be used to predict the temperature of both the bar and the water as a function of time.
Most readers will be familiar with the terms used to denote the three modes of heat transfer: conduction, convection, and radiation.

I.2. Heat Transfer Mechanisms

Heat can be transferred in three different modes: conduction, convection, and radiation. All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one. Below we give a brief description of each mode.

I.2.1. Conduction (Fourier's law)

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.

A cold canned drink in a warm room, for example, eventually warm sup to the room temperature as a result of heat transfer from the room to the drink through the aluminum can by conduction.

The rate of heat conduction through a medium depends on the geometry of the medium, its thickness, and the material of the medium, as well as the temperature difference across the medium.

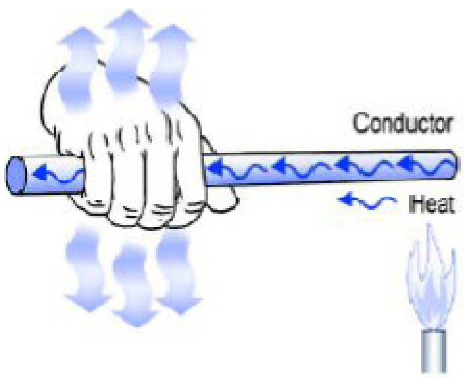
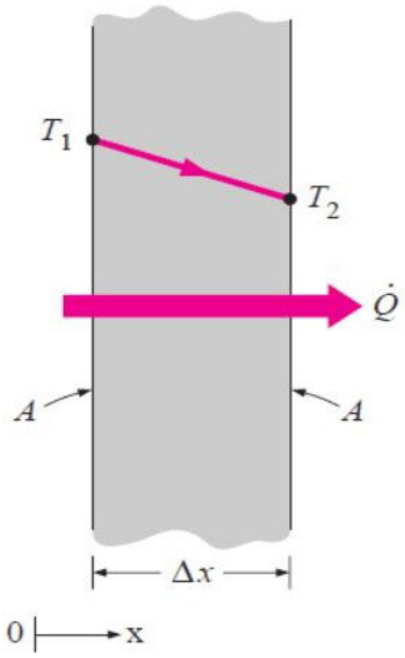


Figure 1.1. Example of conduction heat transfer



Heat conduction through a large plane wall of thickness Δx and area A.

$$\text{Rate of heat conduction} \propto \frac{\text{(Area)(Temperature difference)}}{\text{Thickness}}$$

or,

$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x} \qquad \text{(W)}$$

Where the constant of proportionality k is the **thermal conductivity** of the material, which is a measure of the ability of a material to conduct heat.

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)} \qquad \textbf{(1-1)}$$

(The basic law for conduction heat transfer is **Fourier's law**)

Q: heat transfer rate in watt

k: thermal conductivity of material in w/m°k

A: area in m^2

ΔT: temperature difference in °K

Δx: wall thickness in m

I.2.2. Convection (Newton's law)

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion; and it involves the combined effects of *conduction* and *fluid motion.* The faster the fluid motion, the greater the convection heat transfers. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction. The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates.

$$\dot{Q}_{\text{conv}} = hA_s\,(T_s - T_\infty) \qquad \text{(W)} \qquad \textbf{(1.2)}$$

Q: heat transfer rate in watt

h: convection heat transfer coefficient in $w/m^2.°k$

A: area in m^2

ΔT: temperature difference in °K = $T_w - T_\infty$

T_w: wall temperature in °K

T_∞: fluid temperature in °K

We distinguish between two types of convection:

- **Natural convection**: the movements are due to variations in density within a fluid subjected to the gravitational field. Density variations can be generated by temperature gradients (warm air is lighter than cold air) and/or by composition gradients.

- **Forced convection**: the movement of the fluid is caused by external mechanical actions (pump, fan...).

- We will talk about **mixed convection** when both types of convection coexist in a system.

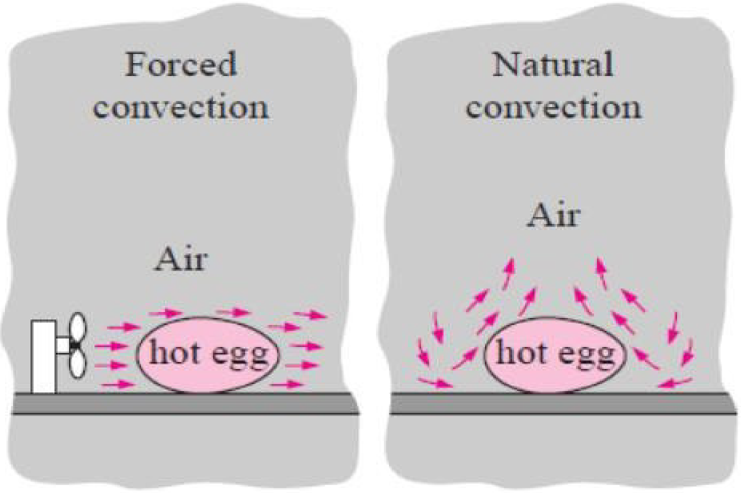


Figure 1.2. Examples of heat free and forced convection

I.2.3. Radiation (Stephan Boltzman's law)

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an *intervening medium.*

$$\dot{Q}_{emit} = \varepsilon \sigma A_s T_s^4 \quad (W) \qquad (1.3)$$

Q: heat transfer rate in watt

ε: emissivity of body

σ: Stephan-Boltzman constant =$5.67.10^{-8}$ $w/m^2.^{\circ}k^4$

A: area in m^2

T: absolute temperature in °K

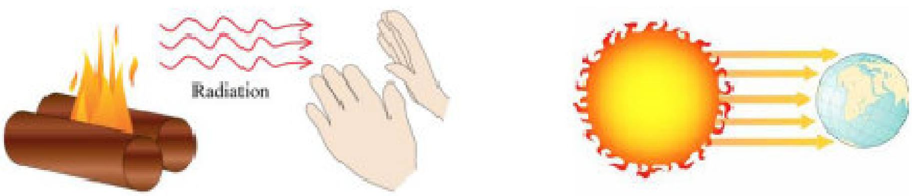


Figure 1.3. Examples of thermal radiation

I.3. Heat transfer parameters

I.3.1. Thermal conductivity

On the basis of this definition, experimental measurements may be made to determine the thermal conductivity of different materials. For gases at moderately low temperatures, analytical treatments in the kinetic theory of gases may be used to predict accurately the experimentally observed values. In some cases, theories are available for the prediction of thermal conductivities in liquids and solids, but in general, many open questions and concepts still need clarification where liquids and solids are concerned.

The mechanism of thermal conduction in a gas is a simple one. We identify the kinetic energy of a molecule with its temperature; thus, in a high-temperature region, the molecules have higher velocities than in some lower-temperature region. The molecules are in continuous random motion, colliding with one another and exchanging energy and momentum.

The molecules have this random motion whether or not a temperature gradient exists in the gas. If a molecule moves from a high-temperature region to a region of lower temperature, it transports kinetic energy to the lower-temperature part of the system and gives up this energy through collisions with lower-energy molecules.

Table 1-1 lists typical values of the thermal conductivities for several materials to indicate the relative orders of magnitude to be expected in practice. In general, the thermal conductivity is strongly temperature-dependent.

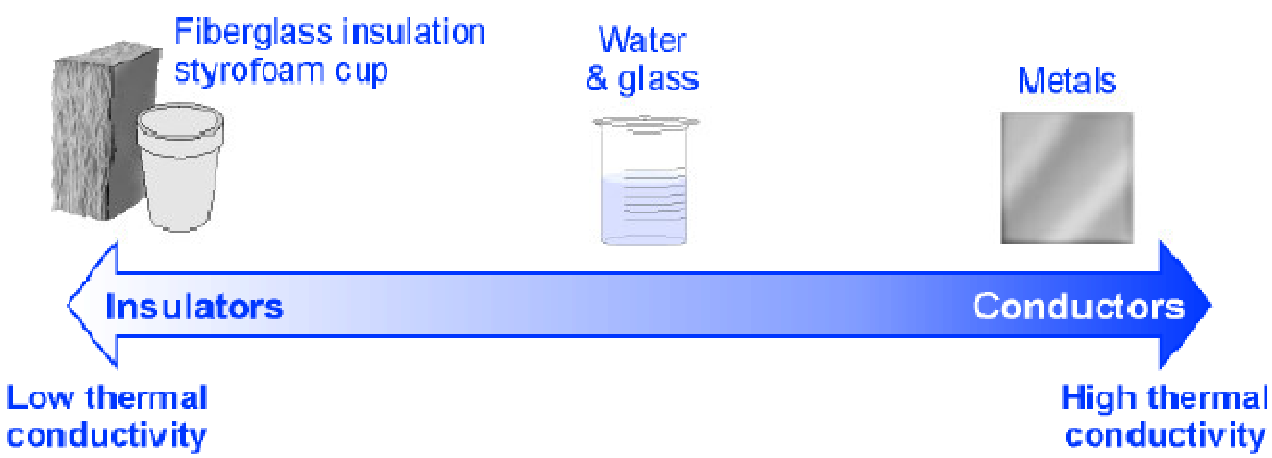


Figure 1.4. Thermal conductivity evolution

Material	Thermal cond. (W/m°C)
IIa diamond	2,650
Copper	401
Aluminum	226
Steel	43
Rock	3
Glass	2.2
Ice	2.2
Liquid water	0.58
Wood	0.11
Wool fabric	0.038
Fiberglass insulation	0.038
Styrofoam	0.025
Air	0.026

Table 1.1: Thermal conductivity value of different materials

I.3.2. Coefficient of heat transfer by convection

This coefficient depends on numerous parameters (fluid, type of flow, surface condition...) and is therefore extremely difficult to quantify precisely.

Process	h ($W/m^2 \cdot K$)
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

Table 1.2: Convection heat transfer coefficient values

I.3.3. Temperature Field

Energy transfers are determined based on the evolution of temperature in space and time: $T = f(x, y, z, t)$. The instantaneous value of the temperature at every point in space is a scalar called the temperature field. We distinguish two cases:

- Temperature field independent of time: the regime is said to be **permanent or stationary.**
- Evolution of the temperature field over time: the regime is said to be **variable or unsteady.**

I.3.4. Temperature Gradient

If we gather all the points in space that have the same temperature, we obtain a surface called an isothermal surface. The variation in temperature per unit length is maximum along the normal to the isothermal surface. This variation is characterized by the temperature gradient:

$$\overrightarrow{\text{grad}}(T) = \vec{n}.\frac{\partial T}{\partial n}, \quad \vec{\nabla}T = \overrightarrow{\text{grad}}T = \left| \begin{matrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{matrix} \right. \qquad \textbf{(1.4)}$$

I.3.5. Heat Flux

A heat flux is an amount of energy transferred in the form of heat per unit of time. It is therefore a power, expressed in Watts (J/s):

$$\varphi = \frac{dQ}{dt} \quad [W] \qquad \textbf{(1.5)}$$

I.3.6. Heat flux density

In general, the flux exchanged through a surface is not uniform across the entire surface. We then define a heat flux density ϕ, which corresponds to a heat flux per unit area.

$$\phi = \frac{1}{S}\frac{dQ}{dt} \quad \left[\frac{W}{m^2}\right] \qquad \textbf{(1.6)}$$

1.3.7. Specific Heats

Specific heat is defined as the energy required raising the temperature of a unit mass of a substance by one degree.

In thermodynamics, we are interested in two kinds of specific heats: specific heat at constant volume Cv and specific heat at constant pressure Cp. The specific heat at constant volume Cv can be viewed as the energy required to raise the temperature of a unit mass of a substance by one degree as the volume is held constant. The energy required to do the same as the pressure is held constant is the specific heat at constant pressure Cp.

Exercises

Exercise 1-1

Calculate the thermal flux (φ) as well as the thermal flux density (Ø) through a flat and homogeneous plate with a thickness of 50mm if it is:

- ✓ In stainless steel (λ_a=16W/m.K) with dimensions 3m x 2m.
- ✓ In concrete (λ_b=0.92W/m.K) with dimensions 30m x 20m.
- ✓ In both cases, the temperatures at the surfaces of the plate are maintained constant and equal to: Tp_1=100°C, Tp_2=90°C.

Solution

- ✓ In stainless steel:

$$\varphi_a = \frac{\lambda_a}{e}.S.(T_1 - T_2) = \frac{16}{50.10^{-3}}.(3.2)(100-90) = \mathbf{19200W}$$

$$\phi_a = \frac{\varphi_a}{S} = \frac{19200}{3.2} = 3200W/m^2$$

- ✓ In concrete

$$\varphi_b = \frac{\lambda_b}{e}(T_1 - T_2) = \frac{0,92}{50.10^{-3}}(30.20)(100-90) = \mathbf{110400W}$$

$$\phi_b = \frac{\varphi_b}{S} = \frac{110400}{30.20} = \mathbf{184W/m^2}$$

Exercise 1-2

a) Determine the heat flux through a flat wall with a thickness of e=10cm and a conductivity of λ=8.5W/m.K. The temperatures on the limiting faces of the wall are respectively equal to 100°C and 30°C. The surface area of the wall S=3m².

b) Find the temperature gradient in the direction of flow.

c) Calculate the depth of the wall where the temperature is 60°C.

Solution

a) The temperature gradient is :

$$\varphi = \frac{\lambda}{e}.S.(T_1 - T_2) = \frac{8,5}{10.10^{-2}}.(3)(100-30) = \mathbf{17850W}$$

b) the temperature gradient is :

$$\varphi = -\lambda.S.\frac{dT}{dx}$$

$$\Rightarrow \frac{dT}{dx} = \frac{-\varphi}{\lambda.S} = \frac{-17850}{8{,}5.3} = \mathbf{-700K/m}$$

c) The depth of the wall is :

$$\mathbf{T}(x) = \frac{(T_2 - T_1)}{e}x + T_1 \Rightarrow T(x) = -700.x + 100$$

$$x = \frac{100 - T(x)}{700} = \frac{100 - 60}{700} = 0{,}057m = \mathbf{5,7cm}$$

Exercise 1-3

The heat flux density Φ is 5000 W.m^{-2} at the surface of an electric heating element. The temperature of this same element is 110°C when it is cooled by forced convection in air with a temperature of 60°C.

- What is the average exchange coefficient *h*?
- What will be the temperature of the heating element if the flux density is reduced to 2000 W.m^{-2}?

Solution

- the average exchange coefficient is:

$$\bar{h} = \frac{\varphi}{\Delta T} = \frac{5000}{110 - 60} = \mathbf{100W.m^{-2}.K^{-1}}$$

- the temperature of the heating element is:

$$\Delta T = T_{element} - 60 = \frac{\varphi}{h} = \frac{2000}{100} = \mathbf{20°C}$$

So the temperature

$$\mathbf{T_{element}} = \Delta T + 60 = 20 + 60 = \mathbf{80°C}$$

Exercise 1-4

A pipe transports steam through a large room where the air and walls are at a temperature of 25°C. The external diameter of the pipe is 70mm and the surface temperature is 200°C. Its emissivity is 0.8.

1. What is the power emission flux density of the surface of the duct?
2. What is the density of the radiation power flux from this surface?
3. The convection coefficient from the surface to the ambient air is 15W/m²·K and the surface is considered gray, calculate the heat transfer flux density of this surface per unit length of the pipe?

Solution

1. The power emission flux density is:

$$q_{surface} = \varepsilon.\sigma.T_p^4 = 0.8x5.67x10^{-8}x(200+273.15)^4 = 2270W/m^2$$

2. The density of the radiation power flux is:

$$q_{max} = \sigma.T^4 = 5.67x10^{-8}x(25+273.15)^4 = 447W/m^2$$

The heat transfer flux density is:

$$Q_{tot} = Q_{ray} + Q_{conv} = h.S.(T_p - T_\infty) + \varepsilon.S.\sigma.\left(T_p^4 - T_\infty^4\right)$$

$$S = \pi x D x L = \pi x 70x10^{-3} xL$$

$$Q_{tot} = \pi x D x L x\left[h.(T_p - T_\infty) + \varepsilon.\sigma.\left(T_p^4 - T_\infty^4\right)\right]$$

Heat transfer flux density of this surface per unit length of the pipe:

$$q_{tot}\frac{Q_{tot}}{L} = \pi xDxLx\left[h.(T_p - T_\infty) + \varepsilon.\sigma.\left(T_p^4 - T_\infty^4\right)\right]$$

$$= \pi x70x10^{-3}x[15.(200-25) + 0.8x5.67x10^{-8}((200+273.15)^4 - (25+273.15)^4)]$$

$$= 998W/m$$

Chapter 2

Heat Transfer by Conduction

2.1. General equation of conduction

Consider the one-dimensional system shown in Figure 2-1. If the system is in a steady state, i.e., if the temperature does not change with time, then the problem is a simple one, and we need only integrate Equation (1-1) and substitute the appropriate values to solve for the desired quantity. However, if the temperature of the solid is changing with time, or if there are heat sources or sinks within the solid, the situation is more complex. We consider the general case where the temperature may be changing with time and heat sources may be present within the body. For the element of thickness *dx*, the following energy balance may be made:

Energy conducted in left face + heat generated within element = change in internal energy + energy conducted out right face

2.1.1. In Cartesian coordinates

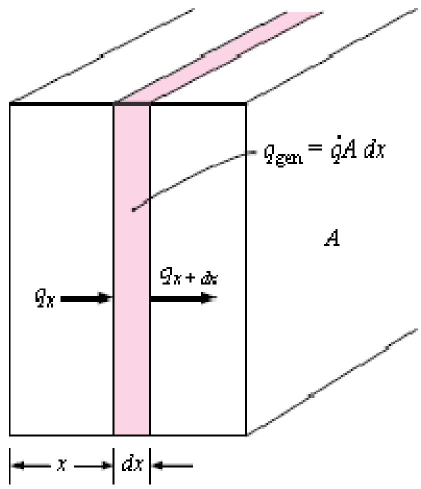


Figure 2-1. Elemental volume for one-dimensional heat conduction analysis.

These energy quantities are given as follows:

$$\text{Energy in left face} = q_x = -kA\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$

$$\text{Change in internal energy} = \rho c A\frac{\partial T}{\partial \tau}dx$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{\partial T}{\partial x}\right]_{x+dx}$$

$$= -A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

Where

$\dot{q}$ = energy generated per unit volume, W/m^3

C = specific heat of material, J/kg °C

ρ = density, kg/m^3

Combining the relations above gives

$$-kA\frac{\partial T}{\partial x} + \dot{q}A\,dx = \rho cA\frac{\partial T}{\partial \tau}dx - A\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

Or **(2.1)**

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

This is the one-dimensional heat-conduction equation. To treat more than one-dimensional heat flow, we need consider only the heat conducted in and out of a unit volume in all three coordinate directions, as shown in Figure 2-2. The energy balance yields:

$$q_x + q_y + q_z + q_{gen} = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{dE}{d\tau} \quad \textbf{(2.2)}$$

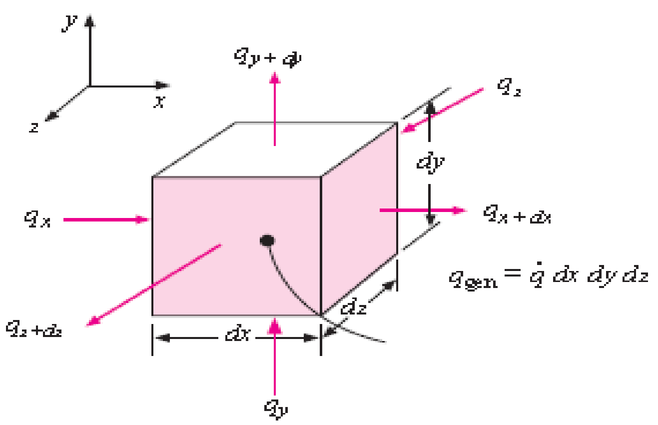


Figure 2-2 Elemental volume for three-dimensional heat-conduction analysis (Cartesian coordinates)

And the energy quantities are given by

$$q_x = -k\,dy\,dz\,\frac{\partial T}{\partial x}$$

$$q_{x+dx} = -\left[k\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]dy\,dz$$

$$q_y = -k\,dx\,dz\,\frac{\partial T}{\partial y}$$

$$q_{y+dy} = -\left[k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)dy\right]dx\,dz$$

$$q_z = -k\,dx\,dy\,\frac{\partial T}{\partial z}$$

$$q_{z+dz} = -\left[k\frac{\partial T}{\partial z} + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)dz\right]dx\,dy$$

$$q_{gen} = \dot{q}\,dx\,dy\,dz$$

$$\frac{dE}{d\tau} = \rho c\,dx\,dy\,dz\,\frac{\partial T}{\partial \tau}$$

So that the general three-dimensional heat-conduction equation is

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial \tau}$$

For constant thermal conductivity, this equation is written:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad \textbf{(2.3)}$$

Where the quantity $\alpha = k/\rho c$ is called the *thermal diffusivity* of the material

Equation may be transformed into either cylindrical or spherical coordinates by standard calculus techniques. The results are as follows:

2.1.2. In Cylindrical coordinates

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \qquad \textbf{(2.4)}$$

2.1.3. Spherical coordinates

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial\tau} \quad \textbf{(2.5)}$$

Many practical problems involve only special cases of the general equations listed above. As a guide to the developments in future chapters, it is worthwhile to show the reduced form of the general equations for several cases of practical interest.

2.2. Steady State and Unsteady State Heat Transfer

Heat transfer is the transfer of thermal energy from a body, at a high temperature, to another at a lower temperature. This transfer of thermal energy may occur under steady or unsteady state conditions. Under Steady state conditions the temperature within the system does not change with time. Conversely, under unsteady state conditions the temperature within the system does vary with time.

Unsteady state conditions are a precursor to steady state conditions. No system exists initially under steady state conditions. Sometime must pass, after heat transfer is initiated, before the system reaches steady state. During that period of transition, the system is under unsteady state conditions.

Clearly, no system can remain under unsteady state conditions perpetually. The temperature of the system will eventually reach the temperature of the heat source, and once this happens, the system will be at steady state. Even if the amount of heat being transferred into the system is increased, at some point the system reaches its critical temperature and the energy transfer red into it the starts causing phase changes within the system rather than temperatures increases.

2.3. Conduction of Heat Transfer in the Steady State Conduction Through a Homogeneous Plans Wall

Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2. For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as:

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

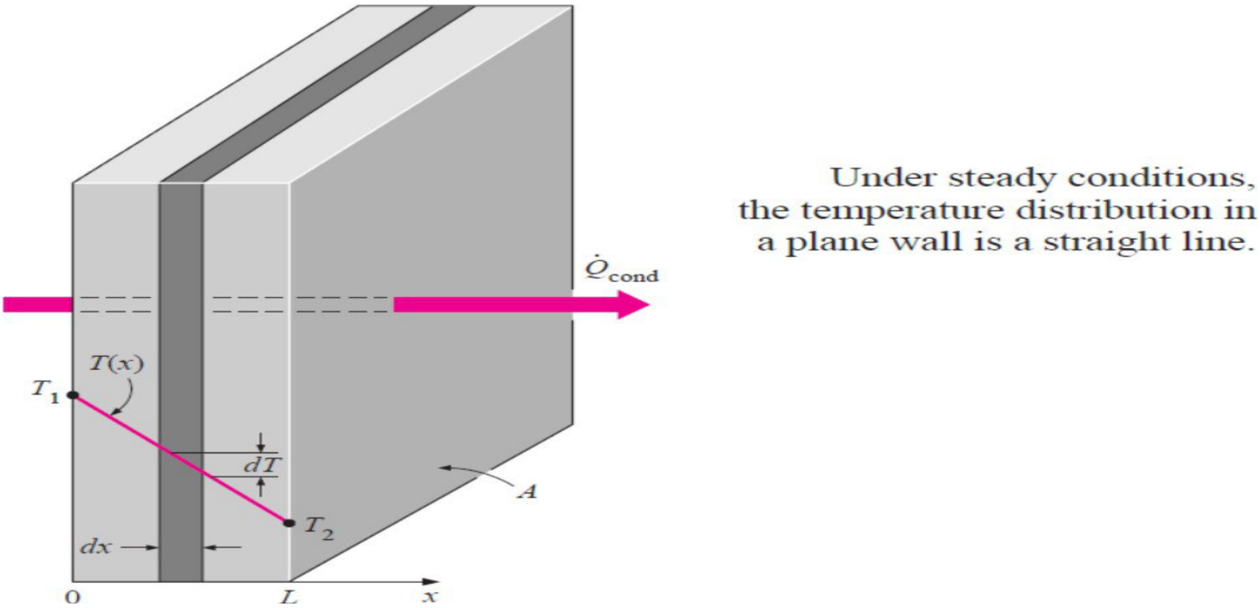


Figure 2-3 Conduction of Heat Transfer in the Steady State Conduction

Where the rate of conduction heat transfer $Q_{condwall}$ and the wall area A are constant. Thus, we have *dT/dx* = constant, which means that the temperature through the wall varies linearly with *x*. That is, the temperature distribution in the wall under steady conditions is a straight line.

Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)} \qquad (2.6)$$

Again, the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness. Also, once the rate of heat conduction is available, the temperature T(x) at any location x can be determined by replacing T_2 in Eq. above by T, and L by x.

***E*xample**: Consider a 3m high, 5m wide, and 0.3m thick wall whose thermal conductivity is k =0.9 W/m·°C. On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively.
Determine the rate of heat loss through the wall on that day?

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.
Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.
Properties The thermal conductivity is given to be k = 0.9 W/m · °C.
Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is A = 3 m × 5 m = 15 m^2, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9\ \text{W/m} \cdot {}^\circ\text{C})(15\ \text{m}^2)\frac{(16 - 2)^\circ\text{C}}{0.3\ \text{m}} = \mathbf{630\ W}$$

Home Work

Consider a 4m high, 6m wide, and 0.3m thick brick wall whose thermal conductivity is k =0.8 W/m·°C. On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 14°C and 6°C, respectively.
Determine the rate of heat loss through the wall on that day?

2.4. Conduction Through a Composite Plans Wall

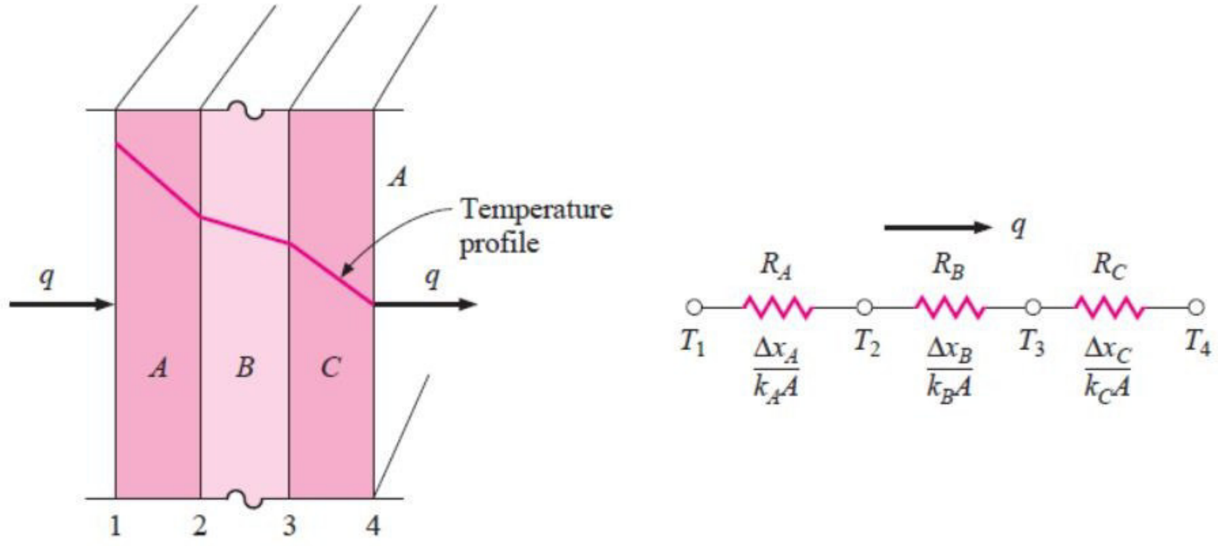


Figure 2-4 Conduction of Heat Transfer Through a Composite Plans Wall

If more than one material is present, as in the multilayer wall shown in Figure (2-4), the analysis would proceed as follows: The temperature gradients in the three materials are shown, and the heat flow may be written:

$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}$$

Note that the heat flow must be the same through all sections.
Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A} \tag{2.7}$$

At this point we retrace our development slightly to introduce a different conceptual viewpoint for Fourier's law. The heat-transfer rate may be considered as a flow, and the combination of thermal conductivity, thickness of material, and area as a resistance to this flow. The temperature is the potential, or driving, function for the heat flow, and the Fourier equation may be written :

$$\text{Heat flow} = \frac{\text{thermal potential difference}}{\text{thermal resistance}} \tag{2.8}$$

a relation quite like Ohm's law in electric-circuit theory. In precedent equation the thermal resistance is *Δx/kA*, it is the sum of the three terms in the denominator.

We should expect this situation in Equation because the three walls side by side act as three thermal resistances in series. The equivalent electric circuit is shown in Figure bellow (2-5).

The electrical analogy may be used to solve more complex problems involving both series and parallel thermal resistances. A typical problem and its analogous electric circuit are shown in Figure (2-5). The one-dimensional heat-flow equation for this type of problem may be written

$$q = \frac{\Delta T_{overall}}{\sum R_{th}} \tag{2.9}$$

where the R_{th} are the thermal resistances of the various materials. The units for the thermal resistance are °C/W or °F .h/Btu.

2.5. Insulation and resistance values *R*

In Chapter 1 we noted that the thermal conductivities for a number of insulating materials was given. In classifying the performance of insulation, it is a common practice

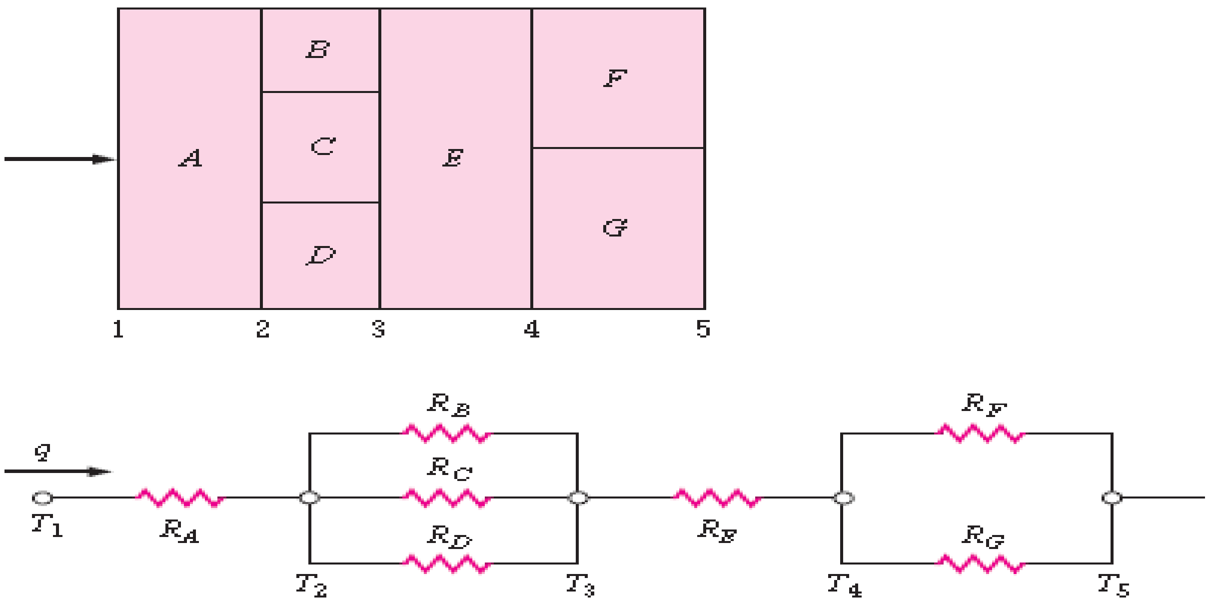


Figure 2-5 Series and parallel one-dimensional heat transfer through a composite wall and electrical analog.

$$q = \frac{k_A A(t_1 - t_2)}{(\Delta x)_A} = \frac{k_B A(t_2 - t_3)}{(\Delta x)_B} = \frac{k_C A(t_3 - t_4)}{(\Delta x)_C}$$

$$\Rightarrow q = \frac{A(t_1 - t_2)}{\frac{(\Delta x)_A}{k_A} + \frac{(\Delta x)_B}{k_B} + \frac{(\Delta x)_C}{k_C}}$$

This equation can be generalized:

$$q = \frac{A \sum \Delta T}{\sum \frac{(\Delta x)}{k}} \tag{2-14}$$

This equation is identical to that of the electric equation:

$$I = \frac{E}{R} \tag{2-15}$$

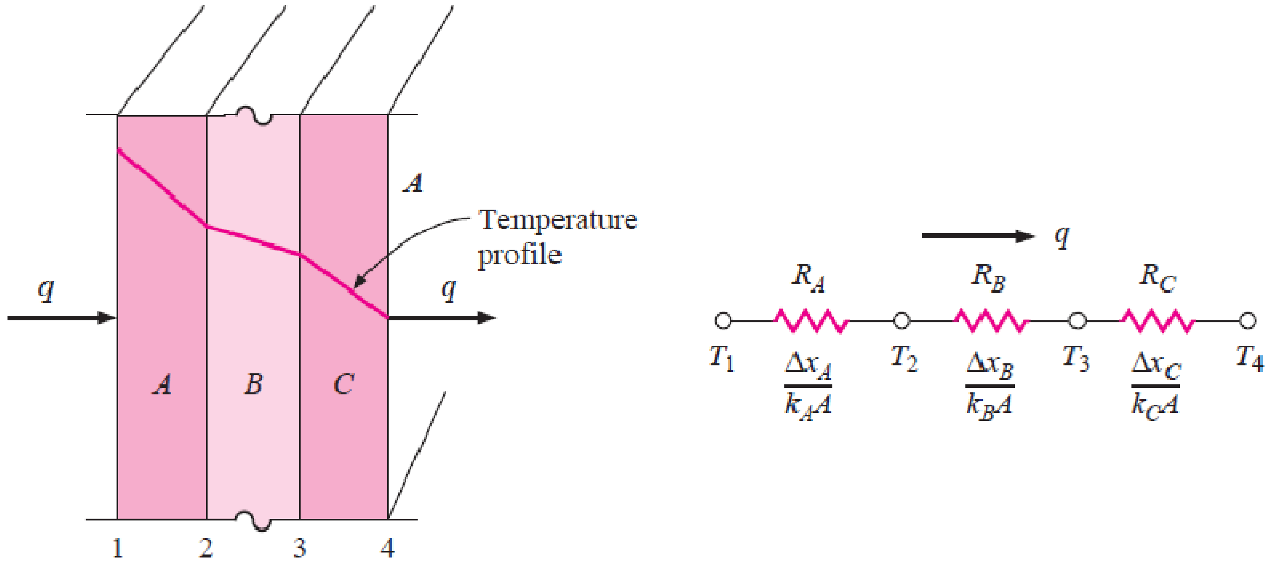


Figure 2-6 One-dimensional heat transfer through a composite wall and electrical analog

$$\text{Heat flow} = \frac{\text{thermal potential difference}}{\text{thermal resistance}}$$

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}} \tag{2-16}$$

2.6. Examples

- **Example 1**

A wall of furnace of 22cm thickness with thermal conductivity k=0.312w/m.°C. Calculate the heat loss from 1m^2 if the outside and inside temperatures of wall are 205°C and 816°C respectively.

Solution

$$q = \frac{\Delta T}{\frac{\Delta X}{K * A}} = \frac{816-205}{\frac{0.22}{0.312*1}} = 866.5 \text{ w}$$

- **Example 2**

A plane slab of thickness (L = 60 cm) is made of a material of thermal conductivity (K= 17.5 W/m.°C). The left side of the slab absorbs a net amount of radiant energy from a radiant source at rate (q=530 W/m^2). The right side of the slab is at constant temperature (T_2 = 38°C). Find the temperature of the left side?

Solution

$$q = \frac{KA(T_1 - T_2)}{L} \Rightarrow (T_1 - T_2) = \frac{qL}{KA} \Rightarrow T_1 = T_2 + \frac{qL}{AK}$$

$$\Rightarrow T_1 = 38 + \frac{530(0.6)}{17.5} = 56.17\ °C$$

2.7. Conduction Heat Transfer Through a Cylindrical Wall

Consider a long cylinder of inside radius r_i, outside radius r_o, and length L,

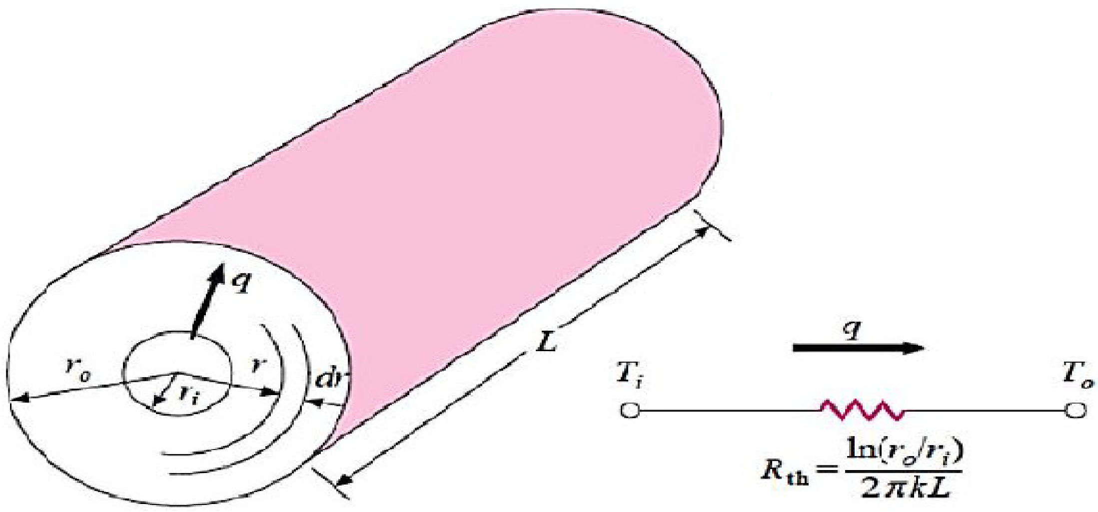


Figure 2-7 One-dimensional heat transfer through a cylinder and electrical analog

We expose this cylinder to a temperature differential ($T_i - T_o$) and ask what the heat flow will be. For a cylinder with length very large compared to diameter, it may be assumed that the heat flows only in a radial direction, so that the only space coordinate needed to specify the system is r. Again, Fourier's law is used by inserting the proper area relation. The area for heat flow in the cylindrical system is:

$$A_r = 2\pi r L$$

so that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi k r L \frac{dT}{dr}$$

$$q \frac{dr}{r} = -2\pi K L \, dt$$

$$\int_{r_1}^{r_2} q \frac{dr}{r} = -2\pi K L \int_{T_1}^{T_2} dt \longrightarrow q\,(\ln r_2 - \ln r_1) = -2\pi K L (T_2 - T_1)$$

$$q \left(\ln \frac{r_2}{r_1}\right) = 2\pi K L (T_1 - T_2) \longrightarrow \boxed{q = 2\pi K L \frac{(T_1 - T_2)}{\left(\ln \frac{r_2}{r_1}\right)}}$$

Thermal resistance can be written:

$$R_{th} = \frac{\ln (r_2 / r_1)}{2\pi k L} \qquad (2\text{-}17)$$

2.8. Conduction through a Composite Cylindrical Wall

The thermal-resistance concept may be used for multiple-layer cylindrical walls just as it was used for plane walls. For the three layers system

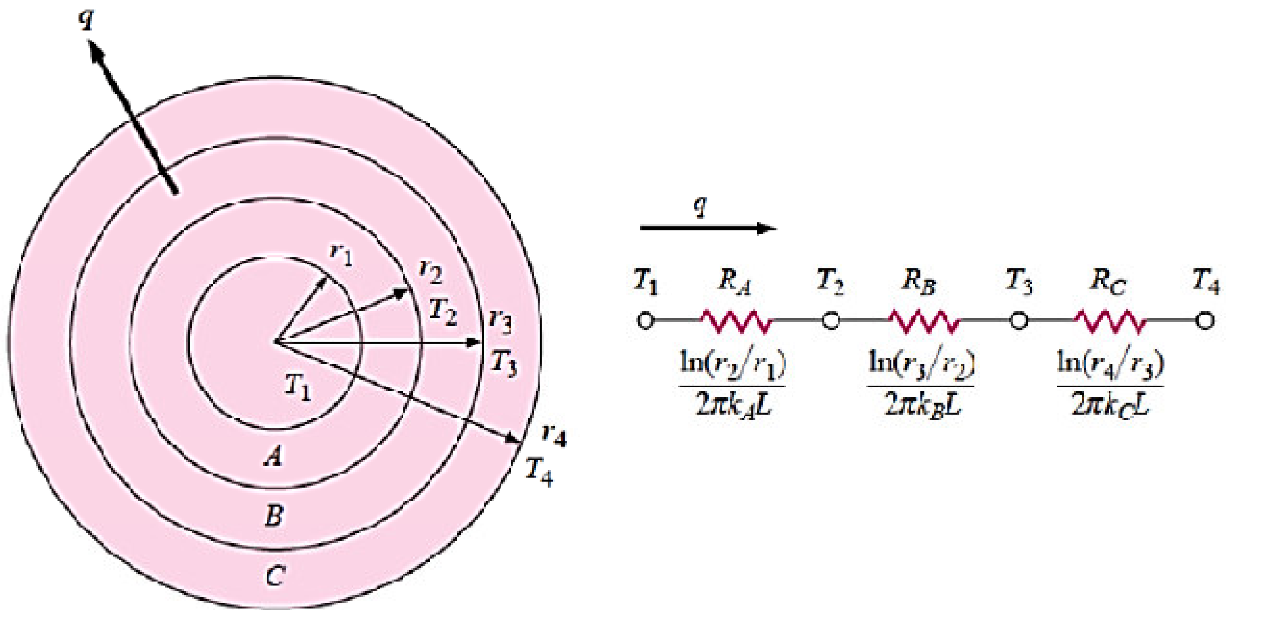


Figure 2-8 One-dimensional heat transfer through a multiple cylindrical sections and electrical analog

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln(r_2/r_1)/k_A + \ln(r_3/r_2)/k_B + \ln(r_4/r_3)/k_C} \qquad (2\text{-}18)$$

2.9. Conduction Heat Transfer Through a Spheres

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o} \qquad (2\text{-}19)$$

2.10. Fins

In the foregoing development we derived relations for the heat transfer from a rod or fin of uniform cross-sectional area protruding from a flat wall. In practical applications, fins may have varying cross-sectional areas and may be attached to circular surfaces. In either case the area must be considered as a variable in the derivation, and solution of the basic differential equation and the mathematical techniques become more tedious. We present only the results for these more complex situations.

To indicate the effectiveness of a fin in transferring a given quantity of heat, a new parameter called fin efficiency is defined by

$$\text{Fin efficiency} = \frac{\text{actual heat transferred}}{\text{heat that would be transferred}} = \eta_f$$
$$\text{if entire fin area were}$$
$$\text{at base temperature}$$

In this case, the fin efficiency becomes

$$\eta_f = \frac{\sqrt{hPkA}\,\theta_0 \tanh mL}{hPL\theta_0} = \frac{\tanh mL}{mL} \qquad (2\text{-}20)$$

The fins discussed were assumed to be sufficiently deep that the heat flow could be considered one-dimensional. The expression for *mL* may be written

$$mL = \sqrt{\frac{hP}{kA}}L = \sqrt{\frac{h(2z+2t)}{kzt}}L$$

Where *z* is the depth of the fin, and *t* is the thickness. Now, if the fin is sufficiently deep, the term *2z* will be large compared with *2t*, and

$$mL = \sqrt{\frac{2hz}{ktz}}L = \sqrt{\frac{2h}{kt}}L$$

Multiplying numerator and denominator by $L^{1/2}$ gives

$$mL = \sqrt{\frac{2h}{kLt}}L^{3/2}$$

L_t is called the profile area of the fin, which we define as

$$A_m = Lt$$

So that

$$mL = \sqrt{\frac{2h}{kA_m}}L^{3/2} \qquad (2\text{-}21)$$

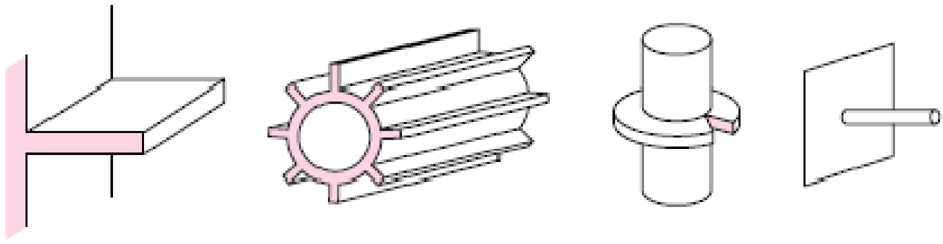


Figure 2-9 Different types of finned surfaces. (*a*) Straight fin of rectangular profile on plane wall, (*b*) straight fin of rectangular profile on circular tube, (*c*) cylindrical tube with radial fin of rectangular profile, (*d*) cylindrical-spine or circular-rod fin.

In some cases a valid method of evaluating fin performance is to compare the heat transfer with the fin to that which would be obtained without the fin. The ratio of these quantities is

$$\frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\eta_f A_f h \theta_0}{h A_b \theta_0}$$

Where A_f is the total surface area of the fin and A_b is the base area. For the insulated-tip fin described by Equation (2-36),

$$A_f = PL$$

$$A_b = A$$

And the heat ratio would become

$$\frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\tanh mL}{\sqrt{hA/kP}} \qquad (2\text{-}22)$$

This term is sometimes called the *fin effectiveness*

In heat-exchanger applications a finned-tube arrangement might be used to remove heat from a hot liquid. The heat transfer from the liquid to the finned tube is by convection. The heat is conducted through the material and finally dissipated to the surroundings by convection. Obviously, an analysis of combined conduction-convection systems is very important from a practical standpoint.

Consider the one-dimensional fin exposed to a surrounding fluid at a temperature T_∞ as shown in Figure 2-10. The temperature of the base of the fin is T0. We approach the problem by making an energy balance on an element of the fin of thickness dx as shown in the figure. Thus:

Energy in left face = energy out right face + energy lost by convection

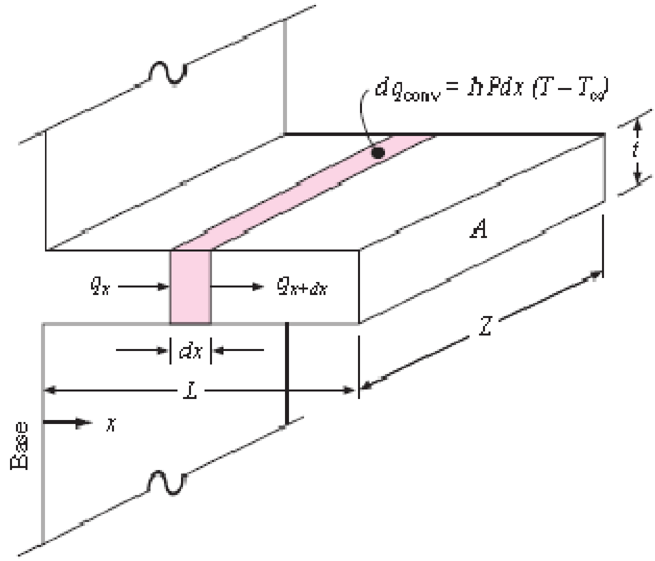


Figure 2-10 Sketch illustrating one-dimensional conduction and convection through a rectangular fin.

The defining equation for the convection heat-transfer coefficient is recalled as

$$q=hA\,(T_w - T_\infty)$$

Where the area in this equation is the surface area for convection. Let the cross-sectional area of the fin be *A* and the perimeter be *P*. Then the energy quantities are

$$\text{Energy in left face} = q_x = -kA\frac{dT}{dx}$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{dT}{dx}\right]_{x+dx}$$

$$= -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right)$$

$$\text{Energy lost by convection} = hP\,dx\,(T - T_\infty)$$

Here it is noted that the differential surface area for convection is the product of the perimeter of the fin and the differential length *dx*. When we combine the quantities, the energy balance yields

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0 \qquad (2\text{-}23)$$

Let $\theta = T - T_\infty$. Then Equation (2-23) becomes

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA}\theta = 0 \qquad (2\text{-}24)$$

One boundary condition is: $\theta = \theta_0 = T_0 - T_\infty$ at $x = 0$

The other boundary condition depends on the physical situation. Several cases may be considered:

3. **CASE 1** The fin is very long, and the temperature at the end of the fin is essentially that of the surrounding fluid.
4. **CASE 2** The fin is of finite length and loses heat by convection from its end.
5. **CASE 3** The end of the fin is insulated so that $dT/dx=0$ at $x=L$.

If we let $m_2 = hP/kA$, the general solution for Equation (2-24) may be written

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \qquad (2\text{-}25)$$

- For **case 1** the boundary conditions are

  $\theta = \theta_0$ at $x=0$

  $\theta = 0$ at $x=\infty$

  And the solution becomes

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx} \qquad (2\text{-}26)$$

- For **case 3** the boundary conditions are

$$\theta = \theta_0 \quad \text{at } x=0$$

$$\frac{d\theta}{dx} = 0 \quad \text{at } x = L$$

Thus

$$\theta_0 = C_1 + C_2$$

$$0 = m(-C_1 e^{-mL} + C_2 e^{mL})$$

Solving for the constants C_1 and C_2, we obtain

$$\frac{\theta}{\theta_0} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}}$$

$$= \frac{\cosh[m(L-x)]}{\cosh mL}$$

The hyperbolic functions are defined as

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

The solution for case 2 is more involved algebraically, and the result is

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL} \qquad (2\text{-}27)$$

All of the heat lost by the fin must be conducted into the base at x=0. Using the equations for the temperature distribution, we can compute the heat loss from

$$q = -kA\left.\frac{dT}{dx}\right]_{x=0}$$

An alternative method of integrating the convection heat loss could be used:

$$q = \int_0^L hP(T - T_\infty)\,dx = \int_0^L hP\,\theta\,dx$$

In most cases, however, the first equation is easier to apply. For case 1,

$$q = -kA\,(-m\theta_0 e^{-m(0)}) = \sqrt{hPkA}\,\theta_0 \qquad (2\text{-}28)$$

- For case 3

$$q = -kA\theta_0 m\left(\frac{1}{1+e^{-2mL}} - \frac{1}{1+e^{+2mL}}\right)$$
$$= \sqrt{hPkA}\,\theta_0 \tanh mL \qquad (2\text{-}29)$$

The heat flow for case 2 is

$$q = \sqrt{hPkA}\,(T_0 - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \qquad (2\text{-}30)$$

Exercises

Exercise 2.1

Calculate the heat loss from a tube of (9m) long and (6cm) outside diameter covered with (2.5cm) thick of insulation with (k=0.065 W/m.°C), if the inside and outside temperatures of insulation are (195°C , 27°C) respectively?

Solution

$$r1=3cm=0.03m$$

$$r2=3cm+2.5cm=5.5cm=0.055m$$

$$q=\frac{\Delta T}{R}=\frac{(t1-t2)}{\frac{\ln\frac{r2}{r1}}{2*\pi*k*L}}$$

$$q=\frac{(195-27)}{\frac{\ln\frac{0.055}{0.03}}{2*\pi*0.065*9}}=1019w.$$

Exercise 2.2 (Home Work)

A tube of 20cm in side diameter and 30cm outside diameter with k=1.42W/m.°C covered with 2cm thick of insulation with k=0.621 W/m.°C. Calculate

1-the heat resistance R_1, R_2 for 5m long of tube

2-the heat loss if the inside and outside temperatures are 600°C, 50°C respectively?

Exercise 2.3

If the rate of heat loss in hour through square meter of furnace wall with (45cm) thick is (5110 kJ), calculate the outside surface temperature of wall if the inside surface temperature is (1040°C) and the mean thermal conductivity of wall is (1.04 W/m.°C)?

Solution

$$\frac{q}{hr} = 5110\ \text{kJ/hr}$$

$$q = \frac{\Delta T}{R} = \frac{(Ti - To)}{\frac{\Delta x}{k.A}} = \frac{(1040 - To)}{\frac{0.45}{1.04 * 1}}$$

$$\frac{5110 * 1000 J}{hr * 3600\ sec} = \frac{(1040 - To)}{\frac{0.45}{1.04 * 1}}$$

$To = 427°C$

Exercise 2.4

Calculate the heat loss per square meter of composite furnace wall made of (20 cm) thick of thermal brick and (15 cm) thick of insulation brick and (10 cm) thick of brick, if the inside and outside temperatures of furnace are (1100°C),(100°C), respectively and the mean thermal conductivity of the three layers are(1.2,0.128,1.62 W/m.°C) respectively?

Exercise 2.5

Calculate the rate of heat loss from a wall of furnace with (20 cm) thick, the inside and outside temperatures of furnace are (980°C, 200°C), respectively if the thermal conductivity of materials of wall is (1.231 W/m.°C), if we use an insulation with (0.75 cm) thick and (k=0.092 W/m.°C), so that the heat loss decrease to 24%. Calculate the outer temperature of insulation?

Solution

$$q=\frac{\Delta T}{R}=\frac{(Ti-To)}{\frac{\Delta x}{k.A}}$$

Before insulation $q_1''=\frac{(Ti-To)}{\frac{\Delta x}{k}}=\frac{(980-200)}{\frac{0.2}{1.231}}=4800.9\ w/m^2$

After insulation $q_2''=q_1''-0.24\,q_1''=0.76\,q_1''=3648.6\ w/m^2$

$$q_2''=\frac{\sum \Delta T}{\sum R}=\frac{T1-T3}{R1+R2}=\frac{T1-T3}{\frac{\Delta X1}{K1}+\frac{\Delta X2}{K2}}$$

$$3648.6=\frac{980-T3}{\frac{0.2}{1.231}+\frac{0.0075}{0.092}}\ \frac{KJ}{hr.m^2} \qquad T3=89.77°C$$

Exercise 2.6

If the inside wall of furnace made of thermal brick with (10cm) thick and the outside wall of red brick with (20cm) thick, the inside and outside temperatures are (700°C), (120°C) respectively. to decrease the heat loss we add an insulation with (5cm) thick and thermal conductivity of (0.063)w/m.°C so that the temperatures will be (730°C), (670°C), (517°C). (77°C) respectively. Calculate

1- The heat loss without insulation?
2- The heat loss with insulation?
3- Express the heat loss in second state as a ratio of heat loss in first state?

Solution

$$q=\frac{\sum \Delta T}{\sum R}=\frac{Ti-To}{R1+R2}$$

$$q_1''=\frac{T1-T3}{\frac{\Delta X1}{K1}+\frac{\Delta X2}{K2}}=\frac{700-120}{\frac{0.1}{K1}+\frac{0.2}{K2}}$$ without insulation

$$q_2''=\frac{517-77}{\frac{0.05}{0.063}}=554.4$$ w/m² with insulation

for steady state $q_2''=q_1''=q_3''$

$$554.4=\frac{730-670}{\frac{0.1}{K1}} \Longrightarrow K_1=0.924 \text{ w/m.°C}$$

$$554.4=\frac{670-517}{\frac{0.2}{K2}} \Longrightarrow K_2=0.724 \text{ w/m.°C}$$

$$q_1''=\frac{T1-T3}{\frac{\Delta X1}{K1}+\frac{\Delta X2}{K2}}=\frac{700-120}{\frac{0.1}{0.924}+\frac{0.2}{0.724}}=1510.41 \text{ w/m}^2$$

$$\frac{q_2''}{q_1''}\times 100\%=\frac{554.4}{1510.4}\times 100\%=36.7\%$$

Exercise 2.7

Starting from the equation for conduction heat transfer per unit length for a tube, prove that heat loss per m^2 from the outside surface of tube which outside diameter is D_2 and inside diameter is $D_1$is given by

Solution

$$q' = \frac{\Delta T}{R} = \frac{(T_1 - T_2)}{\frac{\ln\frac{r_2}{r_1}}{2\pi K}} \quad , \quad q' = \frac{q}{L} \Rightarrow q = q' \cdot L$$

$$q = 2\pi k L \frac{(T_1 - T_2)}{\ln\frac{\frac{D_2}{2}}{\frac{D_1}{2}}} \Rightarrow q = \frac{2\pi k L\,(T_1 - T_2)}{\ln\frac{D_2}{D_1}}$$

$$q'' = \frac{q}{A_2} \quad , \quad A_2 = \pi . D_2 . L$$

$$q'' = \frac{2\pi k L\,(T_1 - T_2)}{\ln\frac{D_2}{D_1}(\pi . D_2 . L)} \Rightarrow q'' = \frac{2k\,(T_1 - T_2)}{D_2 \ln\frac{D_2}{D_1}}$$

Exercise 2.8

Calculate the heat loss per square meter from the outside surface of tube with outside diameter of (10cm) covered with (1cm) thick of insulation of thermal conductivity of (0.07) W/m.°C, assume the inside and outside temperatures of insulation are (274°C), (32°C) respectively?

Solution

$$q = \frac{\Delta T}{R} = \frac{\Delta T}{\frac{\ln\frac{r_2}{r_1}}{2\pi K L}}$$

$$q'' = \frac{q}{A} \quad , \quad A = \pi . 2 r_2 . L$$

r2=5+1=6cm r2=0.06m

Exercise 2.9

A steam passing through a tube of outside diameter of (20cm) covered with (5 cm) of insulation of thermal conductivity of (0.096 W/m.°C). Calculate the heat loss per (100 m) length of the tube if the inside and outside temperatures of insulation are (451°C, 45°C)?

Solution

$$q = \frac{\Delta T}{R} = \frac{(T_1 - T_2)}{\frac{\ln\frac{r_2}{r_1}}{2\pi K L}} = \frac{(451-45)}{\frac{\ln\frac{0.15}{0.1}}{2\pi 0.096 * 100}} = 60398.15 \text{ W}$$

Exercise 2.10

A tube of (25cm) diameter covered with (2.5cm) thick of insulation with (k=0.074 W/m.°C) and (5cm) thick of another insulation with (k=0.061 w/m.°C). If the inside and outside temperatures are (364°C and 46°C) respectively. Calculate the heat loss per 100m^{2}from the outside surface of tube?

Solution

$$q = \frac{\sum \Delta T}{\sum \Delta R} = \frac{(T_i - T_o)}{\frac{\ln\frac{r_2}{r_1}}{2\pi k_1 L} + \frac{\ln\frac{r_3}{r_2}}{2\pi k_2 L}}$$

$A = 2\pi r_3 L$ → $100 = 2 * 3.14 * 0.2\ L$ → $L = 79.5$ m

$$q = \frac{(364-46)}{\frac{\ln\frac{0.15}{0.125}}{2\times 3.14\times 0.074\times 79.5} + \frac{\ln\frac{0.2}{0.15}}{2\times 3.14\times 0.061\times 79.5}} = 22158.64 \text{ W}$$

$$q'' = \frac{q}{A_3} = \frac{q}{100} = 221.586 \text{ W/m}^2$$

Exercise 2.11

A tube of (22cm) outside diameter and (20cm) inside diameter, if the inside and outside temperatures of tube are (226℃ , 222℃) respectively and the thermal conductivity of tube is (43.4 W/m.℃). Calculate:

A-The heat loss per meter length?

B-The reduction in heat loss per meter length if we use an insulation with (4cm) thick and (k=0.0658 W/m.℃) covered the tube, if the inside and outside temperatures of insulation are (270℃ ,64℃)?

C-The inside temperature of tube in case B (H.W)

Solution

$$A)\ q_1' = \frac{(T_1 - T_2)}{\frac{\ln\frac{r_2}{r_1}}{2\pi k_1}} = \frac{(226-222)}{\frac{\ln\frac{11}{10}}{2\pi\times 43.4}} = 11444.32 \text{ w/m}$$

$$B)\ q_2' = \frac{(T_2 - T_3)}{\frac{\ln\frac{r_3}{r_2}}{2\pi k_2}} = \frac{(270-64)}{\frac{\ln\frac{15}{11}}{2\pi\times 0.0658}} = 274.59 \text{ w/m}$$

$$\text{Decreasing in heat loss}\quad q' = \frac{q1' - q2'}{q1'} = \frac{11444.32 - 274.59}{11444.32} = 0.97$$

$$C)\ q_2' = \frac{(t_1' - 64)}{\frac{\ln\frac{r_2}{r_1}}{2\pi k_1} + \frac{\ln\frac{r_3}{r_2}}{2\pi k_2}}$$

$$274.59 = \frac{(t_1' - 64)}{\frac{\ln\frac{11}{10}}{2\pi\times 43.4} + \frac{\ln\frac{15}{11}}{2\pi\times 0.0658}}$$

$$t_1' = 270.09°\text{C}$$

Exercise 2.12

Steam passing through a tube of outside diameter of (60cm) and (410m) long covered with (5cm) thick of insulation with (k=0.0892 W/m.°C) and (4cm) thick of an other insulation with (k=0.0595W/m.°C), if the outside tube and outside insulation temperatures are (405°C and 56°C) respectively. Calculate:

A- The heat loss per hr?

B- The heat loss per m^2 from outside surface of insulation?

C- Heat loss per m^2 of tube?

D- Temperature between two insulations?

Solution

$$q = \frac{\sum \Delta T}{\sum \Delta R} = \frac{(T_1 - T_3)}{\frac{ln\frac{r_2}{r_1}}{2\pi k_1 L} + \frac{ln\frac{r_3}{r_2}}{2\pi k_2 L}}$$

$$q = \frac{(405-56)}{\frac{ln\frac{35}{30}}{2\times 3.14\times 0.089\times 410} + \frac{ln\frac{39}{35}}{2\times 3.14\times 0.059\times 410}} = 252483 \text{ W} = 252.483 \text{ KW}$$

A) $q = 252.483\frac{KJ}{sec} \times 3600\frac{sec}{hr} = 908928\frac{KJ}{hr}$

B) $q'' = \frac{q}{A_3} = \frac{q}{2\pi r_3 . L} = \frac{252483}{2\pi\times 0.39\times 410} = 251.3 \text{ W/m}^2$

C) $q'' = \frac{q}{A_1} = \frac{q}{2\pi r_1 . L} = \frac{252483}{2\pi\times 0.3\times 410} = 326.69 \text{ W/m}^2$

D) q_1 after insulation = q through insulation

$$252483 = \frac{(405 - t_2)}{\frac{ln\frac{35}{30}}{2\pi * 0.089 * 410}}$$

$t_2 = 235.4°C$

Chapter 3

Heat Transfer by Convection

3.1. Introduction

We mentioned earlier that there are three basic mechanisms of heat transfer: conduction, convection, and radiation. Conduction and convection are similar in that both mechanisms require the presence of a material medium. But they are different in that convection requires the presence of fluid motion.

Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity* μ, *thermal conductivity k, density* ρ, and *specific heat Cp,* as well as the *fluid velocity* ν. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent). Thus, we expect the convection heat transfer relations to be rather complex because of the dependence of convection on so many variables.

Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as:

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2)$$

or (3.1)

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W)$$

where

h = convection heat transfer coefficient, $W/m^2 \cdot °C$
A_s = heat transfer surface area, m^2
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C

3.2. Laminar And Turbulent Flows

A careful inspection of flow in a pipe reveals that the fluid flow is streamlined at low velocities but turns chaotic as the velocity is increased above a critical value, as shown in Figure below. The flow regime in the first case is said to be **laminar,** characterized by *smooth stream lines* and *highly-ordered motion,* and **turbulent** in the second case, where it is characterized by *velocity fluctuations* and *highly-disordered motion.* The **transition** from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.

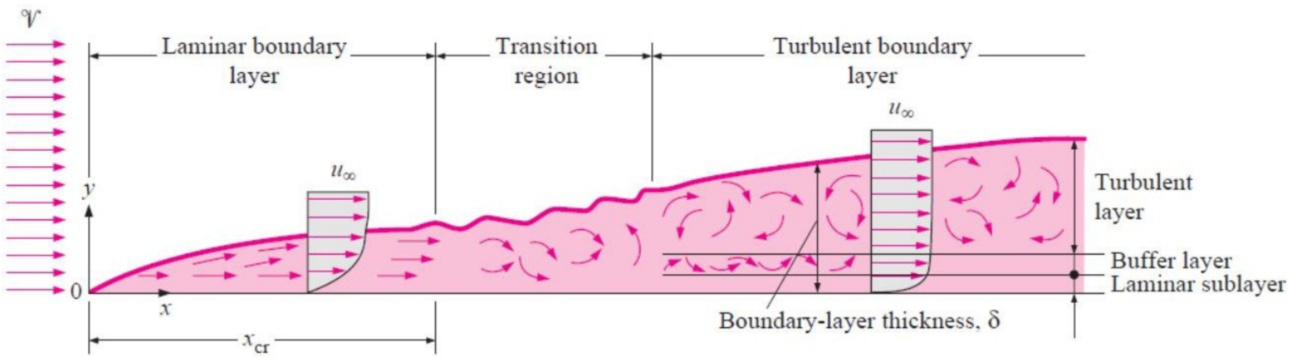


Figure 3.1 the development of the boundary layer for flow over a flat plate, and the different flow regimes

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, free-stream velocity, surface temperature,* and *type of fluid,* among other things. The flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid. This ratio is called the **Reynolds number,** which is a *dimensionless* quantity.

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V} L_c}{\nu} = \frac{\rho \mathcal{V} L_c}{\mu} \tag{3.2}$$

Where:

V: is the upstream velocity (m/sec) (equivalent to the free-stream velocity *u* for a flat plate)

L_C: is the characteristic length of the geometry (m). For a flat plate, the characteristic length is the distance x from the leading edge.

$\nu = \mu / \rho$ is the kinematic viscosity of the fluid.

At *large* Reynolds numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At *small* Reynolds numbers, however, the viscous forces are large enough to overcome the inertia forces and to keep the fluid "in line". Thus, the flow is *turbulent* in the first case and *laminar* in the second.

- For cylindrical pipes of diameter *d*

$$\mathrm{Re} = \frac{u * \rho * d}{\mu} \tag{3.3}$$

$\mathrm{Re} < 2300$	laminar flow
$2300 \leq \mathrm{Re} \leq 10{,}000$	transitional flow
$\mathrm{Re} > 10{,}000$	turbulent flow

- For flat plates

The transition from laminar to turbulent flow occurs when

$$\frac{u_\infty x}{\nu} = \frac{\rho u_\infty x}{\mu} > 5 \times 10^5 \tag{3.4}$$

where

u_∞ = free-stream velocity, m/s

x = distance from leading edge, m

$\nu = \mu/\rho$ = kinematic viscosity, m^2/s

This particular grouping of terms is called the Reynolds number, and is dimensionless if a consistent set of units is used for all the properties:

$$\mathrm{Re}_x = \frac{u_\infty x}{\nu}$$

3.3. Velocity Boundary Layer

Consider the parallel flow of a fluid over a flat plate, as shown in Figure 3.2, the *x*-coordinate is measured along the plate surface from the *leading edge* of the plate in the direction of the flow, and *y* is measured from the surface in the normal direction. The velocity of the particles in the first fluid layer adjacent to the plate becomes zero because of the no-slip condition. This motionless layers lows down the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities.

This fluid layer then slows down the molecules of the next layer, and soon. Thus, the presence of the plate is felt up to some normal distance δ from the plate beyond which the free-stream velocity u_∞ remains essentially unchanged. As a result, the *x*- component of the fluid velocity, *u,* will vary from 0 at $y = 0$ to nearly u_∞ at $y=\delta$.

3.4. Thermal Boundary Layer

Consider a heated wall surface at temperature t_s over which a fluid is flowing with undisturbed velocity U_∞ and temperature t_∞, a region of fluid motion near the plate in which temperature gradients exist is the thermal boundary layer and its thickness $\boldsymbol{\delta_t}$ is defined as the value of transverse distance y from the plate surface, as shown in Figure below:

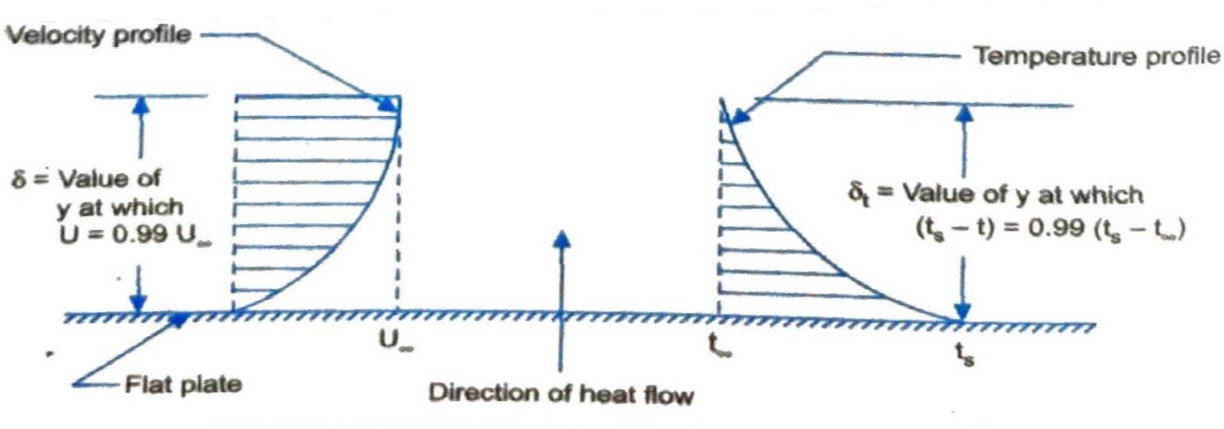


Figure 3.2 Velocity and temperature profiles in convective heat transfer

3.5. Dimensionless Numbers

3.5.1. Prandtl number

The Prandtl number has been found to be the parameter that relates the relative thicknesses of the hydrodynamic and thermal boundary layers. The kinematic viscosity of a fluid conveys information about the rate at which momentum may diffuse through the fluid because of molecular motion. The thermal diffusivity tells us the same thing in regard to the diffusion of heat in the fluid. Thus, the ratio of these two quantities should express the relative magnitudes of diffusion of momentum and heat in the fluid. But these diffusion rates are precisely the quantities that determine how thick the boundary layers will be for a given external flow field; large diffusivities mean that the viscous or temperature influence is felt farther out in the flow field. The Prandtl number is thus the connecting link between the velocity field and the temperature field.

$$Pr = \frac{Momentum\ diffusivity}{Thermal\ diffusivity} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \qquad (3.5)$$

3.5.2. Nusselt number

The Nusselt number is a convenient measure of the convective heat transfer coefficient. For a given value of Nusselt number, the convective surface coefficient *h* is directly proportional to thermal conductivity *k* of the fluid, and inversely proportional to the significant length *l*.

The dimensionless parameter *hl/k* is called Nusselt number. Apparently the Nusselt number may be interpreted as the ratio of temperature gradient at the surface to an overall or reference temperature gradient.

$$Nu = hl/k \qquad (3.6)$$

3.5.3. Grashof number

The Grashof number (*Gr*) represents the natural convection effects, and can calculate by equation:

$$G_{r_L} = \frac{g\beta(T_s - T_\infty)L_C^{\,3}}{\upsilon^2} \qquad (3.7)$$

Where

g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, $1/K$ (β=1/T for ideal gases)

T_s= temperature of the surface, ℃

T_∞= temperature of the fluid sufficiently far from the surface, ℃

$L_C{}^3$ = characteristic length of the geometry, m

υ = kinematic viscosity of the fluid, m^2/s

We mentioned in the preceding chapter that the flow regime in forced convection is governed by the dimensionless ***Reynolds number****, which represents the ratio of inertial forces to viscous forces acting on the fluid*. The flow regime in natural convection is governed by the dimensionless ***Grashof number****, which represents the ratio of the buoyancy force to the viscous force acting on the fluid* show figure 3–3.

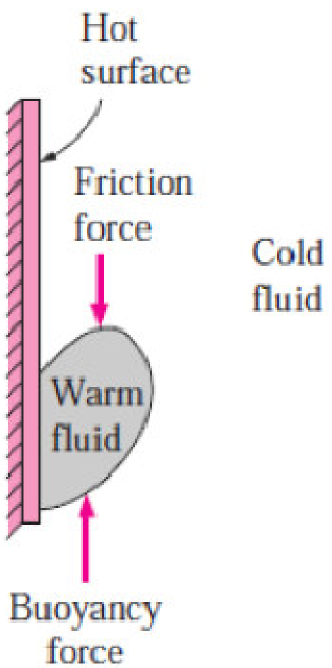


Figure 3-3 The Grashof number *Gr* is a measure of the relative magnitudes of the buoyancy force and the opposing viscous force acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than 10^9.

3.5.4. Rayleigh number

Is a dimensionless number indicating the onset and intensity of natural convection, showing if buoyancy-driven flow overcomes viscous damping and thermal diffusion?

$$Ra = Pr.Gr = \frac{g.\beta.\Delta T.L^3}{a.\nu} \qquad (3\text{-}8)$$

Where

$a = \lambda / (\rho .Cp)$: thermal diffusivity [m²/s],

$\nu = \mu / \rho$: kinematic viscosity of the fluid [m²/s].

3.6. Empirical correlations for the average Nusselt number for natural convection over surfaces

Over the years it has been found that average free-convection heat-transfer coefficients can be represented in the following functional form for a variety of circumstances:

$$\overline{Nu_f} = C\left(Gr_f.Pr_f\right)^m \qquad (3\text{-}9)$$

The values of the constants *C* and *n* depend on the *geometry* of the surface and the *flow regime,* which is characterized by the range of the Rayleigh number. The value of ***n*** is usually ***1/4*** for laminar flow and ***1/3*** for turbulent flow. The value of the constant *C* is normally less than ***1***.

Simple relations for the average Nusselt number for various geometries are given in table (3–1), together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_\infty)/2$.

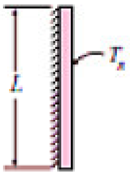
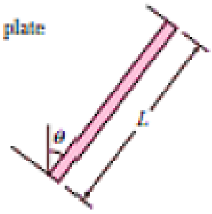
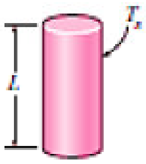
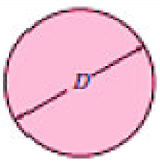
Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{10}–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (20-19) $Nu = 0.1Ra_L^{1/3}$ (20-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1+(0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (20-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ (20-22) $Nu = 0.15Ra_L^{1/3}$ (20-23) $Nu = 0.27Ra_L^{1/4}$ (20-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1+(0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (20-25)
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1+(0.469/Pr)^{9/16}]^{4/9}}$ (20-26)

Table 3-1 Empirical correlations for the average Nusselt number for natural convection over surfaces

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law

Example

Calculate heat transfer by free convection from vertical plate surface (64 * 64 cm) if the temperature of air surrounding is (15 °C) and the plate surface (145 °C)?

Solution:

$T_f = (T_s + T_\infty)/2$ = 160/2=80°C, T_f= 353 K

ΔT=145–15 =130°C, $\beta = 1/T_f$ = 1/353 K

From air properties at 80°C

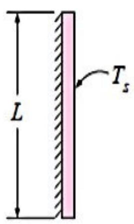
$\nu = 2.097 \times 10^{-5} m^2/s$, Pr=0.7154, K=0.02953W/m.K

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$Ra = \frac{9.18 * 1 * 130 * 0.64^3}{353 * (2.097 \times 10^{-5})^2} * 0.7154 = 1540741041$$

From table 3-1:

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)

$$Nu = \left\{0.825 + \frac{0.387 * (1540741041)^{\frac{1}{6}}}{\left[1 + (\frac{0.492}{0.7154})^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2$$

$$Nu = \left\{0.825 + \frac{0.387 * (1540741041)^{\frac{1}{6}}}{[1 + 0.8101]^{\frac{8}{27}}}\right\}^2$$

$$Nu = \left\{0.825 + \frac{0.387 * (1540741041)^{\frac{1}{6}}}{1.1922}\right\}^2$$

$$Nu = \left\{0.825 + \frac{0.387 * (1540741041)^{\frac{1}{6}}}{1.1922}\right\}^2$$

$$Nu = \left\{0.825 + \frac{0.387 * 33.985}{1.1922}\right\}^2$$

$$Nu = \left\{0.825 + \frac{13.1522}{1.1922}\right\}^2$$

$$Nu = \{11.586\}^2$$

$$Nu = 140.58$$

$$Nu = \frac{h\,L_c}{K}$$

➔ h = 6.486 W/m.

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (W)$$

$A_s = 0.64 * 0.64 = 0.409\ m^2$

$Q = 6.486 * 0.409 * 130 = 34.53\ W$

Free Convection Heat Transfer Exercises

Exercise 3.1 (Home Work)

Along tube of (8cm) outside diameter passing horizontally through water at a temperature of (28°C). if the wall temperature of tube is (132°C), Calculate the heat transfer rate by free convection from unit area of outside surface of tube?

Exercise 3.2

Calculate the value of Reynold's number for water at 30°C flowing in a tube of 5cm diameter at mean velocity of 3m/sec.

Determine whether the flow is laminar or turbulent.

Solution

$$N_{Re}=\frac{\rho\times u\times d}{\mu} \quad \text{or} \quad N_{Re}=\frac{u\times d}{\gamma}$$

From tables of water and at 30°C

$\nu = 8.32*10^{-3}*10^{-4}\ cm^2/sec = 8.32*10^{-7}\ m^2/sec$

$$N_{Re}=\frac{u\times d}{\nu}=\frac{3\times0.05}{8.32*10^{-7}}=1.81*10^{5}$$

$N_{Re} > 2300$

the flow is turbulent

Exercise 3.3

Calculate the coefficient of heat transfer by free convection form and the rate of heat transfer from a vertical surface of 0.46m high and o.61m width at a temperature of 90°C to the air at a temperature of 10°C

Solution:

$$T_f=\frac{T_w+T_\infty}{2}=\frac{90+10}{2}=50°C \quad , B=1/323k$$

K=0.02735 W/m.k $\nu=1.798*10^{-5}$ m^2/sec pr=0.7228

$$Ra=\frac{g.B.\Delta T.L_c^{\ 3}\,Pr}{\nu^2}=\frac{9.81*80*0.46^3*0.7228}{323*(1.798*10^{-5})^2}=528772348$$

$$Nu=\left\{0.825+\frac{0.387*\ (528772348)^{\frac{1}{6}}}{\left[1+(\frac{0.492}{0.7228})^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2=104.7$$

$$h=\frac{Nu*k}{Lc}=\frac{104.7*0.02735}{0.46}=6.225\quad W/m^2k$$

Q = h*A*Δt = 6.225*0.46*0.61*80 = 139.73 W

Exercise 3.4 (Home Work)

Calculate the heat transfer rate from a hot vertical surface of 15cm high and 23cm width at a temperature of 124°C to the air at 38°C?

Exercise 3.5

Calculate the approximate heat loss by free convection from a horizontal cylinder of 15cm diameter and 3m long if the outside temperature of the cylinder is 92°C and the air temperature is 38°C?

Solution:

$T_f=\frac{T_w+T_\infty}{2}=\frac{92+38}{2}=65°C$, B=1/338k

By Interpolation, we found the parameters at the temperature 65°C

T°C	*K (W/m.k)*	*ν (m^2/s)*	*Pr*
60	0.0808	$1.896*10^{-5}$	0.7202
65	*K*	*ν*	*Pr*
70	0.02881	$1.995*10^{-5}$	0.7177

$$\frac{70-60}{65-60} = \frac{0.02881-0.02808}{K-0.02808} \implies K = 0.028445\ \frac{\text{W}}{\text{m}k}.$$

$$\frac{70-60}{65-60} = \frac{0.7177-0.7202}{Pr-0.7202} \implies Pr = 0.71895$$

$$\frac{70-60}{65-60} = \frac{1.995-1.896}{\nu-1.896} \implies \nu = 1.9455 * 10^{-5}$$

$$Ra = \frac{g.B.\Delta T.D^3\ Pr}{\nu^2} = \frac{9.81 * 54 * 0.15^3 * 0.71895}{338 * (1.9455 * 10^{-5})^2} = 10047454.8$$

$$Nu = \left\{0.6 + \frac{0.387 * (10047454.8)^{\frac{1}{6}}}{\left[1 + (\frac{0.559}{0.71895})^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2 = 28.34$$

$$h = \frac{Nu * k}{Lc} = \frac{28.34 * 0.028445}{0.15} = 5.37 \quad W/m^2k$$

$Q = h*A*\Delta T = 5.37 * 3.14 * 0.15 * 3 * 54 =$ **409.7*W***

Exercise 3.6 (Home Work)

Calculate the heat transfer rate by free convection from 1square meter of upward horizontal surface at 150°C to the air at 38°C?

Exercise 3.7 (Home Work)

A long pipe of 8cm outside diameter passing horizontally through a room with air temperature of 22°C, the outside surface temperature of tube is 102°C. Calculate the heat transfer rate per unit area by free convection of tube to the air?

Exercise 3.8 (Home Work)

If the pipe of exercise 6 passing through water at atmospheric pressure and same air temperature, calculate the heat transfer rate by free convection from the outside surface temperature of tube to the water ?

Exercise 3.9

If Grashof and Prandtl numbers of fluid surrounding a horizontal cylinder with 5cm diameter are 10^4 and 10 respectively, assuming the thermal conductivity of fluid is 0.029w/m.k. Calculate the convection heat transfer coefficient between fluid and cylinder also the heat transfer rate by free convection from 55m long of tube if the temperature of tube is 91°C and the air temperature is 21°C?

Solution

Ra=Gr * Pr=10^5

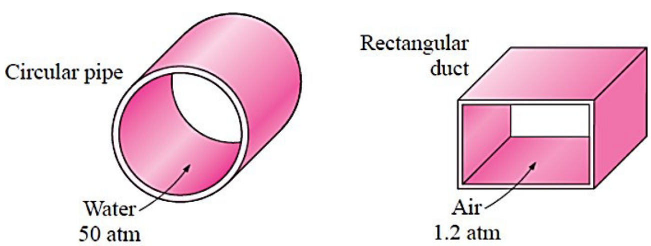
$$Nu = \left\{0.6 + \frac{0.387 * (100000)^{\frac{1}{6}}}{\left[1 + (\frac{0.559}{10})^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right\}^2 = 9.6$$

$$h = \frac{Nu * k}{D} = \frac{9.6 * 0.029}{0.05} = 5.56 \quad W/m^2k$$

$Q=h*A*\Delta T$=5.56*π*0.05*55*70=3362.44W

3.7. Empirical correlations for the average Nusselt number for forced convection

You have probably noticed that most fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross section can withstand large pressure differences between the inside and the outside without undergoing any distortion. Noncircular pipes are usually used in applications such as the heating and cooling systems of buildings where the pressure difference is relatively small and the manufacturing and installation costs are lower.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any distortion, but the noncircular pipes cannot.

In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or fan. Forced convection is a special type of heat transfer in which fluids are forced to move, in order to increase the heat transfer.

For forced convection we have velocity (U_∞) or flow rate (m) of fluid:

$$\dot{m} = \rho . u . A \quad \text{(kg/sec)} \tag{3.10}$$

Where:

ρ: density of fluid (kg/m^3). u:velocity of fluid (m/sec). A: cross section Area (m^2).

Reynolds number is:

$$Re = \frac{\rho\, u}{\nu} = \frac{\rho\, u\, d}{\mu}$$

Where:

d: pipe diameter (m).

μ:viscosity(kg/m.sec)

The flow in pipe is turbulent if ***2300<Re***

For fully developed turbulent flow in smooth tubes, a simple relation for the Nusselt number can be obtained by:

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{n} \tag{3.11}$$

Where n=0.4 for heating and 0.3 for cooling of the fluid flowing through the tube.

The physical properties for fluid are taken at “mixing cup temperature” (T_m) which is the average of inlet and outlet fluid temperatures:

$$T_m = \frac{T_{in} + T_o}{2}$$

In fluid flow it is convenient to work with an **average** or **mean temperature** T_m that remains uniform at a cross section. Unlike the mean velocity, the mean temperature T_m *will change* in the flow direction whenever the fluid is heated or cooled.

For high viscosity fluids like oils:

$$N_u = 0.027 * (Re)^{0.8} * (Pr)^{0.3} * (\frac{\mu}{\mu_s})^{0.14} \qquad (3.12)$$

Where:

μ: viscosity of fluid at mean temperature T_m.

μ_s: viscosity of fluid at surface temperature T_s.

Example

Calculate the heat transfer coefficient by forced convection of water in vapor condenser of single path if the inside diameter of tube is (2.3cm) and the cooling water inter at (17.7°C) and leave at (22.3°C), the mean velocity of water is (2.13 m/s).

Solution:

$T_m = \frac{T_{in}+T_o}{2} = \frac{17.7+22.3}{2} = 20°C$, from the table below

$\nu = 1.006 * 10^{-6}\ m^2/sec$, $k = 0.597\ W/m.°C$, $Pr = 7.02$

$Re = \frac{u\,d}{\nu} = \frac{2.13*0.023}{1.006*10^{-6}} = 48697.8$ turbulent flow

$$Nu = 0.023\, Re^{0.8}\, Pr^{n}$$

The flow is heating so, $n = 0.4$

$Nu = 0.023\,(48697.8)^{0.8} * (7.02)^{0.4} = 282$

$Nu = \frac{h*D}{K}$ ➔ $h = \frac{282*0.597}{0.023} = 7319\ W/m^2.K$

Physical Properties of Saturated Liquid Water

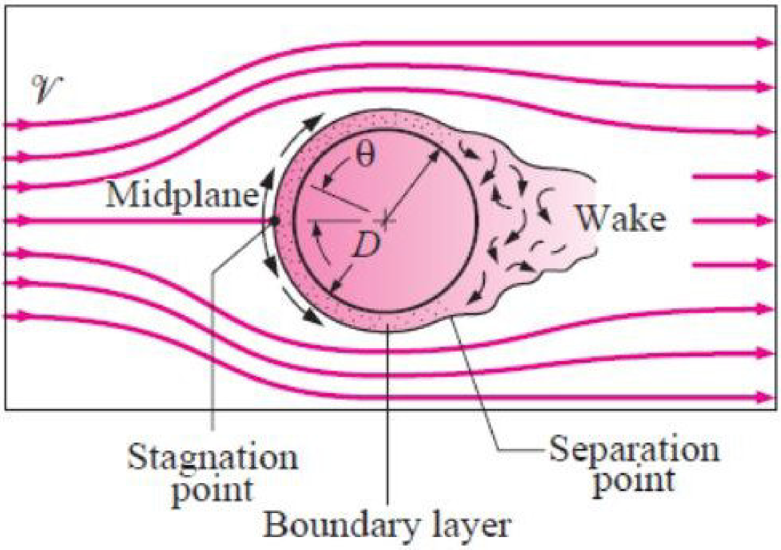
T (°C)	ρ (kg/m³)	c_p (kJ/kg-°C)	ν (m²/s) × 10^{-6}	k (W/m-°C)	α (m²/s) × 10^{-7}	Pr	β (K^{-1}) × 10^{-3}
0	1002.28	4.2178	1.788	0.552	1.308	13.6	
20	1000.52	4.1818	1.006	0.597	1.430	7.02	0.18
40	994.59	4.1784	0.658	0.628	1.512	4.34	
60	985.46	4.1843	0.478	0.651	1.554	3.02	
80	974.08	4.1964	0.364	0.668	1.636	2.22	
100	960.63	4.2161	0.294	0.680	1.680	1.74	
120	945.25	4.250	0.247	0.685	1.708	1.446	
140	928.27	4.283	0.214	0.684	1.724	1.241	
160	909.69	4.342	0.190	0.680	1.729	1.099	
180	889.03	4.417	0.173	0.675	1.724	1.004	
200	866.76	4.505	0.160	0.665	1.706	0.937	
220	842.41	4.610	0.150	0.652	1.680	0.891	
240	815.66	4.756	0.143	0.635	1.639	0.871	
260	785.87	4.949	0.137	0.611	1.577	0.874	
280	752.55	5.208	0.135	0.580	1.481	0.910	
300	714.26	5.728	0.135	0.540	1.324	1.019	

Notes: T = temperature, ρ = density, c_p = specific heat capacity, ν = μ/ρ = kinetic viscosity, k = thermal conductivity, α = $c_p\rho/k$ = heat (thermal) diffusivity, Pr = Prandtl number, β = coefficient of volumetric expansion of fluid

3.7.1. The Heating of Fluids Flowing Normal to Single Wires

The tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes, and both flows must be considered in the analysis of the heat exchanger.

Cross flow over a cylinder exhibits complex flow patterns, as shown in Figure below. The fluid approaching the cylinder branches out and encircles the cylinder, forming a boundary layer that wraps around the cylinder. The fluid particles on the mid plane strike the cylinder at the stagnation point, bringing the fluid to a complete stop and thus raising the pressure at that point. The pressure decreases in the flow direction while the fluid velocity increases.



Typical flow patterns in cross flow over a cylinder.

The expression of Nusselt number for Gases is:

$$Nu = 0.3\,(Re)^{0.57} \qquad (3.13)$$

For the liquid, the number of Nusselt is:

$$Nu = 0.6\,(Re)^{0.5} * (Pr)^{0.31} \qquad (3.14)$$

$$50 < Re < 10^4$$

Example

Calculate the heat transfer coefficient of air which flow normal to single pipe (D= 2.5 cm)? If Reynold's number is (8000) and mean surface temperature of tube is (84 °C).

Solution

$Nu - 0.3\,(Re)^{0.57}$ ➔ $Nu - 0.3\,(8000)^{0.57}$ ➔ Nu = 50.33

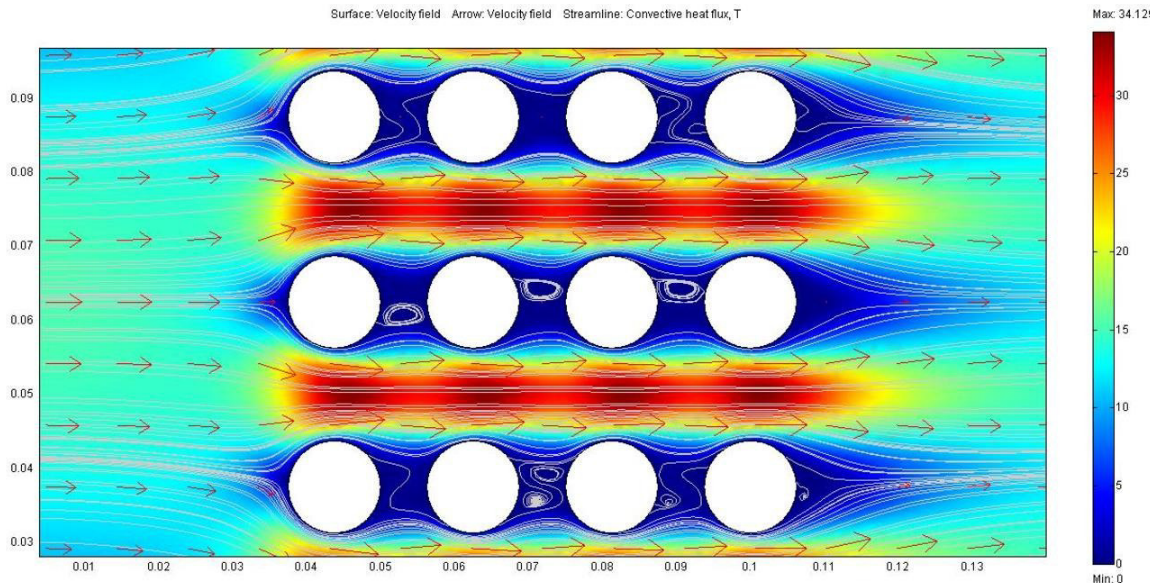
From air properties table at 84 °C

$\frac{90-80}{84-80} = \frac{0.03024-0.02953}{K-0.02953}$ ➔ K =0.029814 W/m.°C

$Nu = \frac{h\,D}{K}$ ➔ $50.33 = \frac{h*0.025}{0.029814}$ ➔ h= 60 $W/m^2.°C$

3.7.2. Flow Across Tube Banks

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.



In a heat exchanger that involves a tube bank, the tubes are usually placed in a shell (and thus the name shell-and-tube heat exchanger), especially when the fluid is a liquid, and the fluid flows through the space between the tubes and the shell. There are numerous types of shell-and-tube heat exchangers.

The expression of Nusselt number for Gases is:

$$Nu = 0.3\ (Re)^{0.57}$$

For the liquid, the number of Nusselt is:

$$Nu = 0.33\ (Re)^{0.6} * (Pr)^{0.33}$$

Forced Convection Exercises

Exercise 3.9

Calculate the mean coefficient of heat transfer for water flowing through a tube of 5cm inside diameter ?

Water inters at 21°C and leave at 54°C if the mean velocity of water is 6.1 m/sec.

Solution

$$t_m = \frac{t_i + t_o}{2} = \frac{21+54}{2} = 37.5°C$$

From water tables and at 37.5°C we find:-

$$\frac{40-35}{37.5-35} = \frac{992.1-994}{\rho-994} \rightarrow \rho = 993 \text{ kg/m}^3$$

$$2 = \frac{0.653*10^{-3} - 0.72*10^{-3}}{\mu \quad 0.72*10^{-3}} \rightarrow \mu = 0.686 * 10^{-3} \text{ kg/m.s}$$

$$2 = \frac{0.31-0.623}{K-0.623} \rightarrow K = 0.627 \text{ W/m.K}$$

$$2 = \frac{4.32-4.83}{Pr-4.83} \rightarrow Pr = 4.575$$

$$Re = \frac{\rho * u * d}{\mu} = 993*6.1*0.05/0.686*10^{-3} = 441494.1$$

Re > 2300 So the flow is turbulent

$$Nu = 0.023\, Re^{0.8}\, Pr^{n} \qquad , n = 0.4 \quad \text{(Heating)}$$

$$Nu = 0.023 * (441494.1)^{0.8} * (4.575)^{0.4} = 1386.1$$

$$Nu = \frac{h*d}{K} \rightarrow h = \frac{1386.1*0.627}{0.05} = 17382.8 \text{ W/m}^2\text{.k}$$

Exercise 3.10

Water flow with flow rate of 2192 kg/hr in a tube with inside diameter of 5cm if the interring and leaving temperatures of water are 27°C and 49°C respectively. Calculate the heat transfer coefficient beside water?

Solution

$t_m = \frac{t_i + t_o}{2} = \frac{27+49}{2} = 38°C \sim \mathbf{40}°C$

From water tables and at 40°C: Pr =4.32

ρ= 992.1 kg/m^3, K= 0.631W/m.K, μ=0.653* 10^{-3}kg/m.s

Exercise 3.11

Calculate heat transfer coefficient for convection of water flow normal to a row of tubes (D=5 cm)? Assume the mean water velocity between tubes is (62 cm/s) and the film temperature is (50°C).

Solution

From water table: ρ = 1000 kg/m^3, μ=0.000547 kg/m.s, K = 0.644 W/m.°C, Pr = 3.55

$$Re = \frac{0.62 * 1000 * 0.05}{0.000547} = 56672.7$$

$$Nu = 0.33\ (Re)^{0.6} * (Pr)^{0.33} = 0.33 * (56672.7)^{0.6} * (3.55)^{0.6} = 356.5$$

$$Nu = \frac{h\ D}{K} \Rightarrow 356.5 = \frac{h * 0.05}{0.644} \Rightarrow \quad h = 4591.72\ W/m^2.°C$$

Chapter 4

Heat Transfer by Radiation

4.1. Introduction

Preceding chapters have shown how conduction and convection heat transfer may be calculated with the aid of both mathematical analysis and empirical data. We now wish to consider the third mode of heat transfer—thermal radiation. Thermal radiation is that electromagnetic radiation emitted by a body as a result of its temperature. In this chapter, we shall first describe the nature of thermal radiation, its characteristics, and the properties that are used to describe materials insofar as the radiation is concerned. Next, the transfer of radiation through space will be considered. Finally, the overall problem of heat transfer by thermal radiation will be analyzed, including the influence of the material properties and the geometric arrangement of the bodies on the total energy that may be exchanged.

4.2. Physical Mechanism

There are many types of electromagnetic radiation; thermal radiation is only one. Regardless of the type of radiation, we say that it is propagated at the speed of light, 3×10^8 m/s. This speed is equal to the product of the wavelength and frequency of the radiation,

$$c = \lambda v \tag{4.1}$$

where

c= speed of light

λ= wavelength

v= frequency

The unit for λ may be centimeters, angstroms (1 Å$=10^{-8}$ cm), or micrometers (1μm$=10^{-6}$ m). A portion of the electromagnetic spectrum is shown in Figure 4-1. Thermal radiation lies in the range from about 0.1 to 100 μm, while the visible-light portion of the spectrum is very narrow, extending from about 0.35 to 0.75 μm.

The propagation of thermal radiation takes place in the form of discrete quanta, each quantum having an energy of :

$$E = hv \tag{4.2}$$

Where h is Planck's constant and has the value h=6.625×10^{-34} J.s

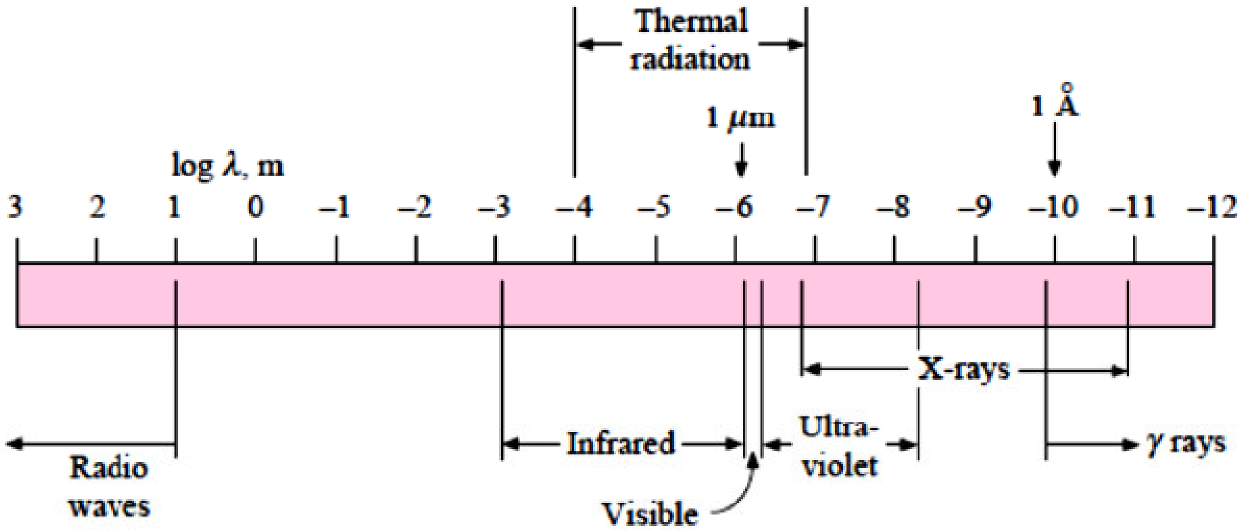


Figure 4-1 Electromagnetic spectrum

In heat transfer studies, we are interested in the energy emitted by bodies because of their temperature only. Therefore, we will limit our consideration to *thermal radiation,* which we will simply call *radiation.* The relations developed below are restricted to thermal radiation only and may not be applicable to other forms of electromagnetic radiation.

4.3. Blackbody Radiation

A body at a temperature above absolute zero emits radiation in all directions over a wide range of wavelengths. The amount of radiation energy emitted from a surface at a given wavelength depends on the material of the body and the condition of its surface as well as the surface temperature. A **blackbody** is defined as ***a perfect emitter and absorber of radiation***. At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs ***all*** incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission. Figure below that is, a blackbody is a ***diffuse*** emitter. The term ***diffuse*** means "independent of direction." The radiation energy emitted by a blackbody per unit time and per unit surface area was determined experimentally by Joseph Stefan in 1879 and expressed as

$$E_b(T) = \sigma T^4 \ (\frac{W}{m^2}) \qquad (4.3)$$

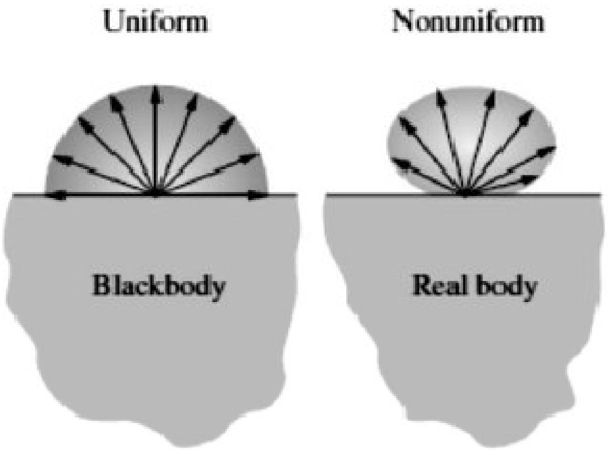


Figure 4-2 Blackbody Radiation

Where σ = 5.67 ×10^{-8} W/m^2.K^4 is the *Stefan–Boltzmann constant* and *T* is the absolute temperature of the surface in *K*. This relation was theoretically verified in 1884 by Ludwig Boltzmann. Equation (4-1) is known as the **Stefan–Boltzmann law** and E_b is called the blackbody emissive power.

Note that the emission of thermal radiation is proportional to the *fourth power* of the absolute temperature. A blackbody is said to be a *diffuse* emitter since it emits radiation energy uniformly in all directions.

The Stefan–Boltzmann law gives the *total* radiation emitted by a blackbody at all wavelengths from λ =0 to λ =∞.

4.4. Emissivity

The emissivity of a surface represents *the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.* The emissivity of a surface is denoted by ε, and it varies between zero and one, 0< ε< 1. Emissivity is a measure of how closely a surface approximates a blackbody, for which ε =1. The emissivity of a real surface is not a constant. Rather, it varies with the ***temperature*** of the surface as well as the ***wavelength*** and the ***direction*** of the emitted radiation.

4.5. Absorptivity, Reflectivity, and Transmissivity

Everything around us constantly emits radiation, and the emissivity represents the emission characteristics of those bodies. This means that every body, including our own, is constantly bombarded by radiation coming from all directions over a range of wavelengths. Recall that radiation flux *incident on a surface* is called **irradiation** and is denoted by ***G***. When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted, as illustrated in Figure. *The fraction of irradiation absorbed by the surface* is called the **absorptivity α**, *the fraction reflected by the surface* is called the **reflectivity ρ**, and *the fraction transmitted* is called the **transmissivity τ**.

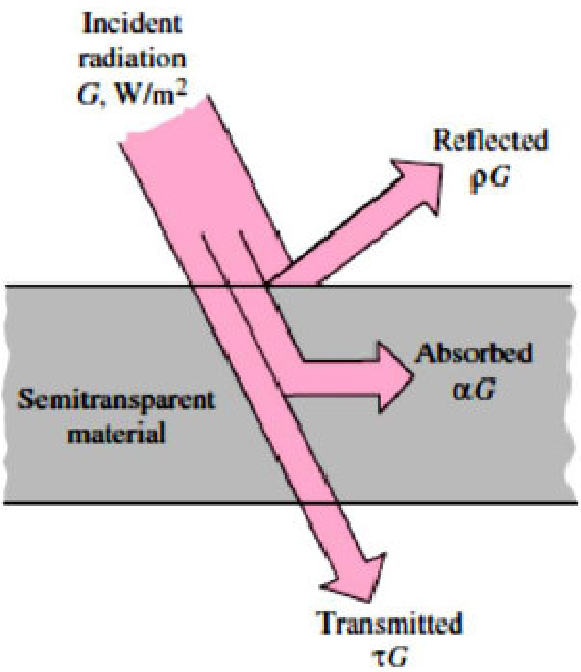


Figure 4-3 Absorptivity, Reflectivity, and Transmissivity

Thus

$$\rho+\alpha+\tau=1 \tag{4.4}$$

Most solid bodies do not transmit thermal radiation, so that for many applied problems the transmissivity may be taken as zero. Then

$$\rho+\alpha=1 \tag{4.5}$$

Two types of reflection phenomena may be observed when radiation strikes a surface. If the angle of incidence is equal to the angle of reflection, the reflection is called *specular*. On the other hand, when an incident beam is distributed uniformly in all directions after reflection, the reflection is called *diffuse*.

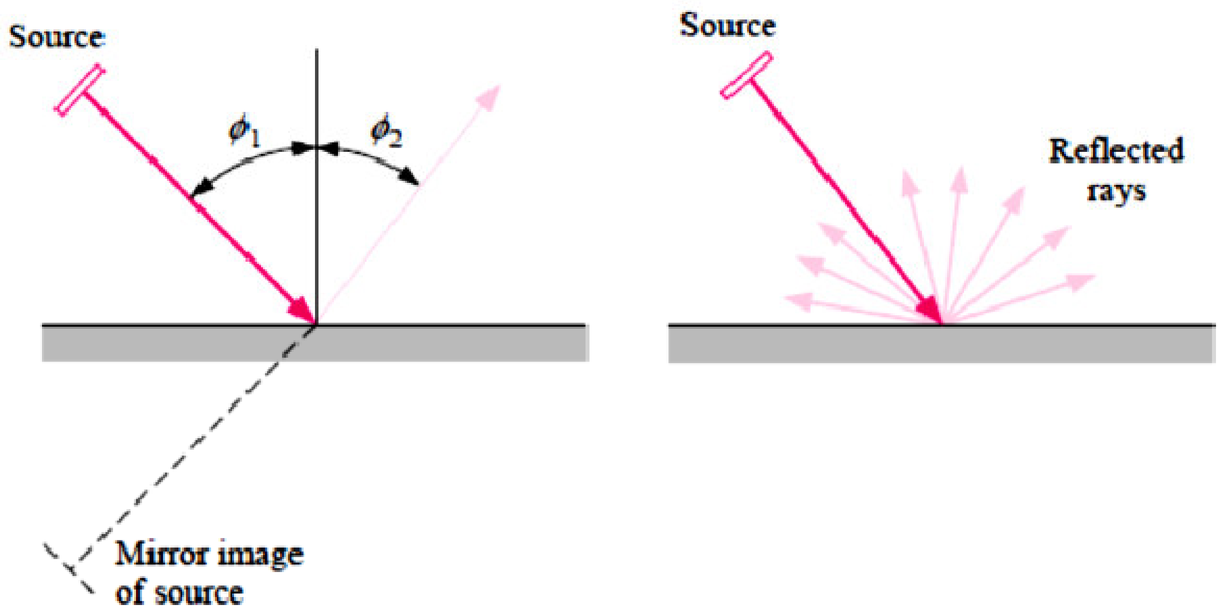


Figure 4-4 Reflection phenomena

4.6. Real surfaces

Thus far we have spoken of ideal surfaces, i.e. those that emit energy according to the **Stefan Boltzman** law:

$$E_b = \sigma \cdot T^4_{abs} \tag{4.6}$$

Real surfaces have emissive powers, E, which are somewhat less than that obtained theoretically by Boltzman. To account for this reduction, we introduce the emissivity, ε.

$$\varepsilon = \frac{E}{E_b} \tag{4.7}$$

Emissive power from any real surface is given by: $E = \sigma \cdot \varepsilon . T^4_{abs}$

4.7. Radiation Shape Factor

Consider two black surfaces $A1$ and $A2$, as shown in Figure below. We wish to obtain a general expression for the energy exchange between these surfaces when they are maintained at different temperatures. The problem becomes essentially one of determining the amount of energy that leaves one surface and reaches the other. To solve this problem the ***radiation shape factors*** are defined as

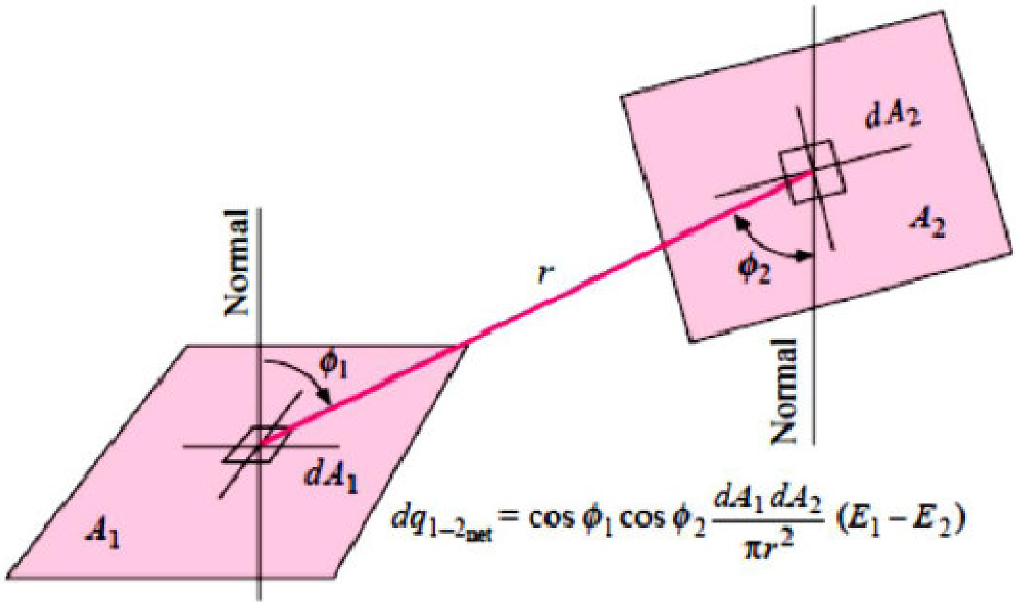


Figure 4-5. Radiation Shape Factor

F_{1-2} = fraction of energy leaving surface 1 that reaches surface 2

F_{2-1} = fraction of energy leaving surface 2 that reaches surface 1

F_{i-j} = fraction of energy leaving surface *i* that reaches surface *j*

Other names for the radiation shape factor are *view factor, angle factor,* and *configuration factor.* The energy leaving surface 1 and arriving at surface 2 is $\boldsymbol{E_{b1}A_1F_{12}}$ and the energy leaving surface 2 and arriving at surface 1 is $\boldsymbol{E_{b2}A_2F_{21}}$

Since the surfaces are black, all the incident radiation will be absorbed, and the net energy exchange is $\boldsymbol{E_{b1}A_1F_{12} - E_{b2}A_2F_{21} = Q_{1-2}}$

If both surfaces are at the same temperature, there can be no heat exchange, that is, Q_{1-2}=0. Also, for $T_1 = T_2$

$E_{b1} = E_{b2}$

So that

$$\boldsymbol{A_1F_{12} = A_2F_{21}} \tag{4.8}$$

The net heat exchange is therefore

$$Q_{1-2} = A_1F_{12}\,(E_{b1} - E_{b2}) = A_2F_{21}(E_{b1} - E_{b2})$$

Equation (4-3) is known as a reciprocity relation, and it applies in a general way for any two surfaces *i* and *j*:

$$A_iF_{ij} = A_jF_{ji}$$

Although the relation is derived for black surfaces, it holds for other surfaces also as long as diffuse radiation is involved.

In the foregoing discussion the tacit assumption has been made that the various bodies do not see themselves, that is, $F_{11} = F_{22} = F_{33} = 0$

To be perfectly general, we must include the possibility of concave curved surfaces, which may then see themselves. The general relation is therefore where F_{ij} is the fraction of the total energy leaving surface i that arrives at surface j. Thus for a three-surface enclosure we would write

$$F_{11} + F_{12} + F_{13} = 1 \qquad (4.9)$$

Two new terms may be defined:

G= *irradiation* = total radiation incident upon a surface per unit time and per unit area

J = *radiosity* = total radiation that leaves a surface per unit time and per unit area

4.8. The Gray Body

A gray body is defined such that the monochromatic emissivity e_y of the body is independent of wavelength. The monochromatic emissivity is defined as the ratio of the monochromatic emissive power of the body to the monochromatic emissive power of a blackbody at the same wavelength and temperature. Thus $e_y = \frac{E_y}{E_{by}}$

The total emissivity of the body may be related to the monochromatic emissivity by noting that

$E = \int_0^\infty e_\lambda\, E_{b\lambda} d\lambda$ and $E_b = \int_0^\infty E_{b\lambda} d\lambda = \sigma T^4$

So that

$$e = \frac{E}{E_b} = \frac{\int_0^\infty e_\lambda E_{b\lambda} d\lambda}{\sigma T^4} \tag{4.10}$$

Where $E_{b\lambda}$ is the emissive power of a blackbody per unit wavelength

If the gray-body condition is imposed, that is, e_λ = constant, Equation (4-9) reduces to

$$e = e_\lambda \tag{4.11}$$

The functional relation for $E_{b\lambda}$ was derived by Planck by introducing the quantum concept for electromagnetic energy. The derivation is now usually performed by methods of statistical thermodynamics, and $E_{b\lambda}$ is shown to be related to the energy density of Equation (4-2) by

$$E_{by} = \frac{c_1 \lambda^{-5}}{e^{c_2/\lambda T} - 1} \tag{4.12}$$

Where

λ = wavelength, μm

T = temperature, K

$C_1 = 3.743\times10^8$ W.μm^4/m^2

$C_2 = 1.4387\times10^4$ μm.K

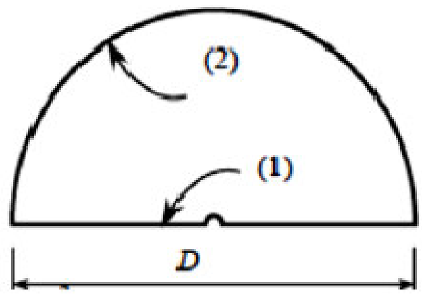
Notice that the peak of the curve is shifted to the shorter wavelengths for the higher temperatures. These maximum points in the radiation curves are related by **Wien's displacement law**

$$\lambda_{max} T = 2897.6\ \mu\text{m.K} \tag{4.13}$$

Exercices

Exercise 4.1

Consider a hemispherical furnace with a flat circular base of diameter *D*. Determine the view factor from the dome of this furnace to its base.



Solution

We number the surfaces as follows:

(1): circular base surface

(2): dome surface

Surface (1) is flat, and thus $F_{11} = 0$.

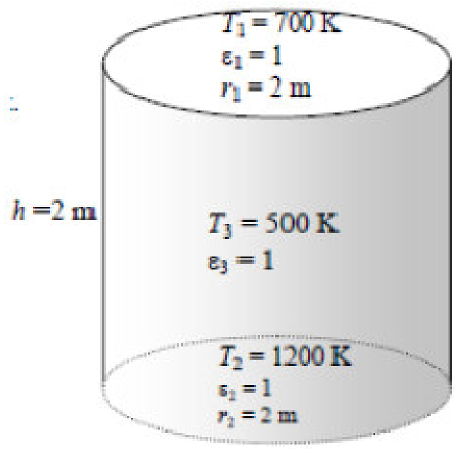
Summation rule : $F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$

Reciprocity rule:

$$A_1F_{12} = A_2F_{21} \Rightarrow F_{21} = \frac{A_1}{A_2}F_{12} = \frac{A_1}{A_2}.(1) = \frac{\pi D^2/4}{\pi D^2/2} = \frac{1}{2}$$

Exercise 4.2

A furnace is of cylindrical shape with R =H =2 m. The base, top, and side surfaces of the furnace are all black and are maintained at uniform temperatures of 500, 700, and 1200 K, respectively. Determine the net rate of radiation heat transfer to or from the top surface during steady operation



Solution

The net rate of radiation heat transfer from the top surface can be determined from

$$\dot{Q} = A_1F_{12}\sigma(T_1^4 - T_2^4) + A_1F_{13}\sigma(T_1^4 - T_3^4)$$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$. The view factor from the base to the side surfaces is determined by applying the summation rule to be

$F_{11} + F_{12} + F_{13} = 1 \rightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$

$$\dot{Q} = A_1 F_{12} \sigma (T_1^4 - T_2^4) + A_1 F_{13} \sigma (T_1^4 - T_3^4) = -762\ KW$$

Exercise 4.3

For a black body maintained at 115°C, determine:

1. The total emissive power
2. The wavelength at which the maximum spectral emissive power occurs.
3. The maximum spectral emissive power.

Solution

$$M^0 = \sigma.T^4 = 5{,}67.10^{-8}.388^4 = 1285 W.m^{-2}$$

$$\lambda_{max}.T = 2897{,}6 \Rightarrow \lambda_{max} = \frac{2897{,}6}{388} = 7{,}47 \mu m$$

$$\left(M_\lambda^0\right)_{max} = \frac{3{,}742 . 10^8 . \lambda^{-5}}{\exp\left(\frac{1{,}439 . 10^4}{\lambda T}\right) - 1} = \frac{3{,}742 . 10^8 . 7{,}47^{-5}}{\exp\left(\frac{1{,}439 . 10^4}{7{,}47 . 388}\right) - 1} = 113{,}06\ W.m^{-2}.\mu m^{-1}$$

Exercise 4.4

A cylindrical furnace with a diameter of 75 mm and a height of 150 mm is open at the top to an ambient temperature of 27°C. The lateral and bottom surfaces (assumed to be black bodies) are electrically heated and maintained at temperatures of 1350°C and 1650°C, respectively.

- What is the radiative heat flux lost by the furnace up wards? What is the radiative heat flux lost by the furnace upwards?

Solution

$$\varphi = \varphi_{13} + \varphi_{23} = S_1.F_{13}.\sigma.(T_1^4 - T_3^4) + S_2.F_{23}.\sigma.(T_2^4 - T_3^4)$$

$$S_1 = \pi.D.L = \pi.0{,}075\,.\,0{,}15 = 0{,}035m^2$$

$$S_2 = \frac{\pi.D}{4} = \frac{\pi(0{,}075)^2}{4} = 0{,}0044m^2$$

$$\left.\begin{array}{l} R_1 = R_2 = \frac{r}{h} = \frac{37{,}5}{150} = 0{,}25 \\ X = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+0{,}25^2}{0{,}25^2} = 18 \end{array}\right\} \rightarrow F_{23} = \frac{1}{2}\left[X - \sqrt{X^2 - 4\left(\frac{R_2}{R_1}\right)^2}\right] = 0{,}056$$

$$F_{21} + F_{23} = 1 \rightarrow F_{21} = 1 - F_{23} = 1 - 0{,}056 = 0{,}944$$

$$F_{12} = \frac{S_2}{S_1}.F_{21} = \frac{0{,}0044}{0{,}035}0{,}944 = 0{,}118 = F_{13}$$

$$\varphi_{13} = S_1F_{13}.\sigma.(T_1^4 - T_3^4) = 0{,}035\,.\,0{,}118\,.\,5{,}67\,.\,10^{-8}[(1923)^4 - (300)^4] = 1639W$$

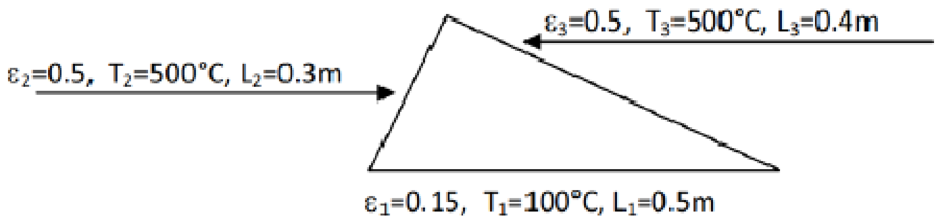
$$\varphi_{23} = S_2F_{23}.\sigma.(T_2^4 - T_3^4) = 0{,}0044\,.\,0{,}056.\,5{,}67\,.\,10^{-8}[(1923)^4 - (300)^4] = 205W$$

$$\varphi = 1639 + 205 = \mathbf{1844W}$$

Exercise 4.5

Let there be three infinite gray flat surfaces (1, 2, and 3) with semi-infinite dimensions (see figure):

- Calculate the following shape factors (see figure): F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}.
- Provide the radiosity of each surface.



Solution

$F_{11} = F_{22} = F_{33} = 0$ flat surfaces.

Application of Hottel's Rule:

$$F_{12} = \frac{S_1 + S_2 - S_3}{2S_1}$$

$$F_{12} = \frac{S_1 + S_2 - S_3}{2S_1} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{0{,}5 + 0{,}3 - 0{,}4}{2\ .0{,}5} = \frac{0{,}4}{1} = 0{,}4$$

$$F_{13} = \frac{S_1 + S_3 - S_2}{2S_1} = \frac{L_1 + L_3 - L_2}{2L_1} = \frac{0{,}5 + 0{,}4 - 0{,}3}{2\ .0{,}5} = \frac{0{,}6}{1} = 0{,}6$$

$$F_{23} = \frac{S_2 + S_3 - S_1}{2S_2} = \frac{L_2 + L_3 - L_1}{2L_2} = \frac{0{,}3 + 0{,}4 - 0{,}5}{2\ .0{,}3} = \frac{0{,}2}{0{,}6} = \frac{1}{3}$$

$$S_2.F_{23} = S_3.F_{32} \Rightarrow F_{32} = \frac{L_2}{L_3}.F_{23} = \frac{0{,}3}{0{,}4}\frac{1}{3} = 0{,}25$$

$$S_2.F_{21} = S_1.F_{12} \Rightarrow F_{21} = \frac{L_1}{L_2}.F_{12} = \frac{0{,}5}{0{,}3}0{,}4 = \frac{2}{3}$$

$$S_3.F_{31} = S_1.F_{13} \Rightarrow F_{31} = \frac{L_1}{L_3}.F_{13} = \frac{0{,}5}{0{,}4}0{,}6 = 0{,}75$$

- Radiosity of different surfaces

$$J_1 = \varphi_1 + (1 - \varepsilon_1)[F_{11}.J_1 + F_{12}.J_2 + F_{13}.J_3]$$
$$J_2 = \varphi_2 + (1 - \varepsilon_2)[F_{21}.J_1 + F_{22}.J_2 + F_{23}.J_3]$$

$$J_3 = \varphi_3 + (1 - \varepsilon_3)[F_{31}.J_1 + F_{32}.J_2 + F_{33}.J_3]$$

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