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*Investigation to Harmonics Identification in Power  
System Based on Advanced Signal Processing  
Techniques*

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## Dedication

*To my mother and my father...*

*To all members of my family...*

*To you especially: Bachir & Hadjer...*

*To all my friends...*

*To all teachers who have contributed to my education since I was 4*

*years old until today...*

*And for all persons who contribute to this work.*

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# List of Acronyms and Symbols

## Acronyms

<b>AC</b>	Alternative Current
<b>ANN</b>	Artificial Neural Network
<b>DC</b>	Direct Current
<b>DFT</b>	Discrete Fourier Transform
<b>DHT</b>	Discrete Hartley Transform
<b>DWT</b>	Discrete Wavelets Transform
<b>ESPRIT</b>	Estimation of Signal Parameters via Rotational Invariance Technique
<b>FFT</b>	Fast Fourier Transform
<b>FT</b>	Fourier Transform
<b>Hz</b>	Hertz
<b>ICA</b>	Independent Component Analysis
<b>IDFT</b>	Inverse DFT
<b>IFFT</b>	Inverse FFT
<b>LES</b>	Least Error Squares
<b>LPM</b>	Linear Prediction Model
<b>LS</b>	Least Squares
<b>MUSIC</b>	Multiple Signal Classification
<b>PCC</b>	Point of Common Coupling
<b>PHD</b>	Pisarenko Harmonic Decomposition
<b>PLL</b>	Phase-Locked Loops
<b>RMS</b>	Root Mean Square
<b>STFT</b>	Short Time Fourier Transform
<b>SVD</b>	Singular Value Decomposition
<b>TDD</b>	Total Demand Distortion
<b>THD</b>	Total Harmonic Distortion
<b>TLS</b>	Total Least Squares
<b>VAR</b>	Volt-ampere Reactive
<b>ZC</b>	Zero Crossing



## Symbols

$f_1$	The fundamental frequency
$f_h$	The $h^{\text{th}}$ harmonic
$H$	The order (or rang) of the harmonic
$M_h$	The RMS value of harmonic component $h$ of the quantity $M$
$M_1$	The RMS value of the fundamental of the quantity $M$
$I_L$	peak demand load current at the fundamental frequency component
$I_h$	The current of the $h^{\text{th}}$ harmonic
$\Omega$	The pulsation
$\Phi$	The phase
$A$	The magnitude
$X[k]$	Frequency response sequence
$x[n]$	Data sequence
$W$	Twiddle Factor
$v[n]$	The window filter
$A$	Scaling parameter in wavelet transform
$B$	Translation parameter in wavelet transform
$G$	Mother wavelet function
$s(t)$	is the instantaneous current/voltage function of time $t$
$S_n$	is the current/voltage amplitude of the $n^{\text{th}}$ harmonic
$f_s$	Sampling frequency
$\Delta T$	Sampling time
$L$	Length of window
$N$	Total number of harmonics

# Introduction

A power system is one of the most complex systems made by man. It consists of a big interconnection between generation units, substations, transmission, distribution lines and loads (consumers). This complexity requires an inevitable use of nonlinear loads, which assure a reduced cost, a fast delivering of power supply to all the power system parts and consumers, but it causes the apparition of some new waveforms superimposed to the principal signal, which distorts its shape. This is called “the harmonic distortion”.

Harmonic analysis becomes day after day a captivating subject for power system researchers and engineers who must ensure the perfect sinusoidal waveform in power supply with 50/60 Hz frequency despite using nonlinear loads - the main cause of harmonic distortion – to keeping abreast the fast developments of technology.

This urgent need of harmonic analysis meets fortunately with the rapid growth of signal processing which can be used as a powerful tool for understanding, correlation, diagnosis and development of key solutions to this complex problem. This intersection between power system analysis and signal processing privileges gives a great interesting for an **“Investigation to Harmonics Identification in Power System Based on Advanced Signal Processing Techniques”**: the topic of our thesis.

As a work plan of this study, we must ask some questions whose answers constitute the different major parts of this study.

The first important question before investigating harmonic identification on power system is: **“what does mean harmonic in power system?”** and this question carries some sub-questions about the nature of harmonics, their sources, and their impacts on the system;

For a detailed answer, the chapter 1 is devoted to defining the problem of harmonics in power system, presenting the different sources of harmonics, their special types called interharmonics and subharmonics, and their harmful effects in all parts of the power network, how to measure the harmonic distortion, as well as a fast presentation of the different available techniques of harmonics mitigation.

It is so important to mention that harmonic identification is also known as spectral analysis.

Secondly, a main question is posed about the second half of the title: **“what are the different techniques based on signal processing which contribute in harmonic identification?”**

A literature review of all signal processing techniques of harmonics identification shows that it is very difficult to cite them in this work. However, when dealing with spectral analysis, methods based on Fourier Transform (FT, DFT, FFT, STFT) are the first and the basic answer. Some disadvantages of these techniques allow the birth of other methods based on time-frequency decomposition as the case of the wavelet transform.

Unfortunately, all the methods based on the Fourier Transform suffer from the major problem of resolution. Because of some invalid assumptions such as the zero data or the repetitive data outside the duration of observation made in these methods, the estimated spectrum can be a smeared version of the true spectrum. They also need a pre-knowledge of the fundamental frequency of the system under study.

These disadvantages orientate the search trend of harmonic identification to another type of techniques called parametric methods such as Prony, MUSIC, ESPRIT or Matrix Pencil. They are then used to detect the meaningful spectral components without any conditions concerning the pre-knowledge of the fundamental frequency.

Plurality of harmonic identification based on signal processing techniques gives an evident other question about: **how to choose the appropriate method for harmonic identification?** An illustrated justification at the end of the second chapter contains the theory of criteria of choice.

The third chapter is focused on Prony's Method as being one of the parametric methods of signal processing techniques for harmonic identification. The choice of this method in our investigation comes from its independence from the pre-knowledge of the fundamental frequency, its ability of development and optimization according to the selection of its parameters. In addition, it represents a good performance in term of accuracy of identification of all spectral components presents in the signal (harmonics, inter-harmonics and sub-harmonics). We start from a background history through the detailed

original method and their variations LS-Prony and TLS Prony, the significant parameters, and algorithm selection criteria towards the limits of this parametric method;

The last chapter is devoted to present the results of simulation of Prony's Method using MATLAB where we discuss about the experimental validation of theoretical criteria of choice of parameters model.

The influence of each parameter on the performance of this method is discussed with a detailed manner.

Also, performance of Prony's Method in harmonics and interharmonics identification appear clearly when comparing it with other signal processing techniques for harmonic identification (LES, FFT...);

Finally, a summary of results of this investigation is shown in the general conclusion with giving new perspectives for harmonic identification using signal processing techniques and especially Prony's method.

# *Chapter I*

## **Harmonic Analysis in Power Systems**

### **I. 1. Introduction**

Harmonic analysis is the study of representation of signals as a superposition of basic signal. In power system, presence of harmonics is considered as the most harmful problem because it affects the original waveform of power supply, that is why the harmonic analysis becomes very important and represents a relevant subject to power engineers, researchers and consumers.

In this chapter, resumed harmonic analysis is given by defining harmonics and interharmonics, their sources, and their harmful impacts as well as some major techniques of mitigation.

### **I. 2. Harmonics in power system**

#### **I. 2. 1. Definition**

The term « harmonic » is used widely in many fields: music, physics, acoustic, seismology... etc., but it was appeared at first in the field of acoustic for expressing the vibration of a string or an air column at a frequency that is a multiple of the base frequency. [1]

Conventionally, harmonic in power system is defined as “a sinusoidal component of a periodic wave or quantity having frequency that is an integral multiple of the fundamental frequency”. [2]

So, it can be expressed mathematically as follows:

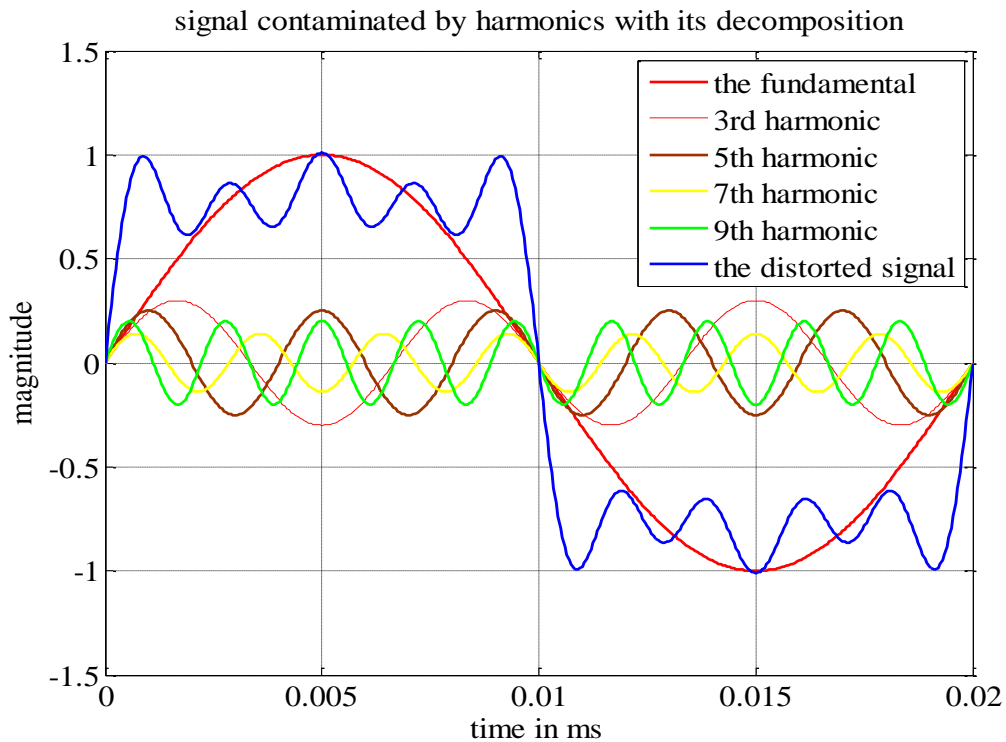
$$f_h = h \times f_1 \quad (1.1)$$

Where :  $f_1$  is the fundamental frequency ;

$f_h$  is the  $h^{\text{th}}$  frequency;

$h$  is an integer, called the order or the rang of the harmonic.

When harmonics are added to the fundamental, its waveform will be distorted and it becomes a non-perfect condition for an electrical grid or a power supply as illustrate the figure below. [1]



**Fig. 1.1 Harmonic distortion illustration.**

### **I. 2. 2. Harmonics sources**

The presence of harmonics in power systems is not a new problem; it began to appear simultaneously with the first development of these systems and still becoming greater and more harmful day after day. In addition, it is not the only disturbance that perturbs the operation of electrical systems but it is the unique problem in light of an increasing use of power electronics [3].

Hence, we can classify its sources into two categories: Old and established sources such as: rotating machines -such as synchronous machines-, transformers, rectifiers, inverters, welders, arc furnaces, voltage controller, frequency converters, power converters, ... etc. [1-3]

While the sources mentioned above still existing on the system, a new harmonic sources also enhances the power network such as:

- Energy conversion measures and devices;

- Motor control devices;
- High voltage direct-current power conversion and transmission;
- Interconnection of wind and solar converters with distribution systems;
- Static VAR compensators;
- AC/DC converters;
- ... etc. [1-3]

### **I. 2. 3. Harmonic effects**

The fast development and progress of power systems requires a widespread use of nonlinear loads that represents a big source of harmonics and leads to the huge increase of harmonic pollution in the system.

Consequently, many effects will appear in all parts of the power network, we can resume them in some points as follows [1-5]:

- Additional losses of the phase and neutral conductors ;
- Appearance of non-active distortion power;
- Additional torque in rotating machines;
- Additional losses of iron and copper of transformer which reduce its lifespan;
- Damages of capacitors because of dielectric breakdown or mis-operation of protective relays;
- Over voltage and excessive currents on the power system from resonance to harmonic voltage/current;
- Excessive losses in - and heating and mechanical oscillation of – induction and synchronous machines;
- Dielectric breakdown of insulated cables;
- Inductive interference with telecommunication systems;
- Signal interference and relay malfunction;
- Data loss in data-transmission systems;
- Harmonic instability which leads to the damage of generator shafts;
- Premature aging of equipment.

The intensity of these effects depends of course on the location of harmonics source in the power network and their characteristics as well as the parity of the harmonics, Odd harmonics distortion is typically dominant in supply voltage and load current. The effect of

odd harmonics is an increase or decrease of the amplitude of the signal with 10%. Usually a third harmonic component has a great influence in the waveform in comparison with the other odd harmonics. The effect of the distortion is the same for the positive and for the negative half of the sine wave. The positive cycle is the same as the negative cycle of the voltage wave as long as only odd harmonics are presents in the voltage [3].

Even harmonics is normally small. That generates by some large converters, transformer energizing (temporary increase). The result of their distortion is that positive and negative half cycles of the signal are no longer symmetrical. They only occur in the presence of a DC component [6, 7].

### I. 3. Interharmonics

A distorted waveform is not generally and only composed of harmonics that are integer multiples of the fundamental. These can also be composed of non-multiples, commonly called interharmonics. Studies of electrical events associated with interharmonics are still in progress, but there is currently a great deal of interest in this phenomenon. Interharmonics have recently become more significant since the many types of power electronic systems, cyclo-converters and similar have led to an increase in their magnitude. [1]

IEC-1000-2-1 standard defines interharmonics as follows: “Between the harmonics of the power frequency voltage and current, further frequencies can be observed which are not integers of the fundamental. They can appear as discrete frequencies or as a wide-band spectrum.”

Harmonics and interharmonics of a waveform can be defined in terms of its spectral components in the quasi-steady state over a range of frequencies. The following explication provides a simple and effective mathematical definition [8]:

Harmonic:  $f = h * f_1$  where  $h$  is an integer  $> 0$

DC:  $f = 0$  Hz ( $f = h * f_1$  where  $h = 0$ )

Interharmonic:  $f \neq h * f_1$  where  $h$  is an integer  $> 0$

Sub-harmonic:  $f > 0$  Hz and  $f < f_1$

Where:  $f_1$  is the fundamental power system frequency.



The term sub-harmonic does not have any official definition but it is referred to special case of interharmonic for frequency components less than the power system frequency [8].

### **I. 3. 1. Interharmonics sources**

Interharmonics types can be divided into two categories. The first type is that interharmonics are located around sidebands of the system frequency and harmonics due to a change of amplitude and/or phase caused by a rapid current variation of facilities; and the second type is asynchronous switching in static converters using semiconductor devices. It is the case that the switching frequency is not synchronised with the power system. Some major sources are mentioned as follows [8, 9]:

- Variable-Load Electric Drives ;
- Double Conversion Systems ;
- Cycloconverters;
- Time-Varying Loads;
- Wind Turbines;
- Unexpected Sources (Nonlinear load itself cannot generate interharmonics).

### **I. 3. 2. Interharmonics impacts**

The interharmonics have several potential adverse impacts to the power system. Firstly, the most common effects of interharmonics are thermal effects observed in the same way as those caused by harmonic currents, low-frequency oscillations in mechanical systems, disturbances in fluorescent lamps and electronic equipment operation.

Others are interference with control and protection signals in power supply lines, overloading passive parallel filters for high-order harmonics, telecommunication interference and acoustic disturbances, CRT flicker, torsional oscillations, overload of conventional series tuned filters, overload of outlet strip filters, communications interference, ripple control (power line carrier) interference.

The presence of interharmonic components strongly increases the difficulties of modelling and measuring the distorted waveforms. An interharmonic has a direct influence on the accuracy of the standard digital signal algorithm, such as frequency estimators and harmonics spectrum analysis [1] [10].

## **I. 4. Harmonics Measurement**

### **I. 4. 1. Interest of measurement**

From the electric utility perspective, the general objectives for conducting harmonic measurements may be summarized as follows:

- To verify the order and magnitude of harmonic currents at the substation and at remote locations where customer harmonic sources may be affecting neighbouring installations;
- To determine the resultant waveform distortion expressed in the form of spectral analysis;
- To compare the preceding parameters with recommended limits or planning levels;
- To assess the possibility of network resonance that may increase harmonic distortion levels, particularly at or near capacitor banks;
- To gather the necessary information to provide guidance to customers in controlling harmonic levels within acceptable limits
- To verify efficacy of implemented harmonic filters or other corrective schemes;
- To determine tendencies in the voltage and current distortion levels in daily, weekly, monthly, etc., basis [1].

### **I. 4. 2. Location of harmonic measurement in power network**

Harmonic distortion occurrence in an electrical installation can sometimes be assessed through a simple inspection of the types of loads at a given customer installation. All this requires is familiarity with the characteristic harmonic spectrum of each type of common nonlinear load. However, considering additional waveform distortion caused by transformer saturation or resonant conditions, a more precise evaluation should be carried out. This involves direct measurements at selected locations — for example, the point of common coupling (PCC) and the node where nonlinear loads are connected.

It is understandable that the main location where measurements are to be conducted is the customer–utility interface. This is so because compliance with IEEE and IEC harmonic limits must be verified at this location. In customer-owned transformer locations, the PCC is the point where the utility will meter the customer, generally the high-voltage side of the transformer. If the utility meters the low voltage side, then this becomes the PCC [1].

### I. 4. 3. Harmonic Measurement indices

The two most commonly used indices for measuring the harmonic content of a waveform are the Total Harmonic Distortion and the Total Demand Distortion. Both are measures of the effective value of a waveform and may be applied to either voltage or current. [7]

#### I. 4. 3. 1. Total Harmonic Distortion (THD)

Periodic signal can be decomposed into a number of harmonics. The signal can be totally characterized by the magnitude and the phase of these harmonics. In power system applications, the fundamental (50 or 60 Hz) component will normally dominate. This holds especially for the voltage. Often it is handy to characterize the deviations from the (ideal) sine wave through one quantity. This quantity should indicate how distorted the voltage or current is. The THD gives the relative amount of signal energy not in the fundamental component. In the mathematical analysis of a continuous signal, an upper limit  $H=\infty$  should be chosen. Otherwise, the upper limit is determined by the sample frequency or by a standard document [6].

The Total Harmonic Distortion is defined as the square root of the sum of the squares of the RMS value of the voltages or currents from 2<sup>nd</sup> to the h<sup>th</sup> harmonic divided by the fundamental value of the voltage or current [6].

$$THD = \frac{\sqrt{\sum_{h>1}^{h_{\max}} M_h^2}}{M_1} \quad (1.2)$$

Where:  $M_h$  is the RMS value of harmonic component  $h$  of the quantity  $M$ .

The THD is a very useful quantity for many applications, but its limitations must be realized. It can provide a good idea of how much extra heat will be realized when a distorted voltage is applied across a resistive load. Likewise, it can give an indication of the additional losses caused by the current flowing through a conductor. However, it is not a good indicator of the voltage stress within a capacitor because that is related to the peak value of the voltage waveform, not its heating value.

### I. 4. 3. 2. Total Demand Distortion (TDD)

THD cannot give a significant threat to the system in some cases, for example, a small current may have a high THD. Some engineers have attempted to avoid this difficulty by referring THD to the fundamental of the peak demand load current rather than the fundamental of the present sample. This is called total demand distortion and serves as the basis for the guidelines in IEEE Standard 519-1992, *Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems*. It is defined as follows:

$$TDD = \frac{\sqrt{\sum_{h=2}^{h_{\max}} I_h^2}}{I_L} \quad (1.3)$$

$I_L$  is the peak, or maximum, demand load current at the fundamental frequency component measured at the point of common coupling (PCC). There are two ways to measure  $I_L$ . With a load already in the system, it can be calculated as the average of the maximum demand current for the preceding 12 months. The calculation can simply be done by averaging the 12-month peak demand readings. For a new facility,  $I_L$  has to be estimated based on the predicted load profiles [7].

## I. 5. Harmonics and interharmonics identification and mitigation

Identification and measurement of harmonic sources has become an important issue in electric power systems, since increased use of power electronic devices and equipment sensitive to harmonics, has increased the number of adverse harmonic related events.

Harmonic measurements are more sophisticated and costly than ordinary measurements because they require synchronization for phase measurements, which is achieved by Global Positioning Systems (GPS). It is not easy and economical to obtain a large number of harmonic measurements because of instrumentation installation maintenance and related measurement acquisition issues.

Because of the critical importance of the harmonic problem, engineers and researchers are working hardly to understand the behaviour of this phenomenon through the detection, measurement, control, and elimination of harmonics and inter-harmonics of electrical signals in power systems [11].

Many algorithms have been proposed to evaluate the frequency content from discrete time samples of a measured signal. Most of them are frequency domain harmonic analysis algorithms and are based on the Discrete Fourier Transform (DFT) or on the Fast Fourier Transform (FFT). These methods however suffer from three main drawbacks, aliasing, leakage and picket fence effect. To overcome them, the Fourier algorithm can be associated to a Zero-Crossing (ZC) technique. Combining both methods allows providing the fundamental frequency of a measured signal corrupted by higher-order harmonics.

Unlike the DFT, Discrete Hartley Transform (DHT) can identify the inter-harmonic and harmonic without any restriction on the frequency of the harmonic to be identified. In addition, DHT can easily identify the harmonics and inter-harmonics directly without mathematical models for them. The proposed algorithm uses directly the samples of the voltage or current waveform at the location where the power quality criterion is to meet [12].

The ZC-technique is a simple and well-known method that can be used as a standalone frequency estimator. Its principle relies on calculating the number of cycles within a predetermined time interval. However, this method is sensitive to noise and is often combined with other methods like least squares algorithms. Kalman or extended Kalman filters are other important alternatives for frequency estimation. This technique is more suitable, when the voltage or current signals are time varying during the data window size, and tracks well the harmonic components, but needs appropriate initialization for the filter and accurate model for the waveform under study. Nevertheless, Kalman filtering for frequency estimation is efficient only with three-phase signals.

Phase-Locked Loops (PLLs) are also well-known signal processing techniques used for frequency measurement. For example, a PLL developed for three-phase signals is based on a fictitious instantaneous active power expression and allows frequency estimation under distorted and unbalanced voltage waveforms. It determines automatically the system frequency and the phase angle of the fundamental positive- sequence component of a three-phase generic signal even under highly distorted and perturbed system voltages.

Frequency estimation can also be achieved through an Adaptive Notch Filter (ANF). An ANF is a second-order notch filter that is further furnished with a nonlinear differential equation to update the frequency.

Recently, Artificial Neural Networks (ANNs) approaches have been developed for on-line frequency estimation. Whatever the architecture, ANNs have to be appropriate for a real-time frequency evaluation. The neural network implementation, but also the computational costs of the learning algorithm, must be compliant with the real-time constraint of this application [12-13].

For harmonics mitigation, Filters are used widely and developed rapidly in recent years. Traditionally, power engineers have been used passive filters until the latest years because of economic reasons. They are constructing from resistance, inductor and capacitor and according to their positions, we find resonant passive filters and amortized passive filters.

The use of new technologies in semi-conductors as MOSFET, IGBT allows the apparition of the active filters which has the ability of mitigate (in real-time) one or several parts of voltage/current waveform and/or harmonic signals from the power network.

Recently, Kalman filters, Notch filters and the filter banks are also developed and used for harmonic and interharmonic mitigation.

## **I. 6. Conclusion**

We present in this chapter a general overview concerning one of the most important problem affecting power systems, which is harmonics and interharmonics. Defining these problems and precising their sources and their impacts in all parts of the power systems affirms the emergent need of harmonics and interharmonics identification and mitigation.

Finally, the brief presentation of the existed techniques of harmonics and interharmonics identification gives us a starting point to specialize in those based on advanced signal processing in the next chapter.

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# Chapter II

## Advanced Signal Processing Techniques for Harmonic and Inter-harmonic identification

### II. 1. Introduction

The increasing complexity of the power grids requires intensive and comprehensive signal monitoring followed by the necessary signal processing for characterizing, identifying, diagnosing and protecting and for a more accurate investigation of harmonics and interharmonics presence in power grids [1].

Because of these reasons, the direction of searches in harmonic and interharmonic identification orientate toward the techniques based on advanced signal processing: the subject of this second chapter.

### II. 2. Advanced Signal Processing techniques

A deep review of literature shows that is so hard to cite the entire signal processing techniques used in harmonics identification on one chapter only. However, we try to concentrate on the most widespread techniques starting by the oldest techniques based on Fourier Transform (DFT, FFT,...), through Time-frequency decompositions (STFT, wavelets,...) down to the parametric methods and the new techniques based on mathematical properties.

The figure (2.1) shows the development of some of these techniques ordered in term of time [1].

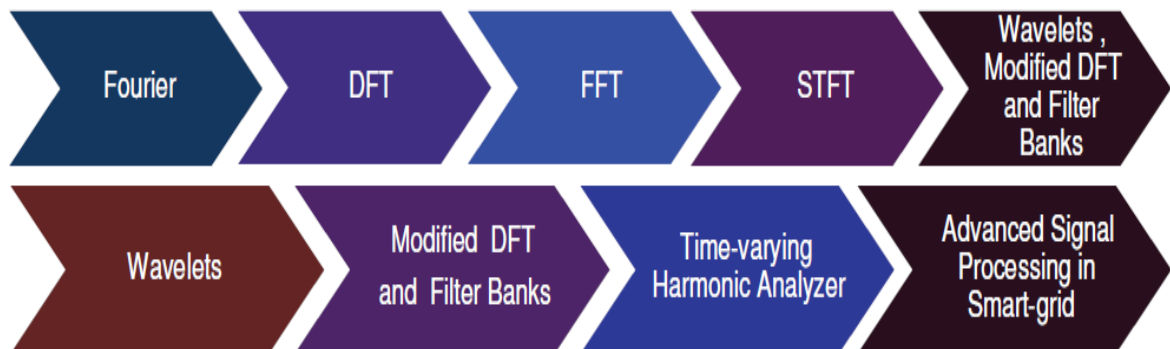


Fig. 2.1 Signal processing development.



### II. 2. 1. Fourier Transform

Fourier analysis took its name from the French mathematician J. Fourier (1768 – 1830) [2]. It is used to convert time domain waveforms into their frequency components and vice versa. When the waveform is periodical, the Fourier series can be used to calculate the magnitudes and phases of the fundamental and its harmonic components. More generally, the Fourier Transform and its inverse are used to map any function in the interval  $-\infty$  to  $+\infty$  in either the time or the frequency domain into a continuous function in the inverse domain. The Fourier series therefore represents the special case of the Fourier Transform applied to a periodic signal [3].

Two equations describe the changes between these spaces: **the analysis equation**, which takes a function in its time space and converts it into a frequency space; and **the synthesis equation** that takes a function in its frequency domain and brings it back to the time domain. Such a transformation has to be bijective.

The analysis equation or Fourier transform is defined as:

$$F(j\Omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\Omega t} dt \quad (2.1)$$

The inverse Fourier Transform or synthesis equation is defined:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\Omega)e^{j\Omega t} d\Omega \quad (2.2)$$

In general,  $F(j\Omega)$  is complex and can be written:

$$F(j\Omega) = A(\Omega)e^{-j\phi\Omega} \quad (2.3)$$

Where:  $A(\Omega) = |F(j\Omega)|$  represents the amplitude spectrum and  $\phi(\Omega) = \angle F(j\Omega)$  is the phase spectrum [1]. In power system case, this representation gives the useful information of each spectral component.

The FT has a prevalent role in signal processing, it allows to release filtering operation by a simple multiplication in frequency domain. However, signals having transient regimes or non-stationary parts (voice signal, seismic signals) are poorly described or represented by a TF [2].

In addition, one of the main limitations of use of FT in harmonic identification is the nature of the investigated signal, which becomes actually discrete and this transform is applicable only for continuous signals.

### II. 2. 2. Discrete Fourier Transform (DFT)

In practice, data are always available in the form of a sampled time function, represented by a time series of amplitudes, separated by fixed time intervals of limited duration. When dealing with such data a modification of the Fourier Transform, the Discrete Fourier Transform (DFT) is used. The implementation of the DFT by means of FFT algorithm forms the basis of the most modern spectral and harmonic analysis systems. DFT transforms a signal from the time domain to the frequency domain. This makes available the amplitude and phase of the fundamental and the harmonics present in the signal. The DC component is also available in the first bin [3].

This gives the representation of the signal in the frequency domain. This tool can be very powerful in signal processing applications because it allows us to examine any given signal (not just a filter) in the frequency domain, which provides the spectral content of a given signal [4].

Therefore, the definition of DFT (Analysis relation of the DFT) is:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad \text{for } 0 \leq k \leq N-1 \quad (2.4)$$

In addition, the synthesis relation of the DFT (also called briefly IDFT) is defined by the following equation [5]:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn} \quad \text{for } 0 \leq n \leq N-1 \quad (2.5)$$

The most common utilization of the DFT is the spectral analysis of signals in order to extract the key information in the sinusoidal waveforms, which is contained in the frequency, the phase and the amplitude of the component sinusoids. This property is exploited widely in power systems to estimate harmonics.

For a signal containing N samples, a total of  $N^2$  multiplication and addition operations are counted to do the entire transform. This can become a computational problem when N grows large, this were one of the most reasons of the development of FFT and IFFT.

Concerning the DFT's properties, first, they are reversible: We can take a signal represented by N samples and perform the DFT on it. We get N outputs representing the frequency response or spectrum on the signal. If we take this frequency response and perform the IDFT on it, we get back our original signal of N samples.

Second, when the DFT output gives the frequency content of the input signal, it assumes that the input signal is periodic in N. To put it another way, the frequency response is actually the frequency response of an infinite long periodic signal, where the N long sequence of  $x_i$  samples repeats repeatedly. Last, the input signal  $x_i$  is usually assumed to be a complex (two-dimensional) signal.

The frequency response samples  $X_k$  are also complex. Often we are more interested in only the magnitude of the frequency response  $X_k$ , which can be more easily displayed. However, to get back the original complex input  $x_i$  using the IDFT, we would need the complex sequence  $X_k$  [4, 6].

### II. 2. 3. Fast Fourier Transform (FFT)

The Fast Fourier Transform (FFT) is considered as an improved version of the DFT and its inverse transform rapidly. The invention of the FFT made the DFT practically applicable to the analysis of data sequences. Cooley and Tukey were the first mathematicians who were published an article concerning this subject in 1965 [5].

The Fast Fourier transform (FFT) can be used for harmonic analysis because of its computational efficiency. FFT can be used to calculate the harmonic distortion and to separate even/odd/inter harmonics etc. Its fast computation is considered as an advantage. With this tool, it is possible to have an estimation of the fundamental amplitude and its harmonics with reasonable approximation [3].

Supposing that we have a data sequence of N element, dividing this sequence into two halves; instead of odd and even sequences can derive another radix 2 FFT algorithm. Thus,

$$X[k] = \sum_{n=0}^{(N/2)-1} x(2n)W_N^{2kn} + W^k \sum_{n=0}^{(N/2)-1} x(2n+1)W_N^{2kn} \quad (2.6)$$

Where:  $W = e^{-j2\pi/N}$  is known as a "twiddle" factor.

Defining the two parts  $X_1(k)$  and  $X_2(k)$  as:

$$X_1(k) = \sum_{n=0}^{(N/2)-1} x(2n)W_N^{2kn} \quad (2.7)$$

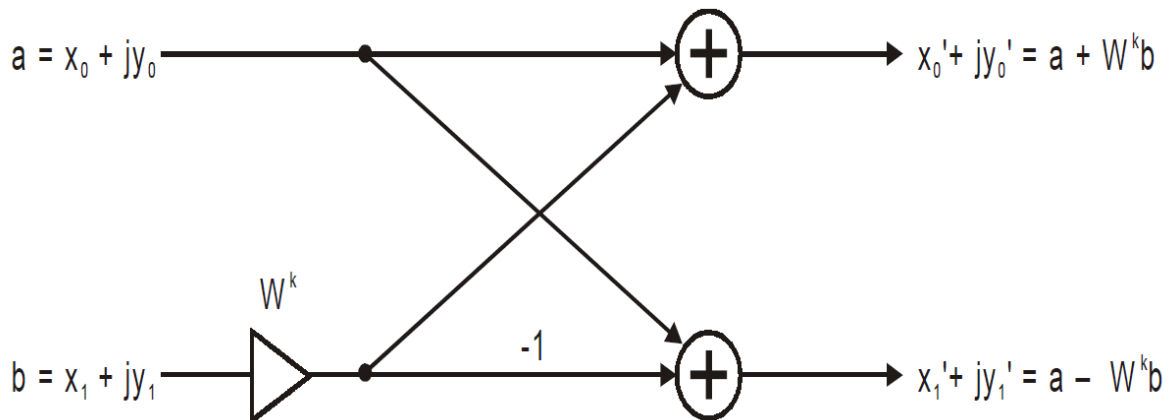
$$X_2(k) = \sum_{n=0}^{(N/2)-1} x(2n+1)W_N^{2kn} \quad (2.8)$$

So, the equation (2.6) can be consequently rewritten as:

$$X[k] = X_1(k) + W^k X_2(k) \quad (2.9)$$

The first part is the DFT of the even sequence and the second part is the DFT of the odd sequence. Notice that the factor  $W^{2nk}$  appears in both DFTs and need only be computed once. The FFT coefficients are obtained by combining the DFTs of the odd and even sequences.

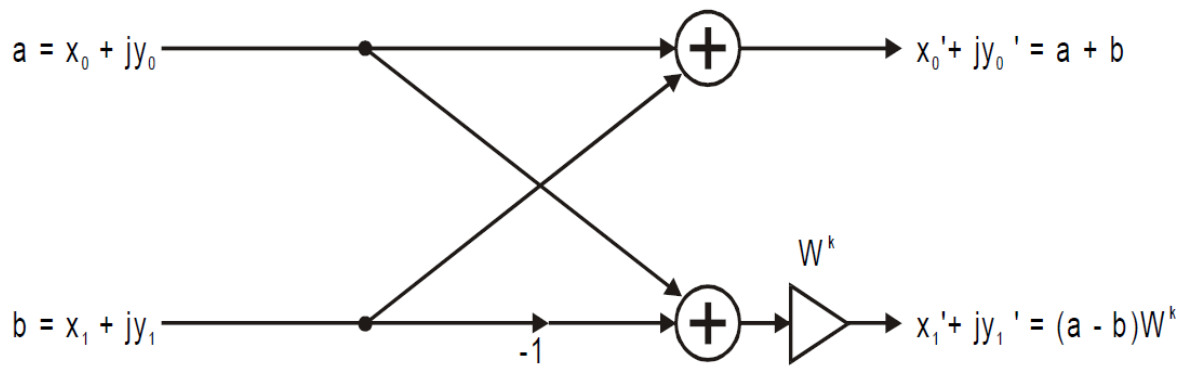
Computation of two-point DFTs is trivial. The basic operation is illustrated in Figure (2.2)



**Fig. 2.2 The butterfly operation of the decimation-in-time FFT**

This FFT algorithm is also referred to as the radix-2 decimation-in-time FFT. Radix 2 refers to the fact that 2-point DFTs are the basic computational block in this algorithm.

Decimation-in-time refers to the breaking up (decimation) of the data sequence into even and odd sequences. Comparing to decimation-in-frequency, which is similar to the butterfly for the decimation-in-time FFT except for the position of the twiddle factor as shown in the figure below [6].



**Fig. 2.3 The butterfly operation of the decimation-in-frequency FFT**

When comparing the FFT by the DFT, Rather than requiring  $N^2$  complex multiplies and additions, the FFT require  $N \log_2 N$  complex multiplication and addition operations. This may not sound like a big deal; the following table explain clearly the difference between DFT and FFT computational effort.

**Tab. 1.1. Comparison between DFT and FFT efficiency.**

<b>N</b>	<b>DFT <math>N^2</math> Complex Multiplication and Addition Operations</b>	<b>FFT <math>N \log_2 N</math> Complex Multiplication and Addition Operations</b>	<b>Computational Effort of FFT Compared to DFT (%)</b>
<b>8</b>	<b>64</b>	<b>24</b>	<b>37.50</b>
<b>32</b>	<b>1024</b>	<b>160</b>	<b>15.62</b>
<b>128</b>	<b>16384</b>	<b>896</b>	<b>5.48</b>
<b>1024</b>	<b>1048576</b>	<b>10240</b>	<b>0.98</b>
<b>4096</b>	<b>16777216</b>	<b>49152</b>	<b>0.29</b>

FFT performs well for estimation of periodic signals in stationary state; however it does not perform well for detection of sudden or fast changes in waveform e.g. transients or voltage sags [3].

The accuracy of FFT depends on the power system fundamental frequency variation when the sampling frequency is fixed. In addition, when the system contains interharmonics the resolution will be insufficient which leads to an inaccurate identification caused by the picket-fence effect. As an appropriate resolution, the IEC-61000-4-7 standard recommends a 200 ms analysis window to obtain a 5Hz FFT resolution [7].

Other algorithms deal with cases where the number of data points is not a power of two, typically when  $N$  is a prime number. One of the most well known in this category is the Winograd Fourier transform (WFT). It does not make use of the butterfly and the idea is completely different from the Cooley and Tukey algorithm. It is very fast in the sense that it requires fewer multiplications and is particularly useful for short length transforms.

However, both the mathematics involved and the implementation are considerably more complex [6].

#### II. 2. 4. Short-Time Fourier Transform (STFT)

In the STFT a signal  $x[n]$  is multiplied by a window  $v[n]$  typically of finite support (finite length in time), generating a signal  $x_0[n] = x[n]v[n]$ . The Fourier transform of  $x[n]$  is computed, the window is shifted in uniform amounts and the Fourier transform of the new data  $x_i[n] = x[n]v[n - ik]$  is computed, where  $K$  is a constant shift imposed on the window. The duration of the window governs the time localization of the analysis, whereas the bandwidth of the window governs the frequency resolution. Figure (2.4) illustrates the basic concept of STFT [1].

The figure shows a  $x[n]$  signal being multiplied by the time-shifted version of the window  $v[n]$  (continuous curves). Note that the shifted windows can overlap, depending on the value of  $K$  and the length of the window  $L$ . The FT of each windowing signal  $x_i[n]$  is then performed. If  $m$  is the center of the window the time-frequency representation is given by  $X_{STFT}(e^{j\omega}, m)$  where  $\omega$  is a digital frequency in the range of  $-\pi \leq \omega < \pi$  and  $m$  is typically an integer multiple of  $K$ . Mathematically, the STFT is defined:

$$X_{STFT}(e^{j\omega}, m) = \sum_{k=-\infty}^{\infty} x[k]v[k-m]e^{-j\omega k} \quad (2.10)$$

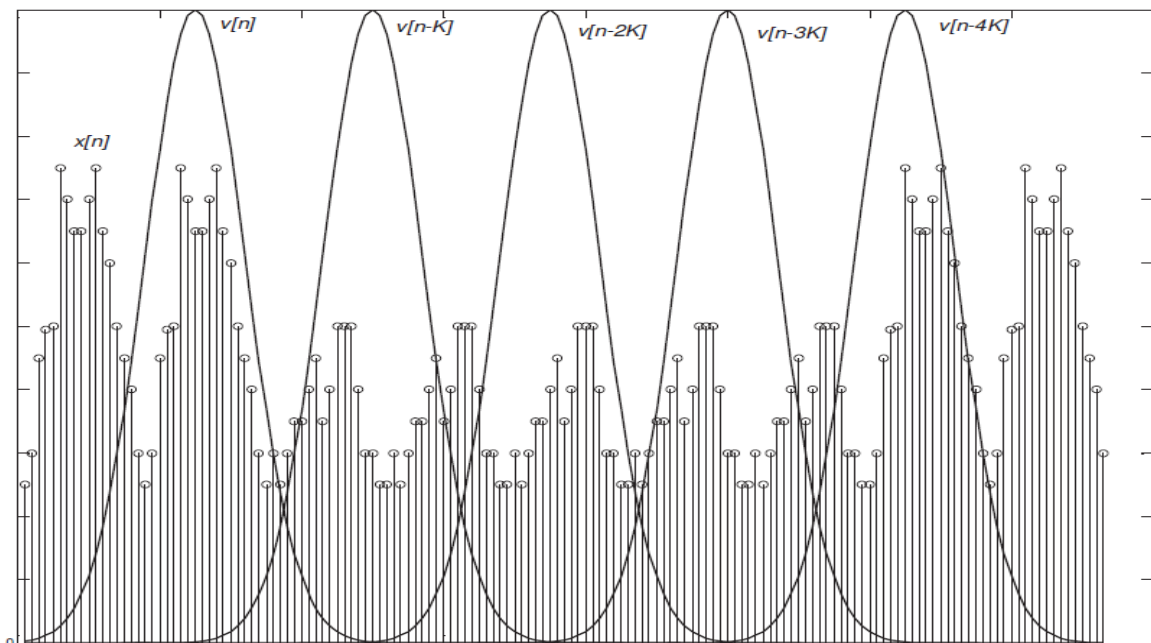


Fig. 2.4 Short-Time Fourier Transform: Illustration.

The interpretation of the STFT technique as a bank of band pass filter gives a practical structure for implementation, equation (2.10) can be rewritten for a specific frequency  $\omega_0$ , for which a graphical representation is given in (2.13) [1]

$$X_{STFT}(e^{j\omega_0}, m) = e^{-j\omega_0 k} \sum_{k=-\infty}^{\infty} x[k]v[k-m]e^{-j\omega_0(k-m)} \quad (2.11)$$

$$y_a[m] = \sum_{k=-\infty}^{\infty} x[k]h_0[m-k] \quad (2.12)$$

$$h_0[m] = v[-m]e^{j\omega_0 m} \quad (2.13)$$

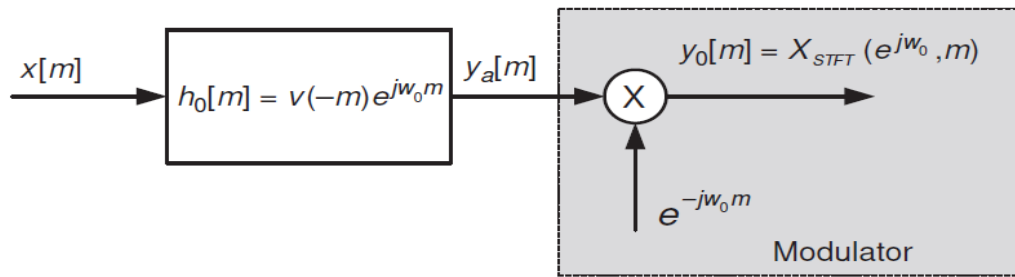


Fig. 2.5 STFT filter interpretation

Hence, advantage is such that voltage magnitude estimation can achieve higher time resolution than that of the RMS. Different authors have studied the application of STFT for the detection and characterization of voltage events in power systems. An advantage the STFT method is that it gives information on the magnitude and phase-angle of the fundamental and harmonic components of the supply [3].

### II. 2. 5. Wavelets

Many power system loads, especially electronically switched industrial devices, are dynamic in nature. With continuous starting and braking operations, these loads may generate time-varying amplitude for the current waveforms. The time-varying current injections may not be an issue of concern in a system with a strong bus. However, the injections may introduce time-varying voltage waveform distortions on a weak bus.

The time-varying nature of waveform distortions requires a precise tool for analysis and its visualization is essential for power quality studies, such as evaluation of the effects of harmonics on electrical devices and setting limits for the time-varying harmonics [1].

Wavelet Transforms were developed in France by J. Morlet in 1987, a geophysicist, to aid seismic analysis. They can be considered as the localised equivalent of Fourier transforms and work on the principle that all signals can be reconstructed from sets of local signals of varying scale and amplitude, but constant shape.

In the wavelet transform, the signal is decomposed into wavelet components in the same way that it is decomposed into harmonic components in the Fourier transform.

Each wavelet component is called a level and when the separate wavelet levels are added together, the original signal is recovered. In the discrete Fourier transform, the sequence length  $N$  of the signal determines how many separate frequencies can be represented. In the wavelet transform when  $N=2^n$  there are  $n+1$  wavelet levels. A set of wavelet components consists of signals of a specified shape that can be scaled and translated. The components of a decomposed signal depend on the shape of the analysing wavelet [8].

Unlike the Fourier transform, the wavelet transform approach is more efficient in monitoring fault signals as time varies. Unlike Fourier analysis, which relies on a single basis function, wavelet analysis uses basis functions of a rather wide functional form such as Haar wavelet and Daubechies wavelet. This new form of signal analysis is far more efficient than Fourier analysis whenever a signal is dominated by transient behaviour or discontinuities. The Wavelet Transform (WT) of a continuous signal  $x(t)$  is defined as:

$$WT(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) g\left(\frac{t-b}{a}\right) dt \quad (2.14)$$

Where  $a$  and  $b$  are the scaling and translation parameters respectively and  $g$  is the mother wavelet function. The Discrete WT is defined by:

$$DWT[m,k] = \frac{1}{\sqrt{a_0^m}} \sum_n x[n] g\left[\frac{k - na_0^m}{a_0^m}\right] \quad (2.15)$$

Where  $g[n]$  is the mother wavelet and the scaling and translation parameters  $a$  and  $b$  are functions of an integer parameter  $m$ .

WT in conjunction with AI/Fuzzy/Expert system/SVM based techniques have the advantage of fast response and increased accuracy in fault type and location identification.



WT has a good ability to separate power quality problems, which overlap in both time and frequency domains. Harmonic analysis, and can extract the fundamental wave in a very good tracking harmonic changes, thus the wavelet theory in harmonic monitoring is promising. However, wavelet transforms have the drawback of high computational burden and high sampling rate to detect a fault.

The type of basic wavelet employed affects the effectiveness in identifying the transient elements and the size of the data to be transformed must be an integer power of 2 i.e. of length  $2^n$  [9].

### II. 2. 6. Prony's Method

Prony's method is a parametric identification technique. The method is based on the assumption that a discrete series of measured values of any sinusoidal or exponentially damped signal can be approximated by a linear combination of exponential terms. The calculated signal components consist of positive, as well as negative frequencies. This can be interpreted as a phasor rotating one or the other way [10].

Therefore, this method needs to get four inputs:

- A sampled data of the signal  $y[1], y[2], \dots, y[N]$ ;
- A specific number of exponential terms knowing as the model order  $p$ ;
- The sampling frequency ;
- Measurement window of the signal.

As result, four extracted parameters are calculated:

- The magnitude  $A_k$ ;
- The frequency  $f_k$ ;
- The damping factor  $\alpha_k$ ;
- The phase angle  $\varphi_k$  of each term where  $k$  varies from 0 to  $p$ .

The following scheme resumes the principle of this parametric method.

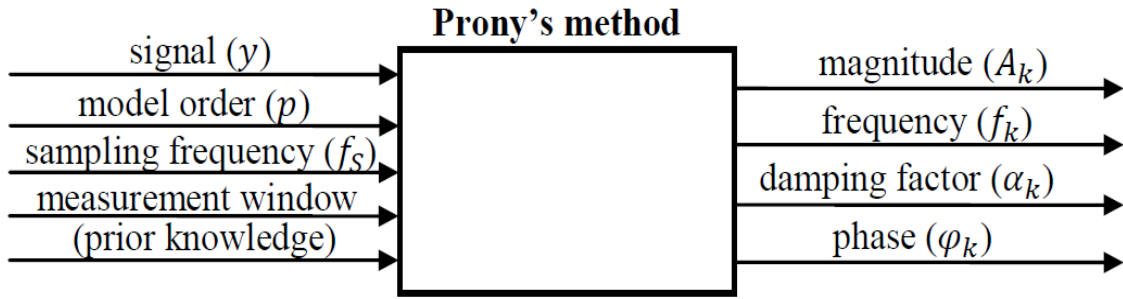


Fig. 2.6 Schematic representation of Prony Method

Prony supposes that any signal can be written as:

$$\hat{y}[n] = \sum_{k=1}^p A_k \cdot e^{(\alpha_k + j\omega_k)(n-1)T_s} \cdot e^{j\varphi_k} \quad (2.16)$$

Partitioning the previous equation into two sub-equations:

$$\underline{h}_k = A_k e^{j\varphi_k} \quad \text{and} \quad \underline{z}_k = e^{(\alpha_k + j\omega_k)T_s} \quad (2.17)$$

This gives another representation to the signal  $\hat{y}[n]$ :

$$\hat{y}[n] = \sum_{k=1}^p \underline{h}_k \underline{z}_k^{n-1} \quad (2.18)$$

This representation means that the problem is nonlinear; the idea of Prony's method is to solve the difficult nonlinear problem based on using linear equations [10].

In general, the Prony's method involves three steps: [10], [11]

- 1) Construction of a linear prediction (LP) model from the observed data and solving it (determining the parameters  $a_k$ ).

$$\begin{bmatrix} y(p) & y(p-1) & \cdots & y(1) \\ y(p+1) & y(p) & \cdots & y(2) \\ \vdots & \vdots & \ddots & \vdots \\ y(2p) & y(2p-1) & \cdots & y(p) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} y(p+1) \\ y(p+2) \\ \vdots \\ y(2p) \end{bmatrix} \quad (2.19)$$

- 2) Computation of the roots of the characteristic polynomial equation generated by the LP model;

$$F_p(z) = \prod_{k=1}^p (z - z_k) \quad (2.20)$$

The  $k^{\text{th}}$  frequency and the  $k^{\text{th}}$  damping factor can be calculated by:

$$f_k = \frac{\arctan\left(\frac{\text{im}(\underline{Z}_k)}{\text{re}(\underline{Z}_k)}\right)}{2.\pi.T_s} \quad (2.21)$$

$$\alpha_k = \text{Re}\left(\frac{\ln(\underline{Z}_k)}{T_s}\right) \quad (2.22)$$

- 3) Solving the original set of linear equations (eq 2.23) in order to estimate the exponential amplitude  $A_k$  and the sinusoidal phase  $\varphi_k$ .

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ Z_1^1 & Z_2^1 & \dots & Z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ Z_1^{p-1} & Z_2^{p-1} & \dots & Z_p^{p-1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[p] \end{bmatrix} \quad (2.23)$$

$$A_k = |\underline{h}_k| \quad (2.24)$$

$$\psi_k = \arctan\left(\frac{\text{im}(\underline{h}_k)}{\text{re}(\underline{h}_k)}\right) \quad (2.25)$$

After extracting these four parameters, all spectral components contained in signal can be identified accurately without any need for knowledge for the fundamental frequency and without any about windowing condition. This may be considered as a great benefit for interharmonics identification.

According to the algorithm selected to solve the two linear problems of this method, a variety of methods are derived from the original one (LS-Prony, TLS-Prony). Moreover, the selection of its parameters influences the performance of the detection using this method.

This parametric Method has some limits of use such as the presence of noise or the non-stationary of the signal of the study. At the other hand, it is a fertile subject for optimisation and development.

### II. 2. 7. Matrix Pencil

The Matrix Pencil is also a parametric method for estimation of magnitudes, damping factors, phases and frequencies of spectral components. It is based on the same model of the previous method (Prony Method):

$$y(t) = \sum_{k=1}^M R_k . e^{(\alpha_k + j\omega_k)t} \quad (2.26)$$

Where its form for the sampled signal is written as:

$$y(nT) = \sum_{k=1}^M R_k \cdot e^{(\alpha_k + j\omega_k)nT} = \sum_{k=1}^M R_k \cdot z_k^n \quad (2.27)$$

Where  $T$  is the sampling period,  $R_k = A_k \cdot e^{j\phi_k}$  are the complex amplitudes,  $\alpha_k$  are the damping factors,  $\omega_k = 2\pi f_k$  are the frequencies and  $z_k = e^{(\alpha_k + j\omega_k)nT}$  are the poles of  $y(t)$ .

MPM finds estimates for the amplitudes  $R_k$  and poles  $z_k$  from samples  $y(nT) = y_n$ ,  $n = 0, 1, \dots, N-1$  in two steps:

First, it finds the poles  $z_k$  as the solution of a generalized eigenvalues problem by using the matrix pencil formed from the sampled values  $y_n$ ;

In the second step, it uses these poles to estimate  $R_k$  by solving a least squares problem.

We define two  $(N-L) \times L$  matrices  $Y_1, Y_2$  as follows:

$$Y_1 = \begin{pmatrix} y_{L-1} & \cdots & y_1 & y_0 \\ y_L & \cdots & y_2 & y_1 \\ \vdots & \ddots & \vdots & \vdots \\ y_{N-2} & \cdots & y_{N-L} & y_{N-L-1} \end{pmatrix} \quad (2.28)$$

$$Y_2 = \begin{pmatrix} y_L & \cdots & y_2 & y_1 \\ y_{L+1} & \cdots & y_3 & y_2 \\ \vdots & \ddots & \vdots & \vdots \\ y_{N-1} & \cdots & y_{N-L+1} & y_{N-L} \end{pmatrix} \quad (2.29)$$

Where  $M \leq L \leq N-M$  is the pencil parameter. It has been in many literature that the best choices for  $L$  is  $\frac{N}{3} \leq L \leq \frac{2N}{3}$ .

In the noise-free case, the poles  $z_k = e^{(\alpha_k + j\omega_k)nT}$  can be found as the generalized eigenvalues of the matrix pencil  $Y_1 - \lambda Y_2$ . This means that the  $z_k$  are the  $M$  eigenvalues of  $Y_1^T Y_2$ .  $Y_1^T$  denotes the pseudo-inverse of the matrix  $Y_1$ . For noisy data, Singular Value Decomposition (SVD) is used to reduce the noise and to estimate the number  $M$  of the signal poles, thus:

$$Y_1 = USV^T \quad (2.30)$$

Here  $U$ ,  $V$  are the unitary matrices and  $S$  is the diagonal matrix containing the singular values of  $Y_1$ . The superscript  $T$  notes the transpose. Typically, the singular values beyond  $M$  are very close to zero, provided that they are arranged from largest to smallest.

The order  $M$  is thus estimated and the pseudo-inverse  $Y_1^T$  is replaced by the rank- $M$  truncated pseudo-inverse:

$$Y_1^T = \sum_{m=1}^M \frac{1}{\sigma_m} v_m u_m^T = V_0 S_0^{-1} U_0^T \quad (2.31)$$

Where  $\sigma_1, \dots, \sigma_M$  are the  $M$  largest singular values of the matrix  $Y_1$ ,  $v_m$  and  $u_m$  are the corresponding singular vectors,  $V_0 = (v_0, \dots, v_M)$ ,  $U_0 = (u_0, \dots, u_M)$ ,  $S_0 = \text{diag}(\sigma_1, \dots, \sigma_M)$ .

Once  $M$  and  $z_k$  are known, The complex amplitudes  $R_k$  are then found by solving the following least squares problem [12]:

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{pmatrix} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_M^{N-1} \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{pmatrix} \quad (2.32)$$

The advantage of this type of method is to exceed the limitations in terms of resolution to which the so-called classical methods based on Fourier analysis are subjected. The Matrix Pencil method is often qualified for this reason of high resolution [2].

However, it is sensitive to noise and the choice of  $L$  and  $M$  influences its performance.

## II. 2. 8. Signal and Noise Subspace Techniques

### Pisarenko Harmonic Decomposition

The Pisarenko harmonic decomposition (PHD) is one of the first procedures for spectral estimation based on Eigen-analysis. The PHD method is based on the observation that the signal subspace is orthogonal to the noise subspace and that the noise subspace is spanned only by one eigenvector [1].

### ESPRIT

The Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) is a parametric method to employ sinusoidal models and estimate the frequencies and

amplitudes of measured signals. By considering the complex form of the signal, containing noise described in the following equation:

$$y[n] = \sum_{h=1}^{2H} A_k e^{jn\omega_h T_s} + w(n) \quad (2.33)$$

The signal subspace  $S_1$  and noise subspace  $S_2$  can be defined by introducing the eigenvector  $U$  of the autocorrelation matrix of the analyzed signal and two selected matrices  $\Gamma_1$  and  $\Gamma_2$  as:

$$S_1 = U \cdot \Gamma_1 \quad S_2 = U \cdot \Gamma_2 \quad (2.34)$$

Then, the rotational invariance between both subspaces would lead to:

$$S_1 = \Phi \cdot S_2 \quad (2.35)$$

Where:  $\Phi = \text{diag}(e^{j\omega_1}, e^{j\omega_2}, \dots, e^{j\omega_{2M}})$  is a diagonal matrix, with  $\omega_1, \omega_2, \dots, \omega_{2M}$  being the estimated frequencies and  $M$  being the estimation order.

After obtaining frequency information from the matrix  $\Phi$  by the least square approaches, the amplitude and phase angle of each harmonic or interharmonic component can be obtained from the eigenvalues and eigenvectors of the signal [7].

## MUSIC

The multiple signal classification (MUSIC) method developed by R. O. Schmidt performs the analysis by employing a harmonic model and estimating frequencies and powers of harmonics in the signal. Different from ESPRIT, which is a signal subspace-based method, the MUSIC technique is a noise subspace-based method [13, 14].

By separating the autocorrelation matrix constituted of the analysed data into signal and noise matrices of eigenvectors, the pseudo-spectrum is computed with the noise subspace as [12]:

$$P_{MUSIC}(e^{j\omega}) = \frac{1}{\sum_{k=m+1}^M |e^H(\omega) s_k|^2} \quad (2.36)$$

The frequencies can be identified by locating the peaks of this spectrum. The key point in the MUSIC method is how to separate the signal and noise subspaces [1].

## II. 2. 9. Methods based on mathematical models

### Least Error Squares

In the general case, the current/voltage waveform can be presented by a periodic non-sinusoidal signal due to the presence of harmonics components with frequencies that are integral multiples of the fundamental frequency. Under this assumption, the current/voltage can be presented by a Fourier-type equation as follows [15]:

$$s(t) = \sum_{n=0}^N S_n \sin(n\omega_0 t + \varphi_n) \quad (2.37)$$

Where:

$s(t)$  is the instantaneous current/voltage function of time  $t$ ;

$S_n$  is the current/voltage amplitude of the  $n$ th harmonic;

$\varphi_n$  is the phase angle of the  $n$ th harmonic;

$f_0 = \frac{\omega_0}{2\pi}$  is the fundamental frequency ;

$n$  is the order of harmonic;

$N$  is the total number of harmonics.

Equation (2.37) can be rewritten in the following form:

$$s(t) = \sum_{n=0}^N (S_n \cos \varphi_n \sin(n\omega_0 t) + S_n \sin \varphi_n \cos(n\omega_0 t)) \quad (2.38)$$

For each harmonic components of the current/voltage, two parameters can be defined  $x_n$  and  $y_n$  as follows:

$$x_n = S_n \cos \varphi_n \quad (2.39)$$

And

$$y_n = S_n \sin \varphi_n \quad (2.40)$$

Based on equations (2.39) and (2.40), equation (2.38) can be rewritten as:

$$s(t) = \sum_{n=0}^N (x_n \sin(n\omega_0 t) + y_n \cos(n\omega_0 t)) \quad (2.41)$$

Then, the signal  $s(t)$  can be sampled with a frequency,  $f_s = \frac{1}{\Delta T}$ , where  $\Delta T$  is the sample rate. Suppose that this signal is sampled  $m$  times a long  $t_m$  at times  $t_1, t_2=t_1+\Delta T \dots, t_m = (m-1)\Delta T + t_1$ . The following equation is obtained based on the expansion of equation (5):

$$\begin{bmatrix} s(t_1) \\ s(t_2) \\ \vdots \\ s(t_m) \end{bmatrix} = \begin{bmatrix} a_{11}(t_1) & a_{12}(t_1) & \cdots & a_{12N+1}(t_1) \\ a_{21}(t_2) & a_{22}(t_2) & \cdots & a_{22N+1}(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}(t_m) & a_{m2}(t_m) & \cdots & a_{m2N+1}(t_m) \end{bmatrix} \begin{bmatrix} y_0 \\ x_1 \\ y_1 \\ \vdots \\ y_N \end{bmatrix} \quad (2.42)$$

This can be simplified as follows:

$$\underline{Z}_s(t) = A(t)\underline{\theta}_s + \underline{\epsilon}_s(t) \quad (2.43)$$

$\underline{Z}_s(t)$  is  $m \times 1$  vector of sampled measurement of  $s(t)$  ;

$\underline{\theta}_s$  is  $(2N+1)$  vector which has to be estimated;

$\underline{\epsilon}_s(t)$  is  $m \times 1$  error vector which has to be minimized;

$A(t)$  is  $m \times (2N+1)$  matrix presenting the measurement coefficients that are the sine and cosine resulting from the expansion of equation (2.41). The elements of this matrix are function of the initial measurement sampling time  $t_1$ , the sampling rate  $\Delta T$  and the date size, which is depending on the range of the estimation process window. These elements can be initially stored based on off-line calculation. It is obvious that the dimension of the matrix  $A(t)$  is depending mainly on the selected number of harmonics to be estimated.

To solve the problem presented in equation (2.43) and to ensure an acceptable estimation under the constraint of error vector minimization, the number of sample has to satisfy the condition  $m \succ (2N+1)$  so based on equation (2.41) an over determined set of equations can be obtained.

### 2.3.1.1. Time domain estimation; least error squares estimation

In order to estimate the elements of the vector  $\underline{\theta}_s$ , the previous problem presented in equation (2.43) has to be solved based on the minimization of the error vector  $\underline{\epsilon}_s(t)$ . Theoretically it is needed that this vector has to be forced to equal zero. The solution of the over determined set of equations based on LES estimation is presented as follows:



$$\begin{aligned}\underline{\theta}_s^* &= [A^T(t)A(t)]^{-1} A^T(t)\underline{Z}_s(t) \\ &= A^+(t)\underline{Z}_s(t)\end{aligned}\quad (2.44)$$

Where  $A^+(t)$  is the left pseudo-inverse of the matrix  $[A] = [A^T(t)A(t)]^{-1} A^T(t)$ .

Since the elements of  $A(t)$  have been calculated on off-line process and stored in advance, the left pseudo inverse of  $A(t)$  can be also calculated on off-line process. As the estimated elements of vector  $\underline{\theta}_v^*$  are obtained, the magnitude of the nth harmonic can be estimated as follows:

$$S_n = [x_n^2 + y_n^2]^{\frac{1}{2}} \quad n = 1, \dots, N \quad (2.45)$$

In the same time, the angle phase of any harmonic of order n can be determined by the following expression:

$$\varphi_n = \tan^{-1} \frac{y_n}{x_n} \quad (2.46)$$

### Independent Component Analysis ICA

Independent Component Analysis (ICA) is a statistical signal processing technique for harmonic source identification and estimation. If the harmonic currents are statistically independent, ICA is able to estimate the currents using a limited number of harmonic voltage measurements and without any knowledge of the system admittances or topology.

To understand the principle of ICA technique, we must define at first Blind source separation algorithms, which estimate the source signals from observed mixtures. The word 'blind' emphasizes that the source signals and the way the sources are mixed, i.e. the mixing model parameters, are unknown.

Independent component analysis is a BSS algorithm, which transforms the observed signals into mutually statistically independent signals [16]. The ICA algorithm has many technical applications including signal processing, brain imaging, telecommunications and audio signal separation [17].

The linear mixing model of ICA is given as

$$x(t_i) = As(t_i) + n(t_i) \quad (2.47)$$

Where  $s(t_i) = [s_1(t_i), \dots, s_N(t_i)]$  is the N dimensional vector of unknown source signals,  $x(t_i) = [x_1(t_i), \dots, x_M(t_i)]$  is the M dimensional vector of observed signals, A is an M×N matrix called mixing matrix and  $t_i$  is the time or sample index with  $i=1,2,\dots,T$ . In (2.47),  $n(t_i)$  is a zero mean Gaussian noise vector of dimension M. Assuming no noise, the matrix representation of mixing model (2.47) is

$$X = AS \quad (2.48)$$

Here  $X$  and  $S$  are M×T and N×T matrices whose column vectors are observation vectors  $x(t_1), \dots, x(t_T)$  and sources  $s(t_1), \dots, s(t_T)$ , A is an M×N full column rank matrix.

The objective of ICA is to find the separating matrix  $W$  which inverts the mixing process such that

$$Y = WX \quad (2.49)$$

Where  $Y$  is an estimate of original source matrix  $S$  and  $W$  is the (pseudo) inverse of the estimate of the matrix A. An estimate of the sources with ICA can be obtained up to a permutation and a scaling factor.

Since ICA is based on the statistical properties of signals, the following assumptions for the mixing and demixing models needs to be satisfied:

- The source signals  $s(t_i)$  are statistically independent.
- At most one of the source signals is Gaussian distributed.
- The number of observations M is greater or equal to the number of sources N ( $M \times N$ ).

There are different approaches for estimating the ICA model using the statistical properties of signals. Some of these methods are: ICA by maximization of non-gaussianity, by minimization of mutual information, by maximum likelihood estimation, by tensorial methods [17].

### II. 3. Criteria of choice

Choosing the appropriate technique for estimating harmonics and/or interharmonics in signal submits to some conditions and limits of use especially in power systems where the signal waveform has specific characteristics.

Starting with the oldest signal processing techniques used on harmonic identification represented in Fourier Transforms. They have been used widely and for many decades for converting the signal from the space domain to its spectral domain, we remark here that the resulting spectrum is independent of time. This leads to two consequences: the first is the impossibility of the inverse operation, i. e. we cannot know the starting time of the analyzed signal or the length of measurement window. The second consequence is the limit of using of these methods only for stationary signals where the frequency information content in the signal is constant in time. This would be needless in power system cases where the signal of power supply can be affected by a small variation at the start time or by some changes in waveform for the dynamic states.

The problem of variation of time or the problem of non-stationary of the signal met when using the FFT and the DFT are solved partially by their replacing with time-frequency decomposition of the voltage/current waveform like the STFT, which analyses the signal as a set of small intervals, and extract the frequency information for each one of them.

The major drawback of the STFT is that the width of the sliding window is fixed, which requires observing slow transients (BF) and fast transients (HF) with the same resolution. What we can know in this case is the time intervals within which certain frequency bands exist, which is therefore a problem of resolution [2].

The wavelet transform has been developed as an alternative to the STFT, it allows to correcting its resolution defects, compressing, expanding, and translating the observation window. The time signal is processed using two filters, one of the high-pass type and the other of the low-pass type. This filtering technique makes it possible to extract the BF part (a) and the HF part (d) of the signal. This procedure can be repeated several times depending on the degree of resolution on each of the two parts BF and HF [2].

The wavelet transform has been frequently utilized in the study of PQ for the assessment, detection, localization, and classification of disturbances [8]–[12]. The wavelet transform analyses the signal using long windows for low frequencies and short windows for high frequencies. This allows good frequency resolution (and poor time resolution) to be obtained at low frequencies and good time resolution (and poor frequency resolution) to be obtained at high frequencies. This approach is especially suitable when the spectrum of the signal contains high frequency components for short durations and low frequency components for long durations (for example, a signal with transient disturbances) [18].

All the previous method can be classified as non-parametric techniques for spectral estimation.

Spectrum estimation using the non-parametric method is mostly used to obtain an estimation of frequency, magnitude and phase of the spectrum content and the power spectral density (PSD) approach hides the phase information. Nonetheless, the non-parametric methods for PSD estimation can be used when the SNR is low and the phase information is not of interest [1].

Non-parametric methods are relatively simple, well understood and easily implemented via the FFT; however, they need long windows to achieve good resolution and they experience spectral leakage due to the use of windows. The major limitation of their use in power quality determination is the finite record length of the signal under analysis for the quality of the spectrum estimation; as such, non-parametric methods are limited to applications in real-time processing [1].

Parametric high-resolution methods (such as the Prony, ESPRIT, and MUSIC methods) have also been proposed to analyze nonstationary waveforms in power systems. These methods analyze waveforms using signal models and furnish a time frequency representation of a waveform providing the time variations of spectral component parameters (for example amplitude and frequency) [18].

The Prony method can be used to estimate harmonic and interharmonic components as well as damping components. The accuracy of the estimation depends on the level of signal distortion, the observation window and the number of samples used in the estimation process, as well as the order of the model.

In most cases, superior performance is achieved when compared with a Fourier algorithm. Its limitations are sensitivity to noise, difficulty to adjust parameters and computational cost [1].

#### **II. 4. Conclusion**

This chapter resumes some advanced signal processing techniques for harmonic and interharmonic identification, especially those that are the most utilized in power applications.

In addition, the criteria of selection of the appropriate technique among them for such application are discussed. It proves that each one of them has some advantages and limits of use and it proves the good performance of the parametric methods in comparison of those based of Fourier aspects despite their high cost of implementation.

The fertility of this type of techniques concentrates our investigation into a deep theoretical and experimental study of Prony method in the next parts of our thesis.

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# *Chapter III*

## **Prony's Method for harmonic and interharmonic identification**

### **III. 1. Introduction**

Among several advanced signal processing techniques used to identify harmonics and interharmonics, parametric methods presents good performance and better characteristics than the other spectral techniques.

This chapter focuses on one of the parametric methods used recently on power systems. Prony Method bases on the idea of composition of any signal into a sum of damped exponentials; this was a great advantage to represent the voltage/current waveform into spectral components.

### **III. 2. Historical Background**

The Prony method is a technique for modelling sampled data as a linear combination of exponentials. It was originally proposed by Gaspard Riche and Baron de Prony in 1795 to model gas expansion by decreasing exponentials while its first examination in electric power systems was published in 1990 [1].

### **III. 3. Prony Method**

Prony method is a technique used for extracting exponential signals from sampled series data. Unlike the auto-regressive (AR) and auto-regressive moving average (ARMA) models that try to adjust the data model using a stochastic approach (second-order statistics), the Prony method is a deterministic method [2].

Prony's method is primarily used for spectral analysis of distorted signals as in [3], as well as in the area of power electronics, to determine the harmonics and inter-harmonics in frequency inverters [4]. In [5] Prony's method is shortly explained and used for the analysis of fault currents. Statements are made about the model order, length of the measuring window, and the sampling frequency. In this context, [6] and [7] should also be mentioned. They also deal with the choice of the sampling frequency, in order to improve the efficiency of Prony's method. Fault currents were also analyzed in [8, 9, 10, 11]. In [8] the authors



analyzed phase-to-ground faults using Prony's method in isolated and compensated power systems. Symmetrical fault-currents are examined in [9]. The authors use the presence of a decaying DC component in the currents as an indication of a symmetrical fault. Inrush currents are examined in [10] with Prony's method and are compared to the FFT-analysis. In [11] Prony's method is brought into conjecture with the reconstruction of currents, which are distorted by transformer saturation.

Prony's method is also examined in the context of fault-location algorithms. In [12, 13] the authors use the calculated frequencies and magnitudes of the currents during the process to calculate the fault location in DC networks. Other methods for determining the fault location in transmission and distribution networks are presented in [14, 15, 16, 17]. The identification of power swing effects, Prony's method appears, among others, in [18].

The methodology can be briefly described in three steps [2]:

- **Step 1:** Determine the coefficients of a linear predictive model that models the samples.
- **Step 2:** Determine the roots of a characteristic polynomial associated with the linear predictive equation. Through this process, the damping factors and the frequency of each exponential term are found.
- **Step 3:** Estimate the amplitude and phase of each exponential term.

These steps can be resumed in the following scheme:

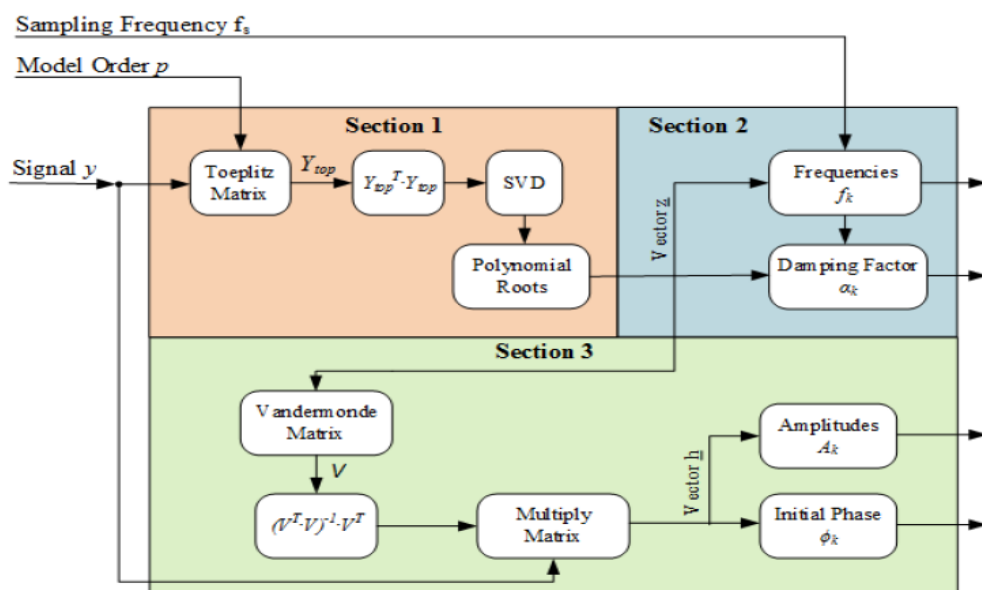


Fig.3. 1 Simplified block diagram of Prony's method

### III. 3. 1. Original Prony

Assuming the N complex data samples  $y[1], \dots, y[N]$  [19]. Each estimated data point  $\hat{y}[n]$  has a relationship with all the p elementary signals presented in the waveform according to Prony model as follows:

$$\hat{y}[n] = \sum_{k=1}^p A_k e^{(\alpha_k + j\omega_k)(n-1)T_s} e^{j\psi_k} \quad (3.1)$$

Where:  $n=1, 2, \dots, N$

$T_s$  : Sampling period,  $A_k$  : Amplitude,

$\alpha_k$  : Damping factor,  $\omega_k$  : Angular velocity,

$\psi_k$  : Initial phase.

Therefore, we can extract the magnitude, frequency, damping factor and initial phase of each modal component of voltage/ current waveform just with a pre-knowledge of the sampling time ( $T_s$ ) of data, the model order p, and the number of samples used for extraction N, which is called "length of measurement window".

Defining the time independent term as follows:

$$\underline{h}_k = A_k e^{j\psi_k} \quad (3.2)$$

And the time dependent term as follows:

$$\underline{z}_k = e^{(\alpha_k + j\omega_k)T_s} \quad (3.3)$$

So, the discrete-time function may be concisely expressed in the following form:

$$\hat{y}[n] = \sum_{k=1}^p \underline{h}_k \underline{z}_k^{n-1} \quad (3.4)$$

In order to represent the signal in a parametric form, the deviation of the model output from the original signal needs to be produced. The model error  $\varepsilon[n]$  calculated for each data point is defined as follows:

$$\varepsilon[n] = \frac{1}{N} \sum_{n=1}^N (y[n] - \hat{y}[n])^2 \quad (3.5)$$

For the calculation of the estimated signal, we must force the error to be equal to zero; so we obtain the following equation:

$$y[n] = \sum_{k=1}^p h_k z_k^{n-1} \quad (3.6)$$

It can be represented in detail by the following system:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ z_1^1 & z_2^1 & \dots & z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{p-1} & z_2^{p-1} & \dots & z_p^{p-1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[p] \end{bmatrix} \quad (3.7)$$

It is obvious that this problem consists of solving a non-linear system. Indeed, the idea of Prony's method is to solve the difficult nonlinear problem based on using linear equations [20]. In our case, the non-linear problem presented in equation (3.7) will be divided into two linear problems:

Firstly, we describe the set of data  $y[N]$  as a Linear Prediction Model (LPM) of order  $p$  as given by equation (3.8) that its coefficients are calculated. Then, computation of the roots of the characteristic polynomial equation generated by the LPM precedes the solution of the original set of linear equations in order to estimate finally the amplitude, the frequency, the damping factor and the phase angle of each elementary component of the test signal. [21]

$$y[N] = a_1 y[p-1] + a_2 y[p-2] + \dots + a_p y[p-N] \quad (3.8)$$

### Obtaining the time dependent part “ $z_k$ ”

The key technique in Prony's method is to introduce a polynomial  $F_p(z)$  that its roots will contain the frequency information, as shown in the following equation [22]:

$$F_p(z) = \prod_{k=1}^p (z - z_k) \quad (3.9)$$

Based on the fundamental theorem of algebra, the previous product can be expressed as a sum as follows:

$$F_p(z) = \sum_{m=0}^p a[m] z^{p-m} \quad (3.10)$$

With:  $a[0] = 1$ . Shifting the index from  $n$  to  $n-m$  and multiplying by the term  $a[m]$  in the equation (7) yields:

$$a[m] y[n-m] = a[m] \sum_{k=1}^p h_k z_k^{n-m-1} \quad (3.11)$$

$$\begin{aligned}
\sum_{m=0}^p a[m]y[n-m] &= \sum_{m=0}^p \left[ a[m] \sum_{k=1}^p \underline{h}_k Z_k^{n-m-1} \right] \\
&= \sum_{k=1}^p \left[ \underline{h}_k \sum_{m=0}^p a[m] Z_k^{n-m-1} \right] \\
&= \sum_{k=1}^p \left[ \underline{h}_k Z_k^{n-p-1} \sum_{m=0}^p a[m] Z_k^{p-m} \right]
\end{aligned} \tag{3.12}$$

The right-hand summation in (3.12) may be recognize as polynomial defined by (3.10), evaluated at each of its roots  $\underline{Z}_k$  yielding the zero result:

$$\sum_{m=1}^p a[m]y[n-m] = -y[n] \tag{3.13}$$

When expanding this equation, we obtain the following system:

$$\begin{bmatrix} y(p) & y(p-1) & \cdots & y(1) \\ y(p+1) & y(p) & \cdots & y(2) \\ \vdots & \vdots & \ddots & \vdots \\ y(2p) & y(2p-1) & \cdots & y(p) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} y(p+1) \\ y(p+2) \\ \vdots \\ y(2p) \end{bmatrix} \tag{3.14}$$

Hence, solving the system above will provide the coefficients  $a_k$  that allow defining the polynomial and extracting its roots  $\underline{Z}_k$ , consequently, the frequency and the damping factor of each component can be easily identified as follows:

$$f_k = \frac{\arctan\left(\frac{\text{im}(\underline{Z}_k)}{\text{re}(\underline{Z}_k)}\right)}{2.\pi.T_s} \tag{3.15}$$

$$\alpha_k = \text{Re}\left(\frac{\ln(\underline{Z}_k)}{T_s}\right) \tag{3.16}$$

### Obtaining the time independent part “ $\underline{h}_k$ ”

When we get all the roots of the polynomial, we can return to the equation (3.8) and solve the system  $Z.h = y$  presented in equation (3.7). Finally, the magnitude and the phase angle of each component can be identified as follows:

$$\psi_k = \arctan\left(\frac{\text{im}(\underline{h}_k)}{\text{re}(\underline{h}_k)}\right) \tag{3.17}$$

$$A_k = |\underline{h}_k| \tag{3.18}$$

In order to find unique solutions for the set of  $\underline{Z}_k$  and  $\underline{h}_k$ , the amount of data  $N = 2p$  can be sufficient. This is the case of the classical variant of Prony's method where both of the two sub-problems can be solved using an appropriate method that can be used for solving a linear system such as SVD (Singular Value Decomposition), Burg, Marple, etc.... If the amount of the data points is greater than the number of the parameters ( $N > 2p$ ), then the sampled data can only be approximated but not interpolated by exponential functions. The identification of the relevant parameters  $a[m]$  corresponds to the classic variant of Prony's method. The zeros of the characteristic polynomial  $\underline{Z}_k$  are extracted. Then, in order to obtain the complex magnitudes  $\underline{h}_k$ , the following calculation is applied [23]:

$$h = Z^{-1} \cdot y \quad (3.19)$$

### III. 3. 2. Least Squares Prony Method

In general situations, the number of data is greater than twice of the model order  $N > 2p$ , so the equation (3.13) will be re-expressed as:

$$y[n] + \sum_{m=1}^p a[m]y[n-m] = \varepsilon[n] \quad \text{for } p+1 \leq n \leq N \quad (3.20)$$

The above equation is recognized as a linear predictor filter from adaptive filter theory; the term  $\varepsilon[n]$  is the prediction error and the summation is the forward predictor filter. Figure (3.2) depicts the forward prediction block diagram [2].

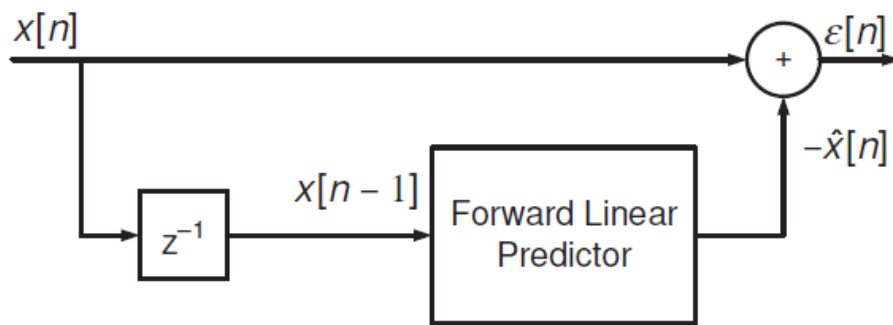


Fig. 3.2 Block diagram of the forward linear predictor filter

The forward linear predictor filter is a FIR filter; its coefficients need to be optimized to minimize the cost function. The cost function can be written:

$$J(a) = \sum_{n=p+1}^N |\varepsilon[n]|^2 \quad (3.21)$$

As the model is linear, the solution is given by:

$$\hat{a} = (H^T H)^{-1} H^T y \quad (3.22)$$

Or

$$\begin{bmatrix} y[p] & y[p-1] & \cdots & y[1] \\ y[p+1] & y[p] & \cdots & y[2] \\ \vdots & \vdots & \ddots & \vdots \\ y[N-1] & y[N-2] & \cdots & y[N-p] \end{bmatrix} \begin{bmatrix} a[1] \\ a[2] \\ \vdots \\ a[p] \end{bmatrix} = \begin{bmatrix} y[p+1] \\ y[p+2] \\ \vdots \\ y[N] \end{bmatrix} \quad (3.23)$$

The matrix H is identified as the first matrix in the previous equation, the vector x is the right-hand side vector and the estimated parameters are the coefficients of the characteristic equation. After finding these coefficients, the zeros of the characteristic equations can be obtained. Equation (3.7) can then be written:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ Z_1^1 & Z_2^1 & \cdots & Z_p^1 \\ \vdots & \vdots & \ddots & \vdots \\ Z_1^{N-1} & Z_2^{N-1} & \cdots & Z_p^{N-1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} y[1] \\ y[2] \\ \vdots \\ y[N] \end{bmatrix} \quad (3.24)$$

Finally, the vector H is obtained by:

$$H = (Z^T Z)^{-1} Z^T y \quad (3.25)$$

### III. 3. 3. Modified Least Squares Prony Method

The Prony method can be applied to signals where some frequency components are known a priori, as in power system applications. This assumption reduces the computational complexity when compared to the conventional method. The procedure is developed in reference [6] and is summarized below [2].

Assume that q exponential components  $\underline{Z}_k$  are known, that is  $\underline{Z}_1, \underline{Z}_2, \dots, \underline{Z}_q$ . Knowledge of a sinusoidal component corresponds to knowledge of two exponential components. The characteristic polynomial associated with q known components is:

$$\prod_{m=1}^q (z - z_k) = \sum_{k=0}^q c[m] z^k \quad (3.26)$$

Where  $c[q] = 1$ . The characteristic polynomial relating all exponentials can be written:

$$Q(z) = \sum_{m=0}^p a[m]z^m = \left( \sum_{k=0}^q c[m]z^k \right) \left( \sum_{i=0}^{p-q} \alpha[m]z^i \right) \quad (3.27)$$

Where  $\alpha[p-q]=1$ . The coefficients  $a[m]$  are then given by:

$$a[m] = \sum_{k=0}^q c[m] \alpha[m-k] \quad (3.28)$$

Where  $\alpha[i]=0$  for  $i > p-q$  or  $i < 0$ . Applying Equation (3.28) to equation (3.11) yields

$$\sum_{m=1}^p a[m]y[n-m] = \sum_{m=1}^p \left( \sum_{k=0}^q c[m] \alpha[m-k] \right) y[n-m] = 0 \quad (3.29)$$

Which is defined for  $p+1 \leq n \leq 2p$ . Equation (3.28) can be rewritten

$$\sum_{m=1}^{p-q} a[m] \hat{y}[n-m] = 0 \quad (3.30)$$

Where:

$$\hat{y}[n] = \sum_{k=0}^q c[k]y[n-k] \quad (3.31)$$

Note that Equation (3.31) represents a convolution process, that is, a filtering process. The sequence  $\hat{y}[n]$  is therefore obtained by filtering the input signal  $y[n]$  with a filter that has an impulse response of  $c[n]$ . Note that once  $\hat{y}[n]$  is found, the values of  $\alpha[n]$  can be determined by using Equation (3.30).

The procedure can be summarized as follows:

- The data window is filtered through Equation (3.31). The filter coefficients are composed of the known poles obtained from Equation (3.26).
- By using the filtered data  $\hat{y}[n]$ , we can find  $\alpha[n]$  using Equation (3.30). The characteristic polynomial with the  $p-q$  unknown poles is then established.
- After finding the complete coefficients of the polynomial characteristics, Equation (3.25) can be applied to estimate the amplitude and phase of all components [2].

### III. 4. Selection Criteria of Prony parameters

As it has been mentioned previously, the Prony's model needs a precise selection of the  $N$  sample data (also called length of measurement window), the model order  $p$  and the sampling frequency  $f_s$ . This leads to a tertiary problem.

Firstly, we should benefit from the most important characteristic merit in parametric methods and Prony's method in particular, which is their ability to estimate all the modal components of the spectrum under study without any need of the fundamental frequency knowledge. Hence, the parametric methods can be used for the identification of harmonics components and even the inter-harmonics components identification with good resolution.

Secondly, the sampling frequency can be adjusted according to Nyquist sampling theorem that should be at least two times of the highest frequency in the concerned signal. Since the Prony analysis results are not accurate for too high sampling frequency [21,24]. Therefore, two or three times of the highest frequency is considered to guarantee the accurate results based on Prony analysis [25].

On the other side, for the window of measurement, selecting one cycle of the tested signal gives inaccurate results. Therefore, two cycles can be used to allow obtaining precise results under voltage or current waveforms in the presence of harmonics and inter-harmonics. Nevertheless, in real cases the voltage waveform may be influenced by transition phases such as the sudden variation and the long starting time, which can change the nature of signal. So to avoid all these transition phases, 4 to 5 cycles of the lowest frequency can be taken to ensure the requirement of accurate identification of the harmonics and the inter-harmonics in the studied signals.

The shifting time window for Prony analysis has to be filled with sampled data before correct estimation results are derived. The selection of the equal sampling intervals between samples and the data length in an analysis window depends on the simulation time step and the estimated frequency range [26].

The last issue to be performed is the selection of the LPM order, which represents another important Prony analysis parameter. A common principle is the LPM order should be no more than one thirds of the data length [26]. For ensuring the identification of  $m$  spectral components contained in the signal, it was proved that at least  $p=2m$  are intended to fulfill the requirement of accurate detection.



On the other side, theoretically, the accuracy stays high when  $p$  exceeds twice the number of elements to be identified, but in practice, the complexity and the high cost of the filter of order  $p$  limit its incrementing.

It is quite difficult to make the first selection of the LPM order since the exact number of modes of a real system is hard to determine.

### III. 5. Selection of Prony Analysis Algorithm

Three normally used algorithms to derive the LPM coefficients, the Burg algorithm, the Marple algorithm, and the SVD (Singular value decomposition) algorithm are compared for implementing Prony analysis in transient harmonic studies. The non-recursive SVD algorithm utilized the Matlab pseudo-inverse function "pinv". This "pinv" function uses LAPACK routines to compute the singular value decomposition for the matrix. To choose the appropriate algorithm, the three algorithms are applied on the same signals with the same Prony analysis parameters [27].

From comparison on the estimation results of various signals to approximate power system transient harmonics, the SVD algorithm has the best overall performance on all estimation results and thus is selected as the appropriate algorithm for Prony analysis.

**Tab. 3.1: Estimated Dominant Harmonics (EDH) on A Nonstationary Signal**

	EDH	Ideal	Burg	Marple	SVD
Frequencies (Hz)	#1	60	60.1690	59.9986	59.9987
	#2	300	298.2309	279.3917	299.9951
	#3	420	419.3031	420.0081	420.0138
	#4	660	657.8118	659.9380	659.9578
	#5	780	779.1504	779.9914	780.0137
Damping factors ( $s^{-1}$ )	#1	0	-0.0037	-0.0027	-0.0012
	#2	-6	-1.3173	0.2127	-6.0403
	#3	-4	-0.0940	0.1245	-4.0638
	#4	0	-0.5625	-0.1881	-0.1097
	#5	0	-3.4003	-0.6494	-0.1752
Magnitudes (A)	#1	1	1.0001	0.9997	1.0002
	#2	0.2	0.1478	0.1441	0.2002
	#3	0.1	0.0819	0.0809	0.1003
	#4	0.02	0.0184	0.0203	0.0204
	#5	0.01	0.0104	0.0107	0.0103
Phase Angles (Degree)	#1	0	3.1693	0.0320	3.1693
	#2	45	79.0299	44.9906	45.0567
	#3	30	41.4376	30.2150	29.9158
	#4	0	36.8397	-0.8574	0.7913
	#5	0	12.7567	0.1850	0.3631

### III. 6. Conclusion

Theoretical investigation of Prony Method is presented in the third chapter; this study starts with a detailed explication to the original method and some other modified methods based on the same Prony method.

This investigation leads also to the result that the choice of Prony parameters and the algorithm chosen to solve the linear sub-problems in this method influences its performance.

In order to justify and to clarify these criteria, experimental simulations are given and discussed in the next chapter.

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# *Chapter IV*

## **Results and Discussion**

### **IV. 1. Introduction**

The aim of this chapter is the general analysis of Prony Method and the experimental validation of the criteria and the properties of this method.

The first part of this chapter is dedicated to harmonic identification using this method with the detailed analysis of the influence of each parameter in the accuracy and performance of the estimation.

In the second part, the same analysis is repeated but for interharmonics components. After, we apply this method for real data case and we browse a comparison between three signal processing techniques for spectral components estimation in order to prove its efficiency.

### **IV. 2. Harmonic identification using Prony Method**

As Prony Method is a parametric method and the right selection of its parameters influences significantly its accuracy in detection, so an experimental validation of the appropriate parameters is needed to justify all the theoretical criteria of parameters selection for Prony Method before examining the effectiveness of this signal processing method for harmonic and interharmonic identification.

#### **IV. 2. 1. Validation of theoretical choice of parameters**

In order to demonstrate all the criteria detailed in the previous chapter concerning the right selection of the sampling time, the model order and the length of measurement window in Prony analysis, signals are synthesized to be considered as test signals using matlab.

##### **IV. 2. 2. 1. Construction of the test signal 1**

The first type of signal is chosen as a sum of undamped sinusoids (the damping factor is null) in order to estimate only the magnitudes and the frequencies corresponds to the spectral components present in the signal.

So, the signal can be written as:

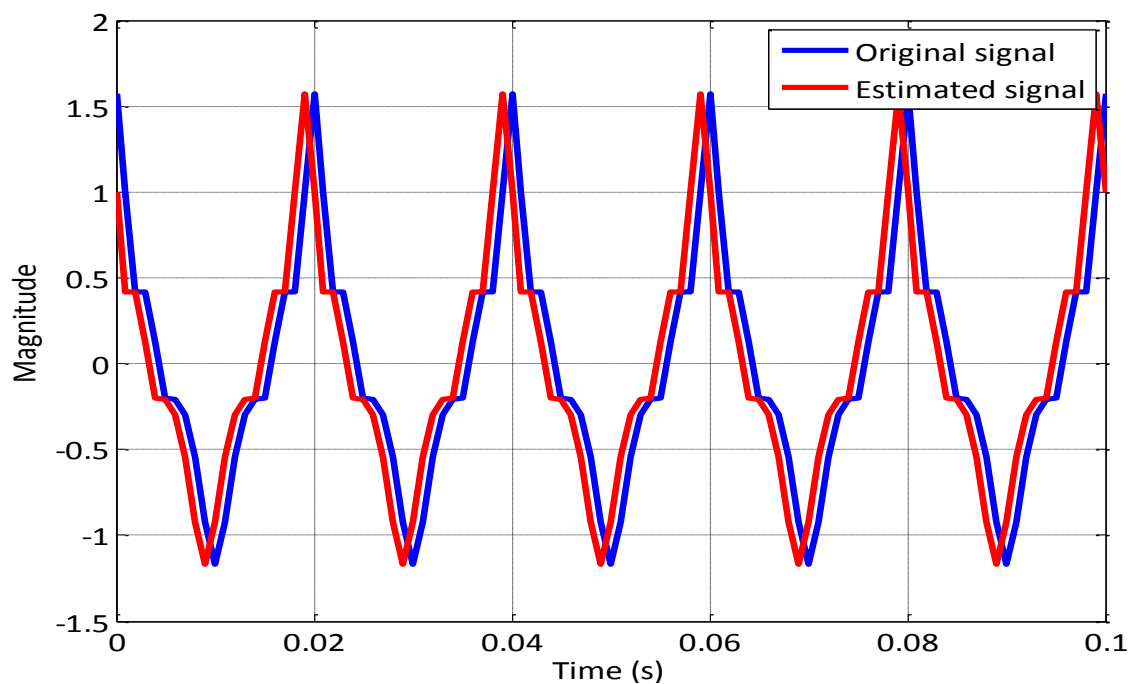
$$y[n] = \sum_{k=1}^m A_k \cdot \cos(2\pi f_k n) \quad (4.1)$$

**Tab. 4.1 Characteristics of synthesized test signal 1**

Spectral component	Frequency (Hz)	Magnitude
1	50	0.9
2	100	0.09
3	150	0.25
4	250	0.15
5	300	0.11
6	350	0.07

As we know the number of the modal components of the signal  $m=6$ , the model order chosen at first will be equal to 12. The highest frequency present in this test signal is 350 Hz, so the sampling frequency must be greater than 700 Hz. For more accuracy we take  $f_s=1$  KHz. For the window of measurement, we select 20 samples per cycle and for an accurate visualise, four cycles of the lowest frequency are present in the signal. The fig 4.1 proves the theoretical suppositions discussed in the previous section.

After obtaining the magnitude, frequency, damping factor and the angle phase of each modal component, we can get consequently a temporal representation of the estimated signal which is pseudo-identical to the original one with a time shifting equal to 1 ms ( $T_s$ ) with an error of 0.1135 as well as a spectral representation shown in fig. 4.2



**Fig. 4.1 Representation of the original signal and the estimated signal in time domain**

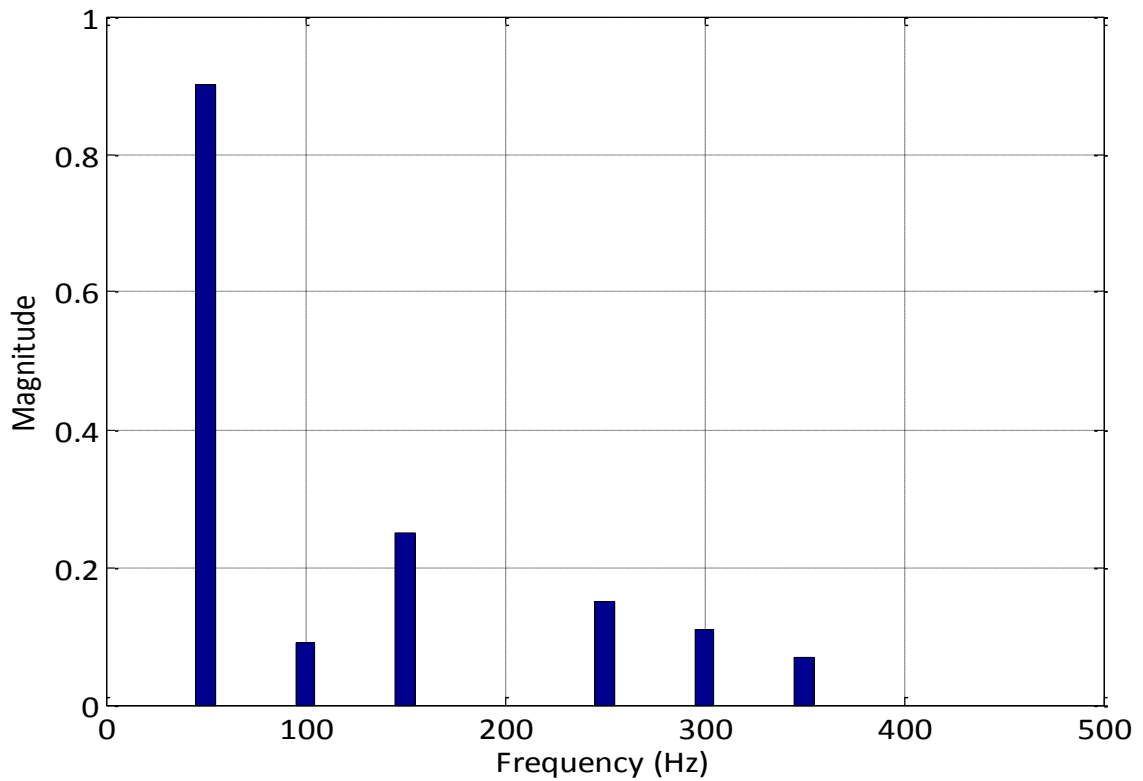


Fig. 4.2 Spectrum of the estimated signal

#### IV. 2. 2. 2. Construction of Test signal 2

In the second case, the test signal is considered as a sum of damped sinusoids. For more simplification of the influence of the damping factor, we keep the same information from the previous signal (magnitudes and frequencies) and we add non-null damping factors for each modal component. Therefore, the test signal 2 has the following form:

$$y[n] = \sum_{k=1}^m A_k \cdot \cos(2\pi f_k nTs) e^{-\sigma_k nTs} \quad (4.2)$$

Tab. 4.2 Characteristics of synthesized test signal 2

Spectral component	Frequency (Hz)	Magnitude	Damping factors (s <sup>-1</sup> )
1	50	0.9	0.1
2	100	0.09	5.4
3	150	0.25	0.2
4	250	0.15	0.5
5	300	0.11	0.2
6	350	0.07	0.41

The temporal and spectral representation of the estimated damped signal is given respectively by figures 4.3 and 4.4.

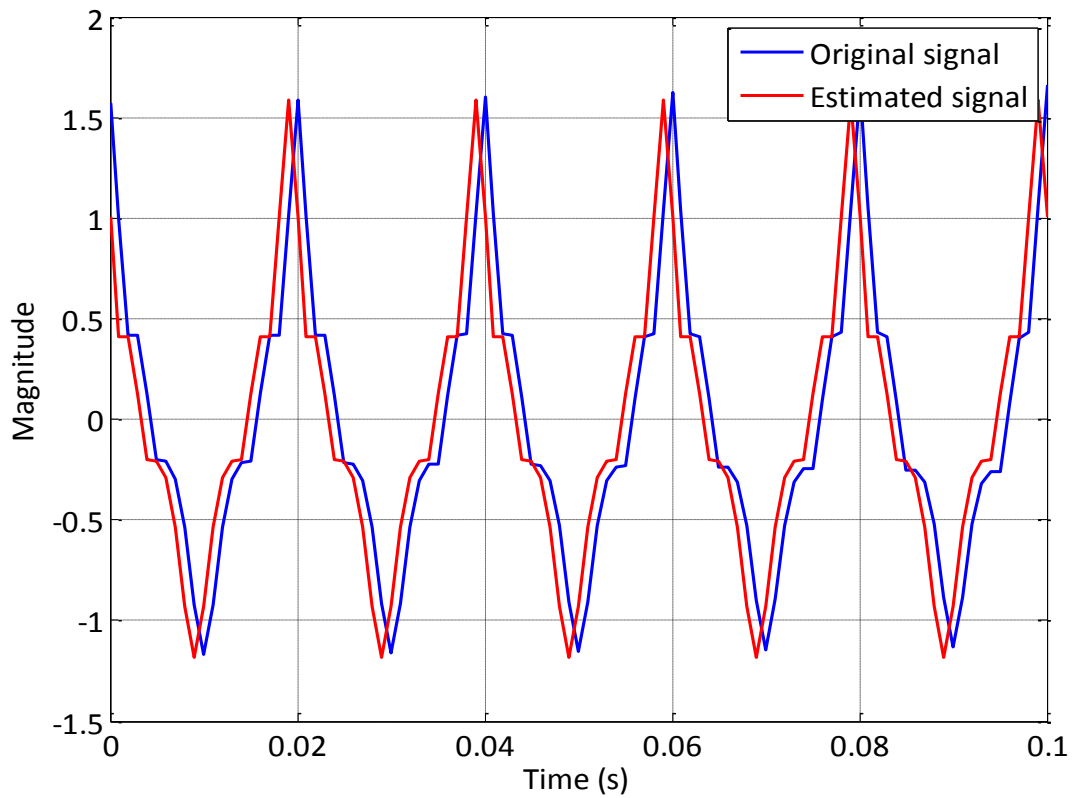


Fig. 4.3 Reconstruction of a damped signal using Prony Method

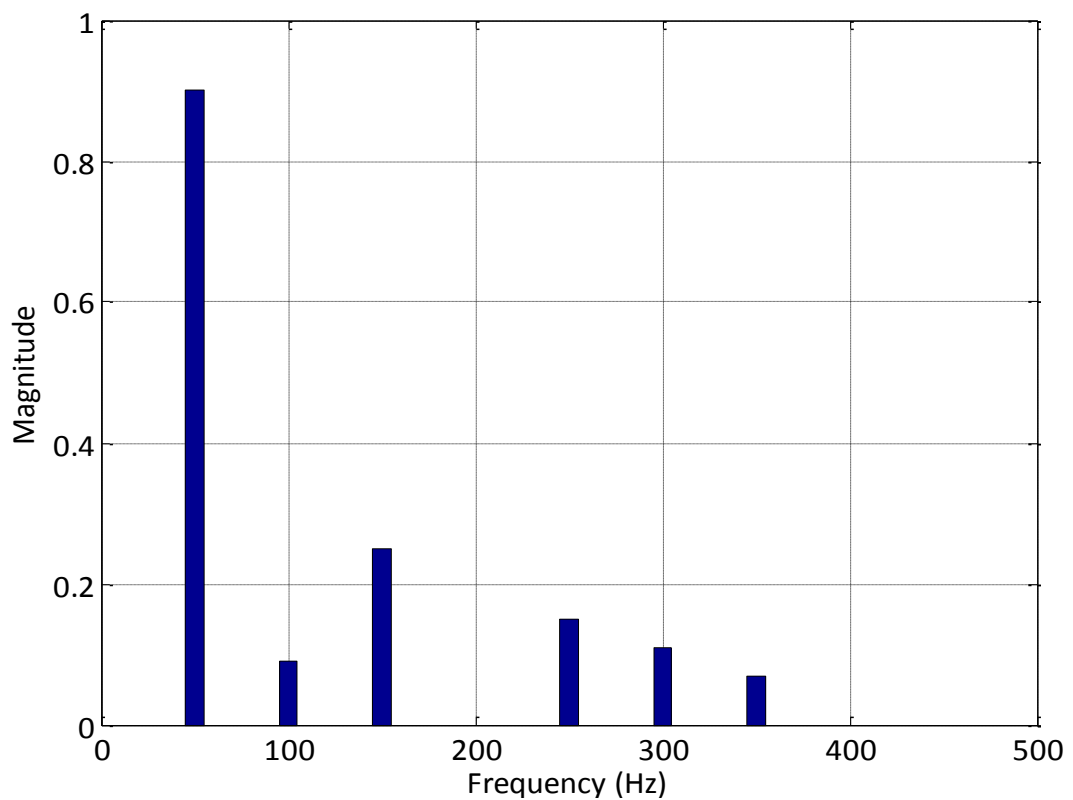


Fig. 4.4 Spectrum of the damped signal

The error in this case is equal to 0.1196. The small deviation between the errors in these cases and the changes appeared at the end of window (between 0.09 and 0.1 s) in the



second case poses a hypothesis that the length of the window affects the performance of Prony method.

#### IV. 2. 2. Influence of the length of measurement window

To prove or disprove the hypothesis mentioned in the section below, a new simulation of the cases above is given with the same model order  $p=12$ , the sampling time of 1 ms, but with new measurement window where its length is 1 second.

Results affirm clearly that the length of measurement window -or the number of data points- used in the identification of spectral components using Prony Method decreases the accuracy of this method in the case of damped signal only, by increasing the amount of error from 0.11 to 18.36. In contrast of the other case -undamped signal-, where the error stays constant (0.11 in average).

The two following figures show the difference between the original and estimated test signals.

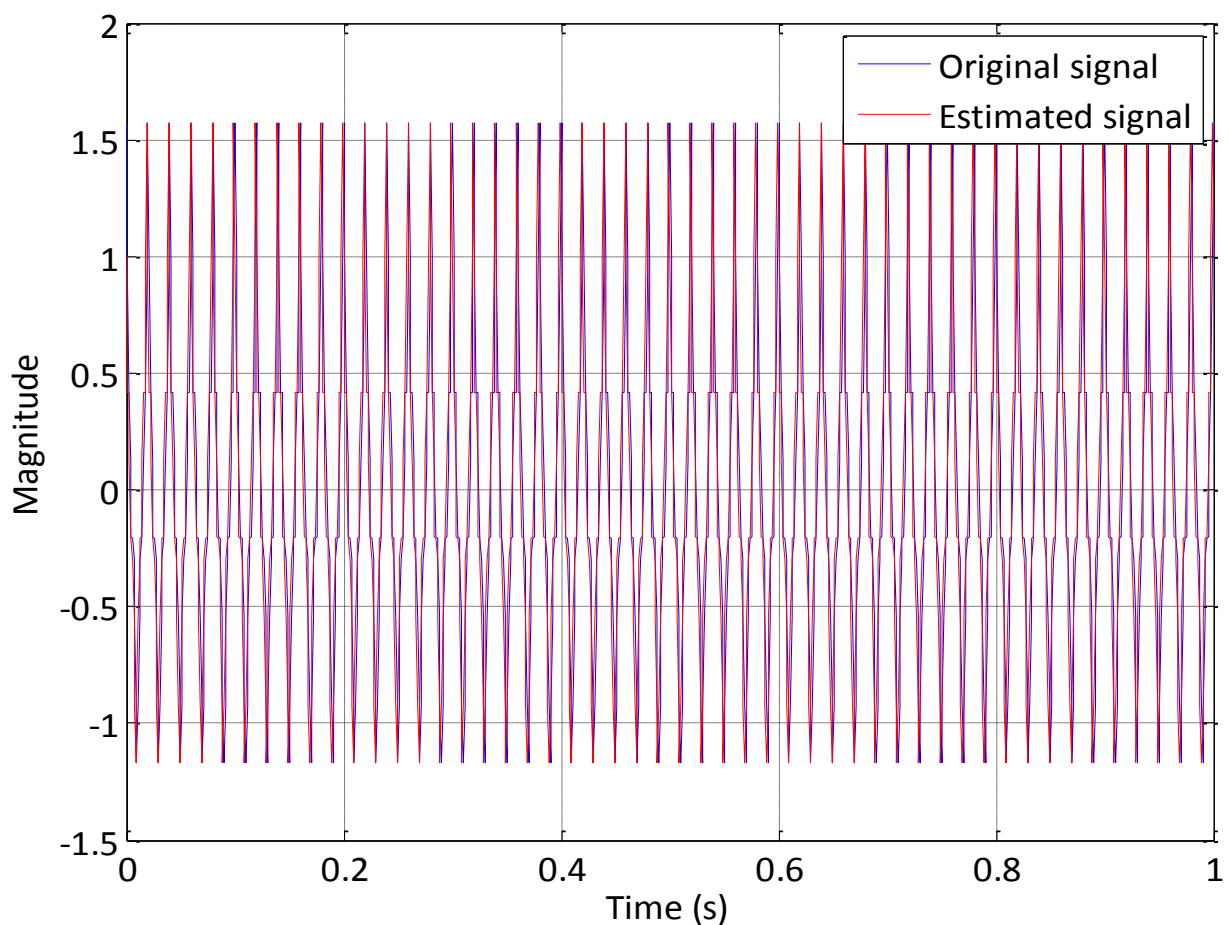
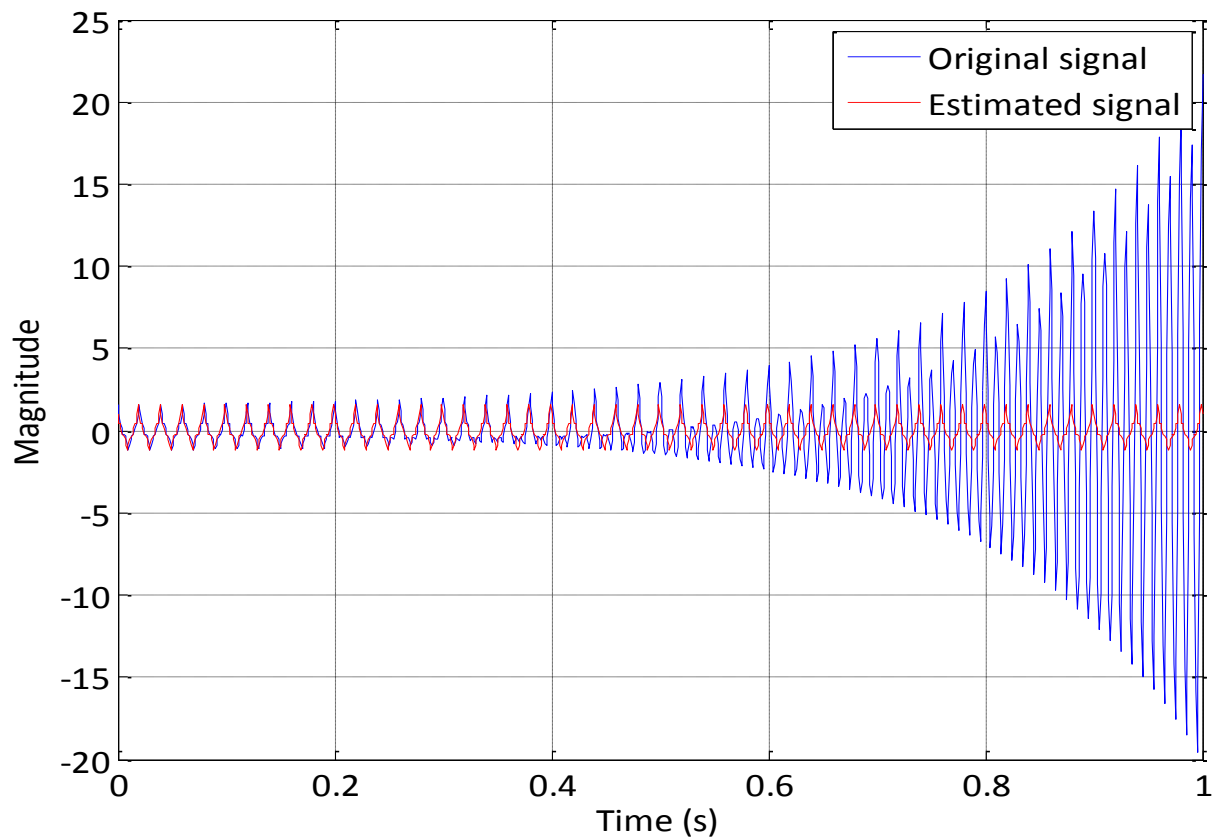


Fig. 4.5 original and estimated signals for non-attenuated signal



**Fig. 4.6 original and estimated signals for attenuated signal**

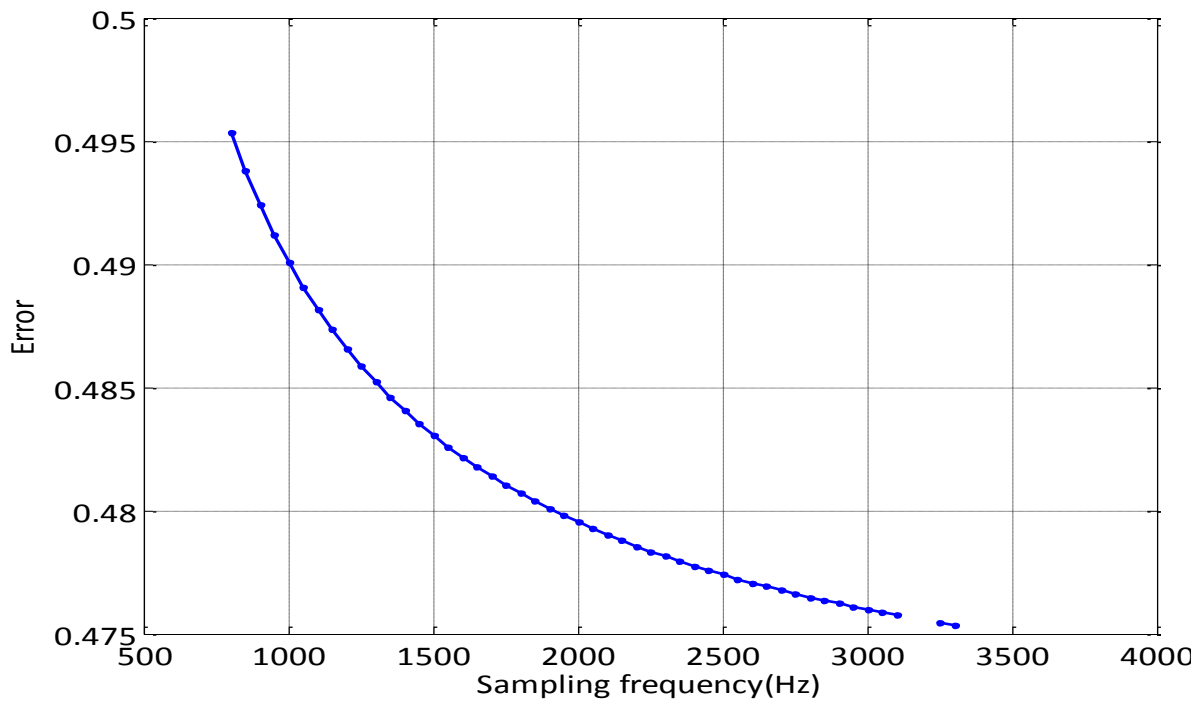
Note that this difference appears only in the temporal reconstruction of the signal after the estimation of its components. The spectral representation stays fixed and accurate in all cases.

The first step in adjustment of parameters indicates that few cycles of signal contains a reasonable numbers of data, which leads to an accurate estimation in both time and frequency domains.

In the following sections, 4 or 5 cycles are sufficient for running the program and extracting the modal components.

#### **IV. 2. 3. Influence of selecting of sampling rate**

In order to visualize the influence of sampling frequency on the accuracy and the effectiveness of this method in modal components identification, we present the curve of the error referred to equation (3.5) in terms of sampling frequency which varies between 500 Hz and 4 KHz with a step of 50 Hz. The obtained results are shown in Figure 4.7.



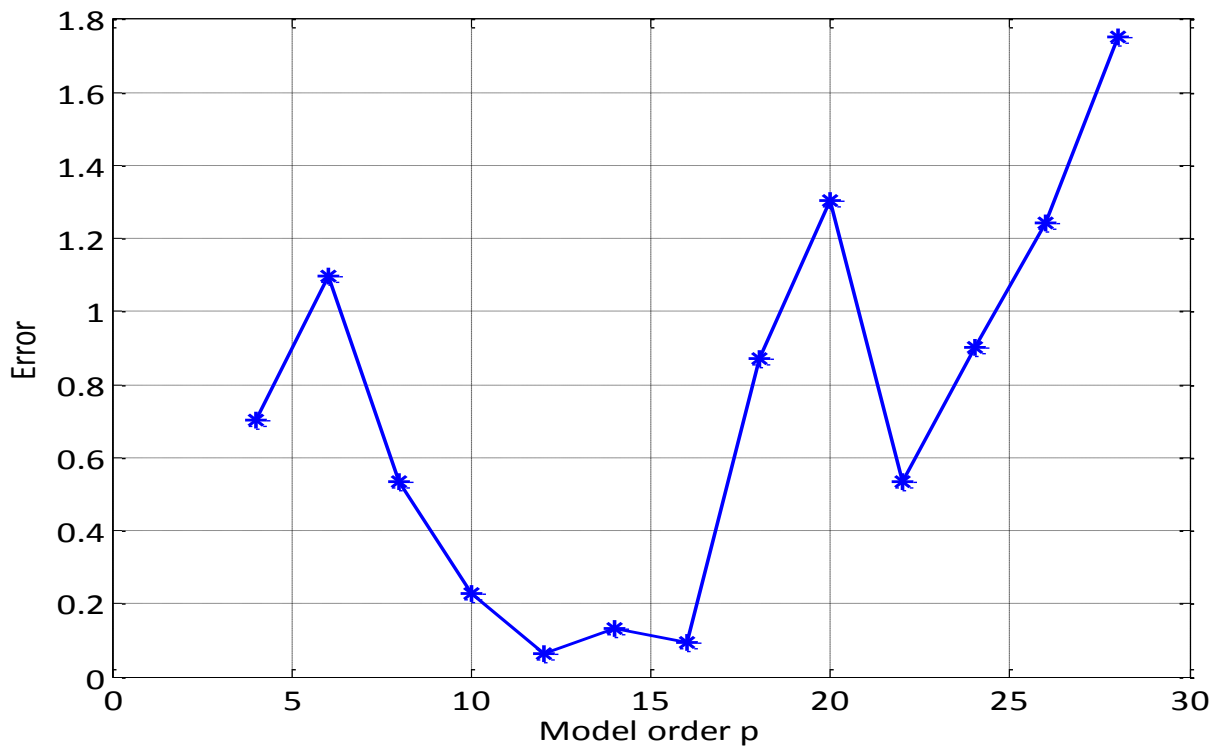
**Fig. 4.7 The influence of selection the sampling frequency for harmonic estimation**

The calculation of the error in term of sampling frequency for the test signal 2 does not start from the lower bound of the chosen interval but it begins to be estimated just from 800 Hz satisfying the Shannon's theorem.

When increasing the sampling rate, the error decreases but in a reasonable amount (difference of 0.02). Also, too high sampling frequency leads to inaccurate simulation, the spectrum becomes just a smeared version of the real one and the values of error will be invalids as the case of our test signal where the simulation stops in 3,7 KHz.

#### **IV. 2. 4. Influence of selection of model order**

Theoretical criterion indicates that the model order must be at least twice of the number of spectral components presented in the concerned signal. To confirm this lower limit of this parameter selection, we fix at first the sampling frequency at 3 KHz as being the best sampling rate in the section above. Then, we try to estimate our test signal with  $p$  varying from 4 to 28.



**Fig. 4.8 The influence of selection of the model order for harmonic estimation**

The figures 4.1 and 4.2 show us that both of temporal and spectral estimation are optimal when the model order is  $p=12$ . However, when increasing the model order, the spectral estimation remains always accurate because the new components have magnitudes with rang of  $10^{-14}$  and the temporal reconstruction becomes imprecise as shown in figure 4.9.

In addition, the realization of a filter with  $p$  order limits the use of high values during selections of Prony's model order because of its complexity and the high cost of fabrication. That is why the model order must be as low as possible; (the algorithm does not work well for higher-order models [1]).

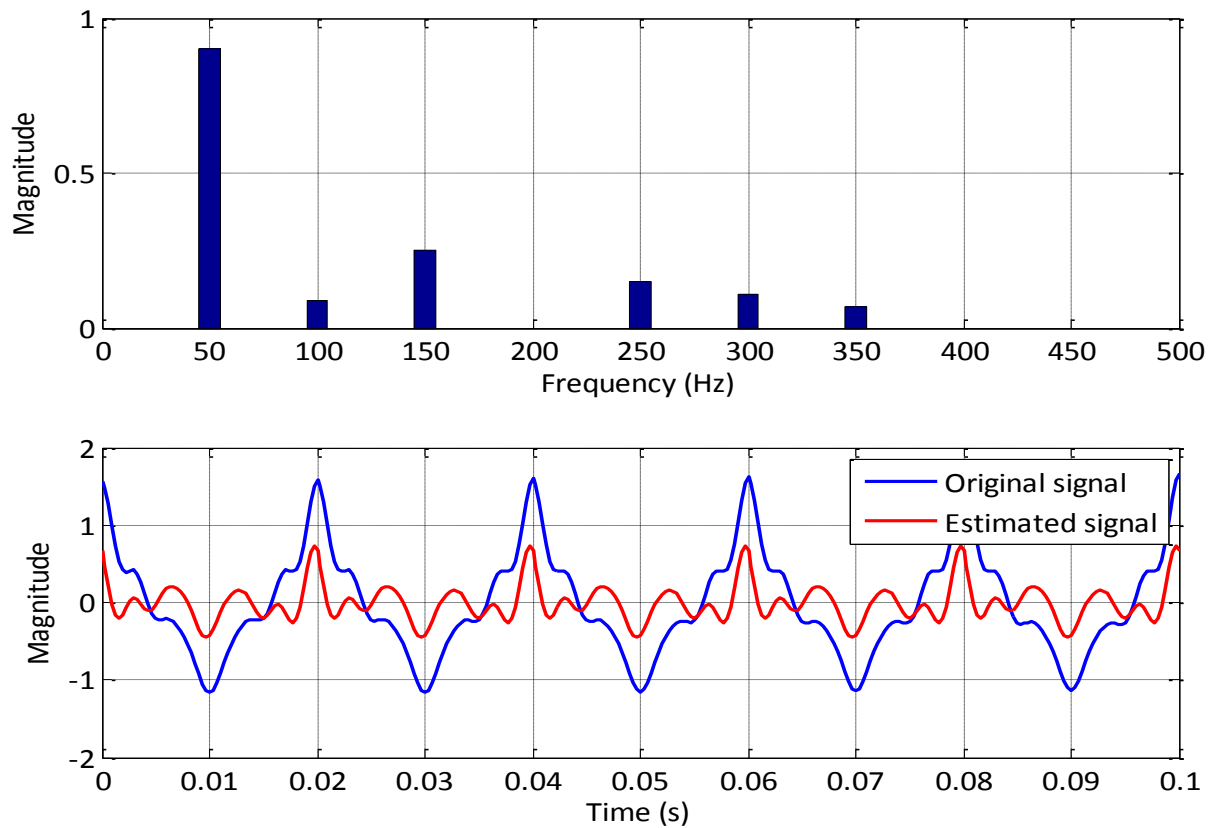


Fig. 4.9 Spectral and Temporal estimation of the test signal with  $p=20$ ,  $F_s=3$  KHz

### IV. 3. Interharmonic identification using Prony Method

In this part, we try to repeat our analysis of spectral components identification with Prony Method but for a test signal which contains interharmonics.

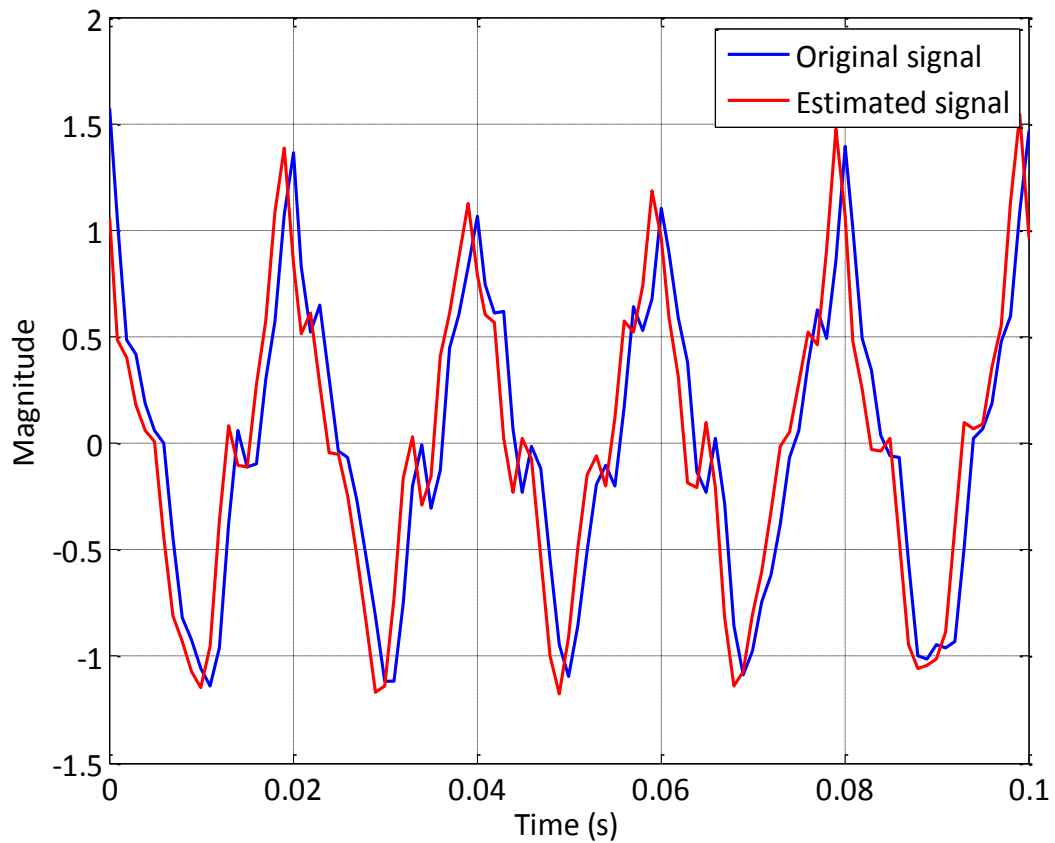
#### IV. 3. 1. Validation of theoretical criteria

##### IV. 3. 1. 1. Construction of test signal 3

The table below contains the characteristics of the elementary components that their sum composes our test signal for the following part of study.

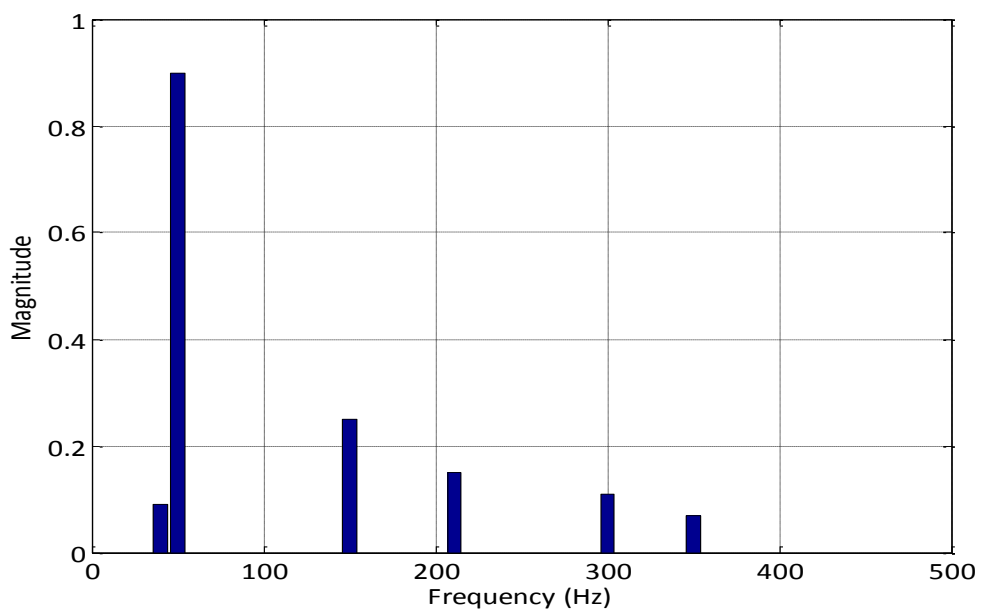
Tab 4.3 the characteristics of the test signal 3

Spectral component	Frequency (Hz)	Magnitude	Damping Factor ( $s^{-1}$ )	Phase (rad)
1	40	0.09	2	0
2	50	0.9	0.01	0
3	150	0.25	-5.4	0
4	211	0.15	0.2	$\pi/4$
5	300	0.11	0.5	-0.22
6	350	0.07	0.2	0



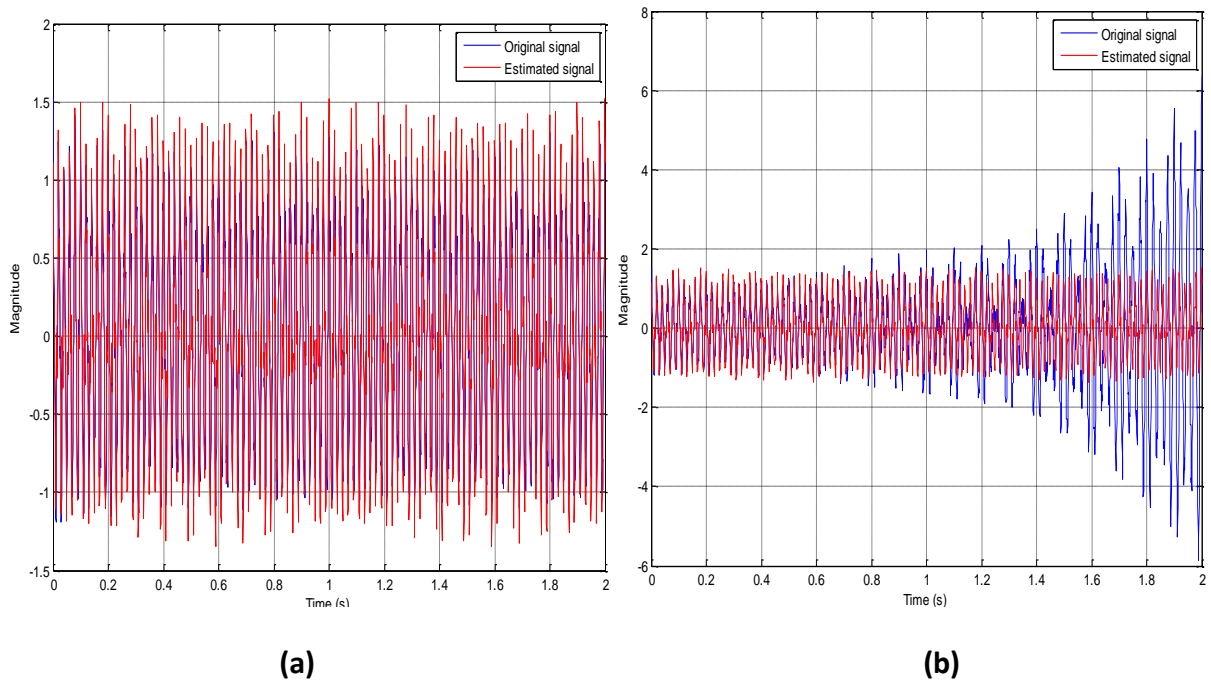
**Fig. 4.10 Reconstruction of a damped signal using Prony Method**

The error in this case is equal to 0.0629. The small deviation between the errors in these cases and the changes appeared at the end of window (between 0.09 and 0.1 s) in the second case poses a hypothesis that the length of the window affects the performance of Prony method.



**Fig. 4.11 Spectrum of the damped signal**

To prove or disprove this hypothesis, a new simulation of the cases above is given with the same model order  $p=12$ , the sampling time of 1 ms, but with new measurement window where its length is 2 seconds.



**Fig. 4.12 original and estimated signals for (a) non attenuated signal (b) attenuated signal**

Note that the error stay small for the first case ( $e_r=0.069$ ) and it increases in the second one ( $e_r=1.5006$ ).

The first step in adjustment of parameters indicates that few cycles of signal contains a reasonable numbers of data, which leads to an accurate estimation in both time and frequency domains.

In the following sections, 4 or 5 cycles are sufficient for running the program and extracting the modal components.

#### **IV. 3. 2. Influence of selection of sampling rate**

In order to visualize the influence of sampling frequency on the accuracy and the effectiveness of this method in modal components identification, we present the curve of the error referred to equation (3.5) in terms of sampling frequency which varies between 750 Hz and 2,5 KHz. The obtained results are shown in Figure 4.6.

Based on the first signal case studied in section 2.1, it is obvious that the optimal value of sampling rate is 1500 Hz with an error of 0,052.

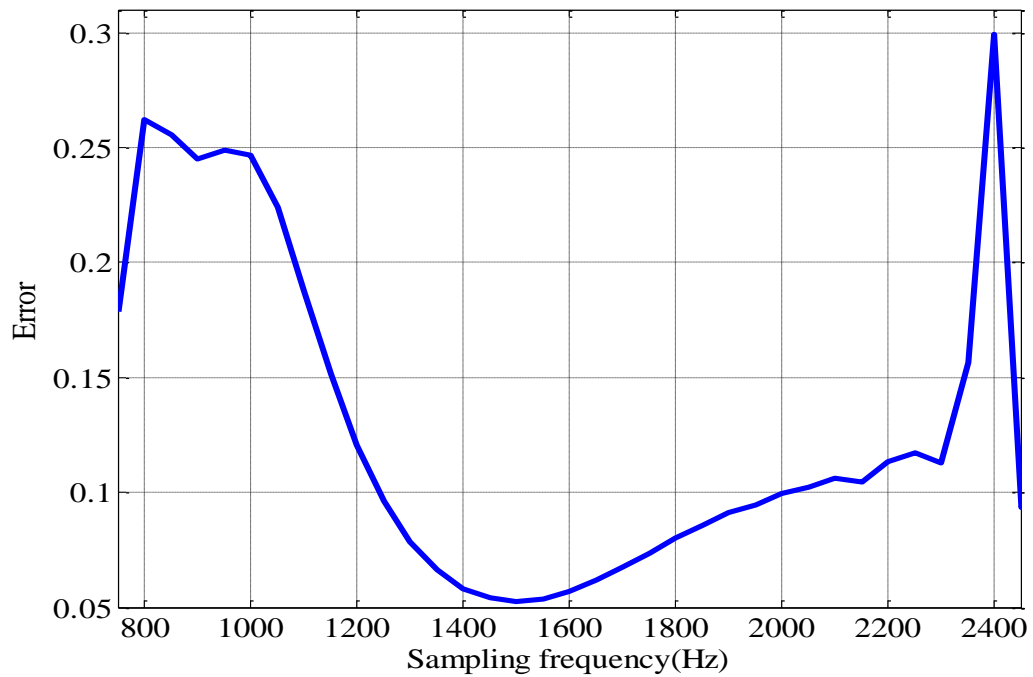


Fig. 4.13 The influence of sampling frequency

#### IV. 3. 3. Influence of selection of model order

Theoretical criterion indicates that the model order must be at least twice of the number of spectral components presented in the concerned signal. To confirm this lower limit of this parameter selection, we fix at first the sampling frequency at 1,5 KHz. Then, we try to estimate our test signal with  $p$  varying from 4 to 28.

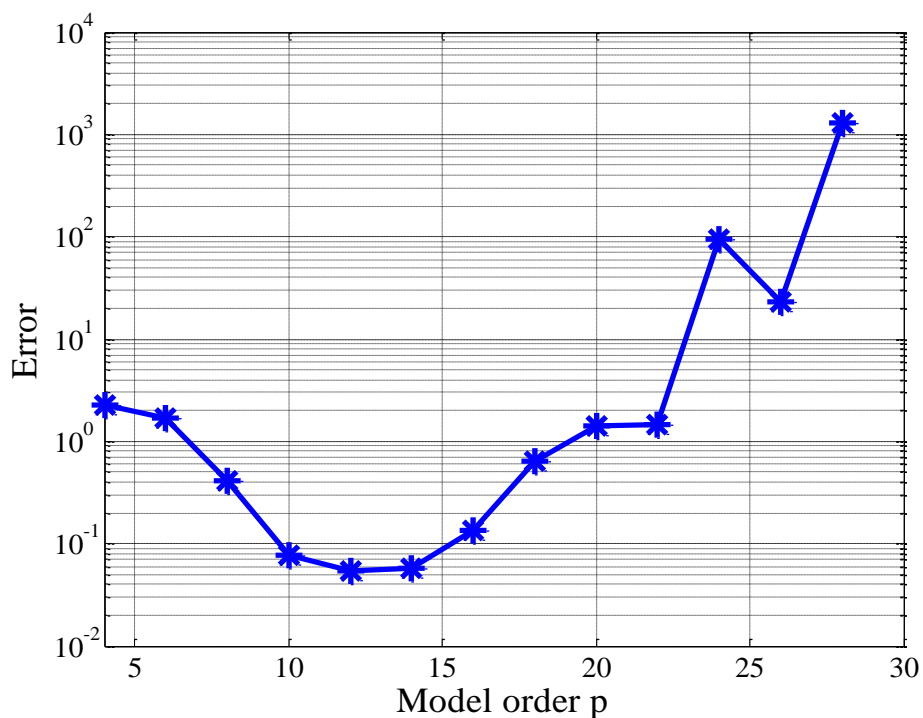


Fig. 4.14 The influence of selection of the model order



The figures 4.1 and 4.2 show us that both of temporal and spectral estimation are optimal when the model order is  $p=12$ . However, when increasing the model order, the spectral estimation remains always accurate because the new components have magnitudes with rang of  $10^{-14}$  and the temporal reconstruction becomes imprecise as shown in figure 4.9.

In addition, the realization of a filter with  $p$  order limits the use of high values during selections of Prony's model order because of its complexity and the high cost of fabrication. That is why the model order must be as low as possible; (the algorithm does not work well for higher-order models [1]).

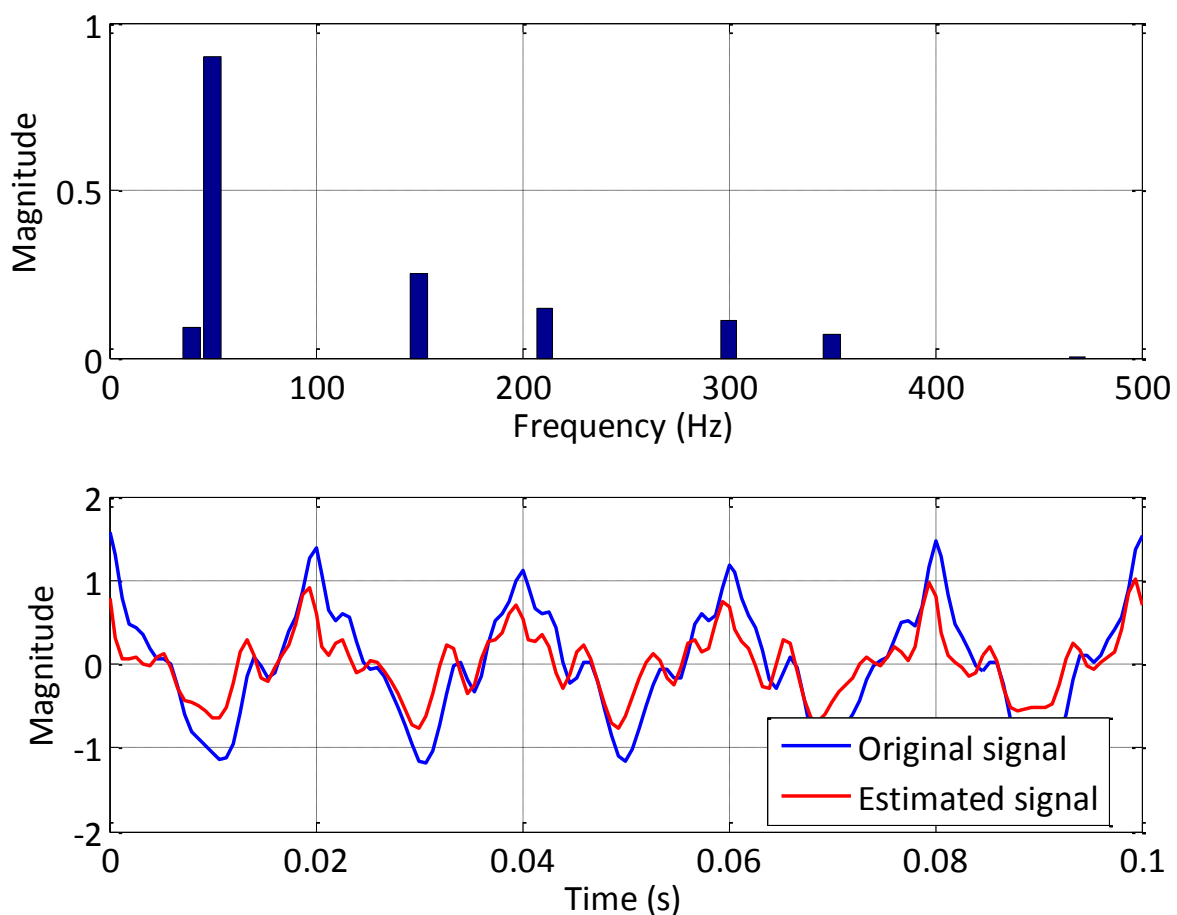


Fig. 4.15 Spectral and Temporal estimation of the test signal with  $p=20$ ,  $F_s=1.5\text{KHz}$

#### IV. 4. Real case study for harmonics and interharmonics identification with Prony Method

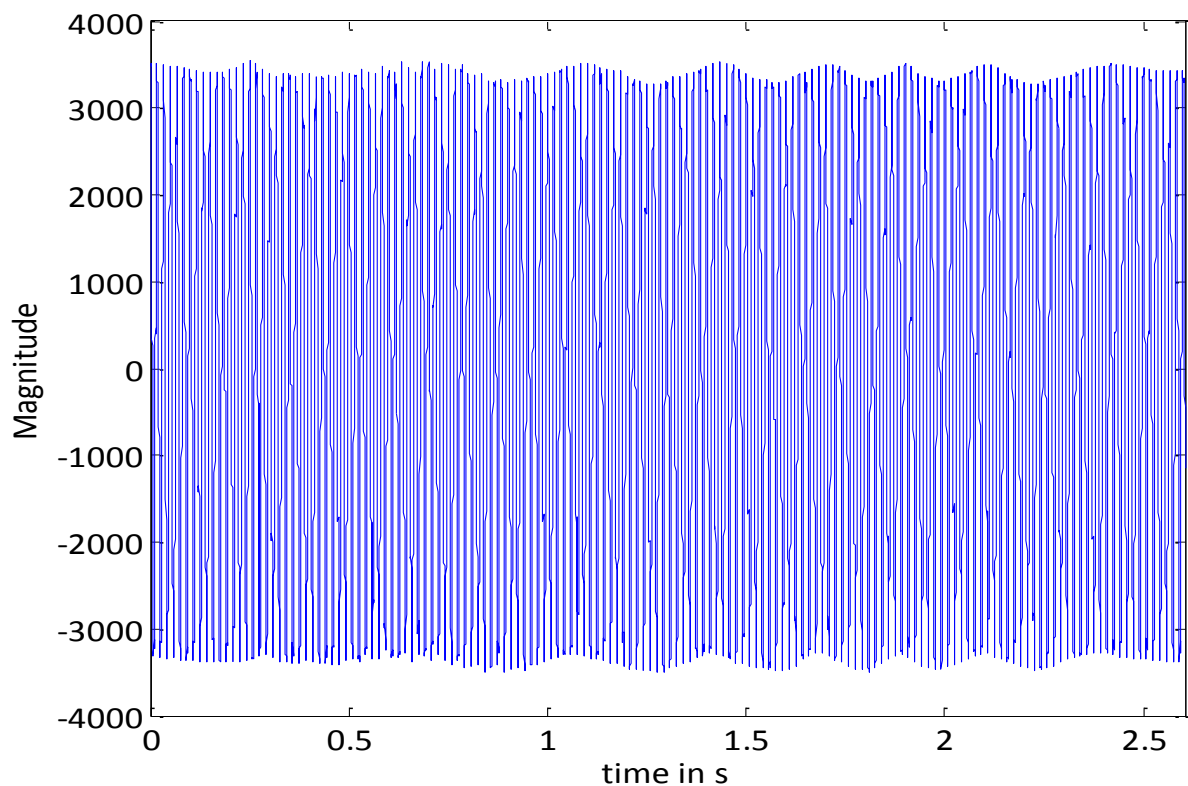
For validate the theoretical results obtained in the sections above, a real data case is studied in this part in order to prove the ability of application of Prony method in power

systems as well as to visualize the practical problems that met the implementation of this parametric method in real systems.

#### IV. 4. 1. Presentation of the test signal 4

The test signal 4 is uploaded from a real machine. It allows consequently to examine the ability of application as well as the efficiency of Prony method to real signals.

- Machine name: Motor Load.
- Trigger time: 13:18:45.037934 Tue 19 Mar 2002 (078).
- Sample rate: 11520 Hz.
- $S_{\text{time}} = 0.000087$  s



**Fig. 4.16 Temporal representation of the test signal 3**

The first use of these data leads to the following results. Note that five cycles are selected in the measurement window and for more visibility.

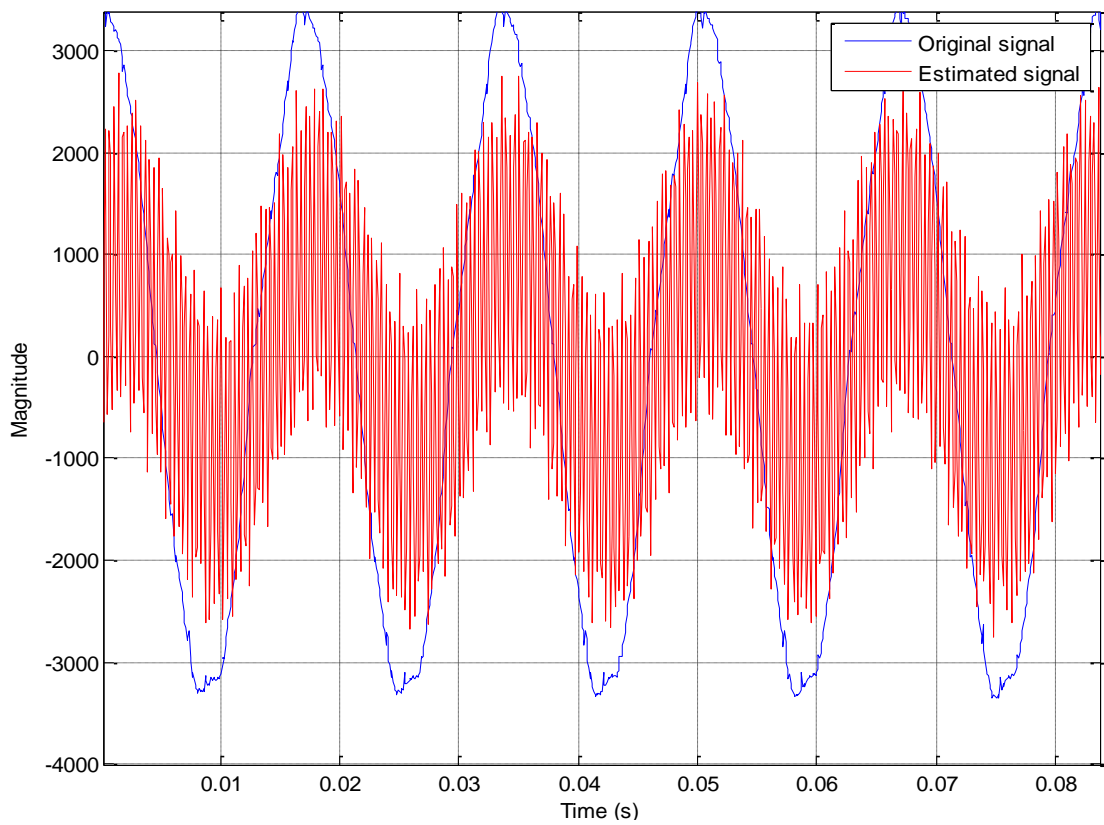


Fig. 4.17 Original and estimated test signal 4 using Prony Method

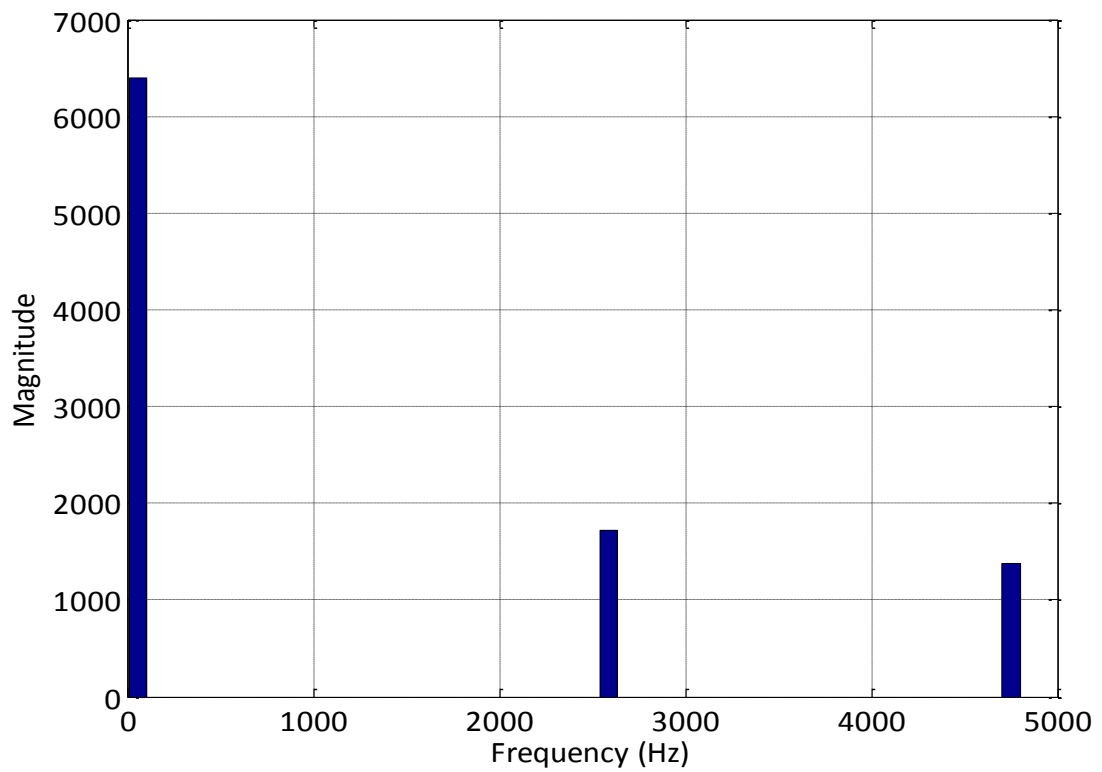


Fig. 4.18 Spectrum of the estimated signal 4

The primary interpretation of this result shows that this signal has only one spectral component at 60 Hz; the two other frequencies appeared in the figure are of very high frequency rang which can be caused by the high sampling rate used in this simulation.

Therefore, the estimation in this case is inaccurate and it does not give any useful information. To solve the problem of the high sampling rate, we use the resampling mode of the test signal 4.

Suppose that the resampled signal  $y_r[n]$  is sampled with  $S'_{\text{time}}=S_{\text{time}}\times 8$ . We obtain finally the following estimation:

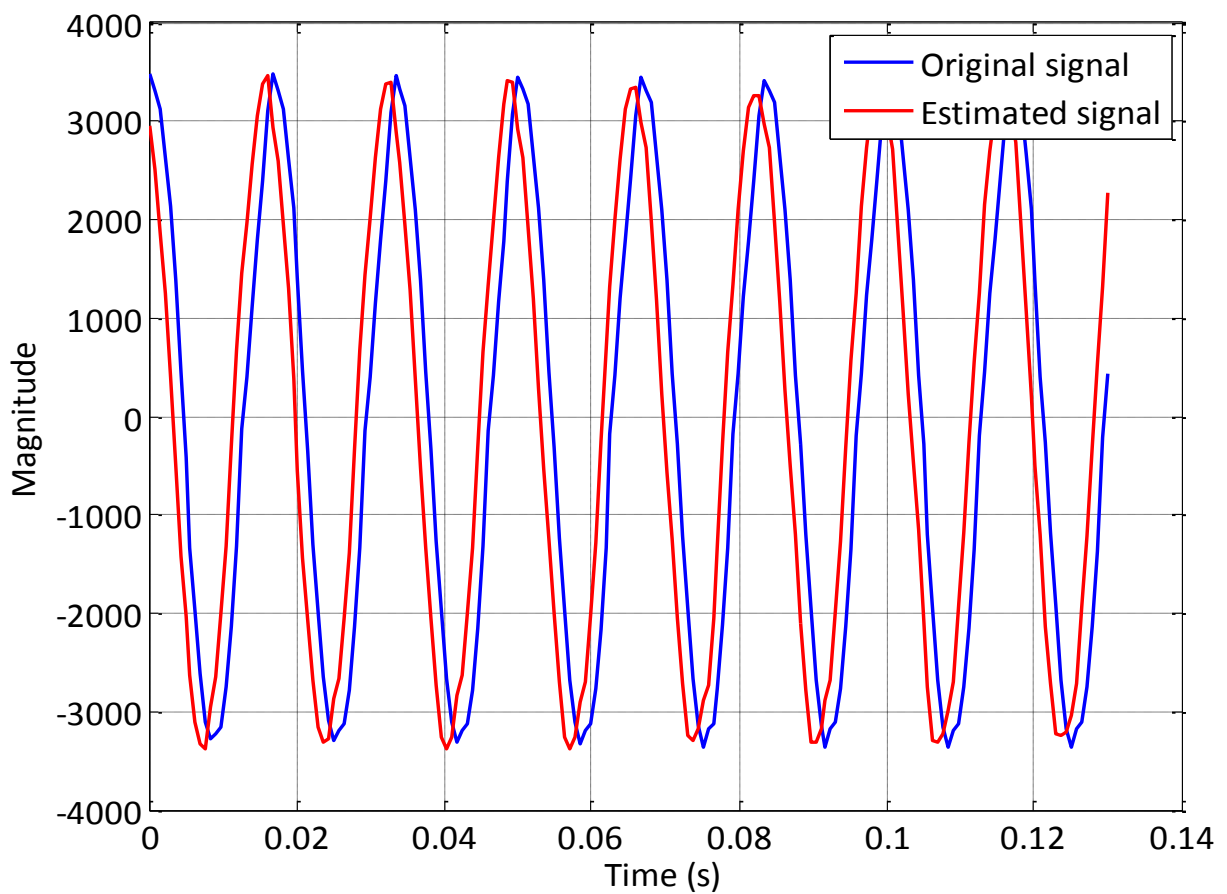
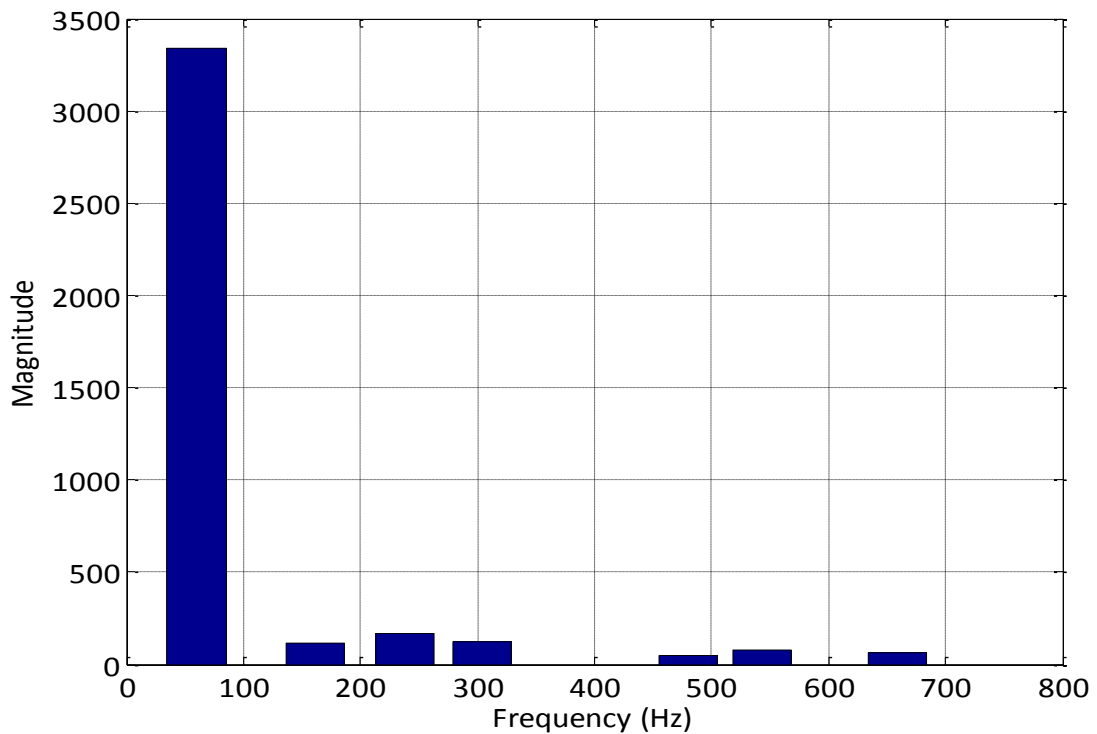


Fig. 4.19 Estimation of the resampled signal test wit  $S'_{\text{time}}=S_{\text{time}}\times 8$ ,  $p=14$ ,  $N=188$ .



**Fig. 4.20 Spectrum of the resampled test signal 4**

Selecting a small measurement window and resampling the signal to solve the problem of sampling rate gives us an accurate detection of the spectral components contained in our test signal.

#### **IV. 5. Comparison between 3 signal processing technique for harmonics and interharmonics identification**

##### **IV. 5. 1. Prony and Least Error Squares methods**

Based on the obtained results, it can be said that the proposed method in this paper with the selected aforementioned parameters, is very reliable in estimating the even and odd harmonic components and the Inter-harmonics components with a high accuracy. On the other side, the proposed method has better advantages in comparison to the Least Error Squares Technique that can ensure only the detection of the harmonics components with an acceptable precision, but it fails in the detection of the inter-harmonics as it is detailed in [2].

Figure 4.20 justifies the limited accuracy of LES technique and the good performance of Prony's method [3]

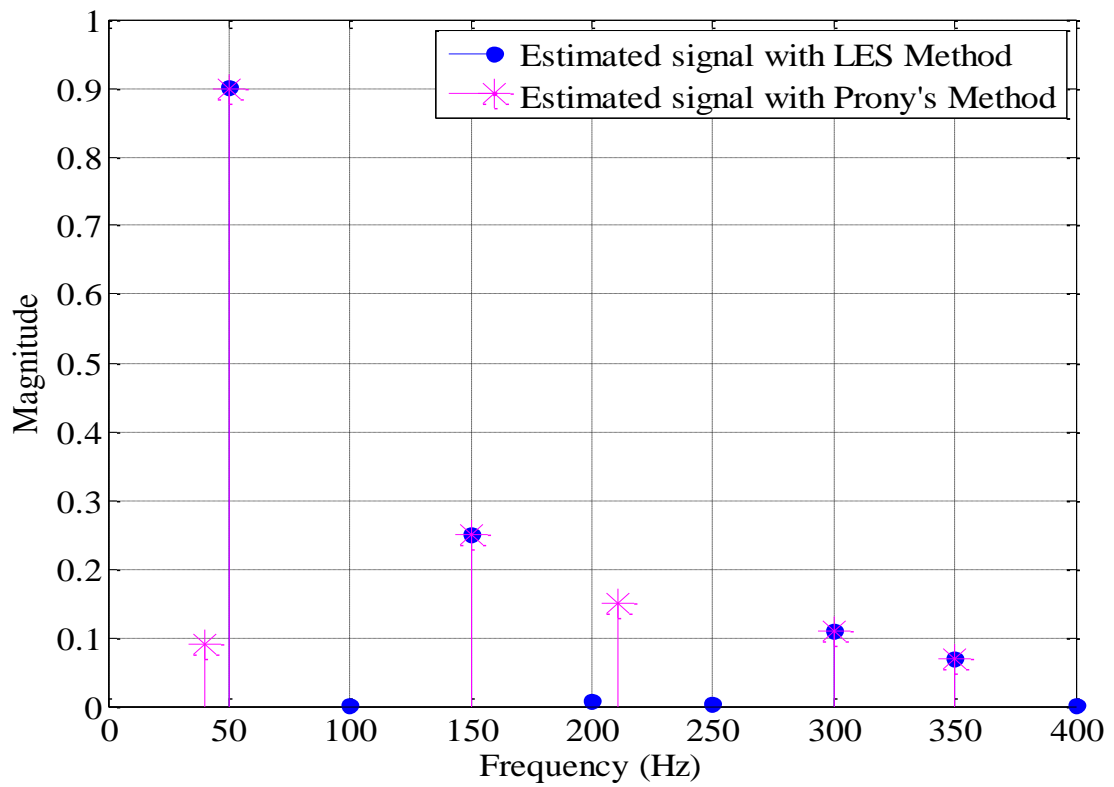


Fig. 4.21 Comparison between LES Method and Prony's Method for the spectral estimation of the test signal 4

#### IV. 5. 2. Prony and FFT

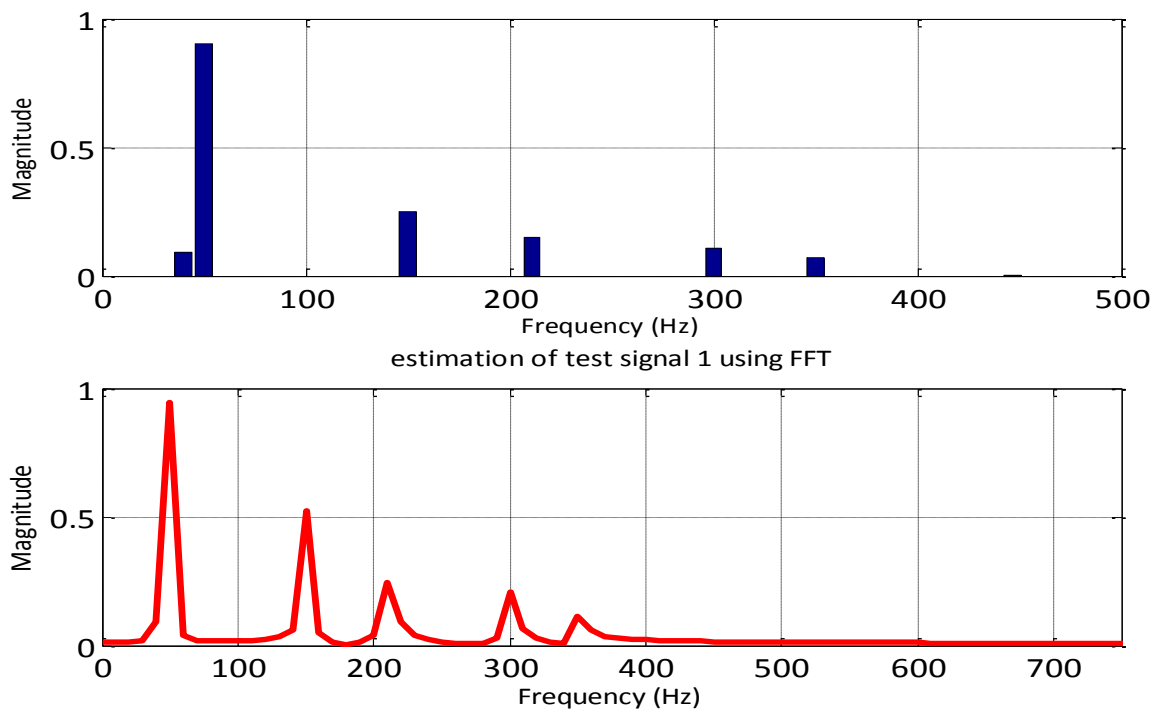


Fig. 4.22 Prony Method vs. FFT for the spectral estimation of the test signal 4

## **IV. 6. Conclusion**

In this chapter, we have investigated the method of Prony for harmonics and interharmonics identification.

The first part of this chapter focuses on the setting of the parameter of the model, this setting open a window of study about the influence of each parameter of them.

After getting set the parameters of the model, the application of Prony method in real case demonstrate its ability to be used widely in power system field.

Validation of results is given by comparing our investigated method with FFT and LES method in order to prove its effectiveness.

## IV. 7. References

- [1] R.Rabehi, A. Kouzou, and all, “harmonics and inter-harmonics estimation of voltage/current based on Least Error Squares Algorithm”, 15th International Multi-coference on System, Signals and Devices SSD, 19-22th March 2018, Tunisia.
- [2] P. F. Ribeiro and all, *Power systems signal processing for smart grids*, WILEY, 2014.
- [3] Rabehi R., Kouzou A., Saadi S.,Hafaifa A., “Parameter selection criteria of Prony method for accurate harmonics and inter-harmonic component identification”, in *Electrotehnica, Electronica, Automatica (EEA)*, 2019, vol. 67, no. 1, pp. 46-53, ISSN 1582-5175.



## Conclusion

Among several problems affecting the power system, harmonics and interharmonics is considered one of the most harmful of them. It can mainly reshape the current/voltage waveform and consequently lose the useful information beside other crucial problems produced in all parts of the power grid.

These reasons yields to raise the interest of the power researchers and engineers about this subject in lest four decades in order to ensure a pseudo-perfect sinusoidal supply for consumers.

For a precise mitigation of these spectral components from the power supply, selective filters must be released. Moreover, for producing such type of filters, an accurate technique of identification must be selected. Harmonic and interharmonic identification based on signal processing techniques presents a great amount of precision and good performance for use in term of accuracy and low cost.

We try in this work to give an investigation to harmonic identification based on signal processing techniques. We start by a global analysis of harmonics in power systems: their sources, their harmful impacts in all parts of the power network and the possible ways to mitigate them.

Then, we resume some advanced signal processing techniques for harmonic and interharmonic identification, especially those that are the most utilized in power applications. We find that the criteria of selection of the appropriate technique among them for such application are not arbitrary. It is also proved that each one of them has some advantages and limits of use, and it is demonstrated that the parametric methods have a good performance in comparison of those based of Fourier aspects despite their high cost of implementation. The fertility of this type of techniques concentrates our investigation into a deep theoretical and experimental study of Prony method in the third chapter of our thesis.

The chapter 3 is devoted to a general investigation of Prony method for harmonics and interharmonics identification. It presents a good performance and it has a high ability to be used of power application as being a parametric method for extracting spectral component, which make it needless for any prior knowledge for some information before using.

This independence of the fundamental frequency solves the problem of the fix window of detection and makes this method able to estimate even the interharmonics and sub harmonics with a significant precision.

Finally, we finish this investigation by experimental simulation using MATLAB. We apply this method on three test-signals: the first in an undamped sinusoid signal, it is used to starting with the adjustment of model parameters. Then, the second one is damped sinusoid signal used for accomplish the setting and visualize the influence of each model parameter on the performance of estimation. The last test signal is a real case uploaded from a motor machine, which allows proving the ability of application of this parametric method on real power application.

This investigation opens a big search axe concerning the optimization of Prony method for more accuracy, and for its use in real time estimation.

## Perspectives

As this work has some limitation

- The parameters of Prony Method for harmonics and interharmonics identification can be selected using laws of mathematical optimization instead of the arbitrary choice or the choice after several tries ,
- The use of this mathematical method in real time for harmonic and interharmonic identification,
- The use of Prony Method for sliding windows to solve its limitation of use in the case of non-stationary signals.

## List of Scientific Productions

### **PAPER:**

Rabehi R., Kouzou A., Saadi S., Hafaifa A., "Parameter selection criteria of Prony method for accurate harmonics and inter-harmonic component identification", in *Electrotehnica, Electronica, Automatica (EEA)*, 2019, vol. 67, no. 1, pp. 46-53, ISSN 1582-5175.

### **INTERNATIONAL CONFERENCES:**

[1]. Ratiba Rabehi, Sarah Abdourraziq and Rachid El Bachtiri, "A variable Conductance MPPT Implementation for PV Pumping Systems". The 1st International Conference on Power Electronics and their Applications ICPEA, Djelfa on 6-7 November 2013, Djelfa, Algeria.

URL: <http://www.univ-djelfa.dz/icpea>

[2]. Ratiba Rabehi, Kouzou Abdellah, Saadi Slami and Ahmed Hafaifa, "Harmonics and sub-harmonics estimation based on Least Error Squares technique". The 3rd International Conference on Power Electronics and their Applications ICPEA2017, Djelfa on 16-17 September 2017, Djelfa, Algeria.

URL: <http://www.univ-djelfa.dz/icpea17>

[3]. Ratiba Rabehi, Kouzou Abdellah, Saadi Slami and Ahmed Hafaifa, "Harmonics and inter-harmonics estimation of voltage/current based on Least Error Squares Algorithm", 15th International Multi-Conference on systems, signals and devices SSD18, 19-22 March 2018, Hammamet, Tunisia.

## ملخص:

يمثل تحديد التوافقيات والتوافقيات الجزئية في جميع مستويات أنظمة الطاقة موضوعا بالغ الأهمية لمختصيه في جميع أنحاء العالم ، لا سيما فيما يتعلق بجودة الطاقة الكهربائية لأن تحقيقها في نظام الطاقة أمر مهم ونقصانها يضر بالمعدات الحساسة في شبكة الكهرباء.

وبالتالي، بات من الضروري تحليل إشارة الطاقة وتحديد التوافقيات والتوافقيات الجزئية لضمان إشارة نظيفة للتشغيل الآمن للمعدات المتصلة بالشبكة. الهدف الرئيسي من هذه الأطروحة هو استخدام التقنيات المتقدمة في معالجة الإشارة لتحديد التوافقيات والتوافقيات الجزئية في إشارات أنظمة القدرة.

**كلمات مفتاحية:** التوافقيات، التوافقيات الجزئية، تقنيات معالجة الإشارة، طريقة بروني.

## Résumé :

L'identification des harmoniques et de l'interharmonique à tous les niveaux des systèmes électriques représente une préoccupation persistante pour ces spécialistes, en particulier en ce qui concerne la qualité de l'énergie électrique, car l'existence des harmoniques et de l'interharmonique dans les systèmes électriques est inévitable et nuisible pour les équipements sensibles du réseau électrique.

Il est donc important d'analyser le signal de puissance et d'identifier les différentes harmoniques et interharmoniques afin d'assurer un signal d'alimentation propre pour un bon fonctionnement de l'équipement connecté au réseau. L'objectif principal de la présente thèse est l'utilisation des techniques avancées pour l'identification des harmoniques et des interharmoniques dans les signaux des systèmes de puissance.

**Mots Clés :** Harmoniques, Interharmoniques, techniques de traitement du signal, La méthode de Prony.

## Abstract:

Identification of harmonic and inter-harmonics in all levels of power systems has been representing a permanent concern of professionals all over the globe, especially for electrical power quality concerns because the existence of harmonics and interharmonics in power system is unavoidable and harmful to the sensitive equipment in the power network.

Hence, It is primordial to analyze the power signal and identify different harmonics and inter-harmonics to ensure a clean signal for safe operation of the grid connected equipment. The main aim of the present thesis is the use of the advanced techniques for the identification of harmonics and inter-harmonics in power systems signals.

**Key Words :** harmonics, interharmonics, signal processing techniques, Prony method.