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Abstract

This thesis provides a safe communication system based on optimal sliding mode step-by-step observers and fractional-order chaotic Chua's systems. On the emitter side, a fractional-order chaotic system is used as the drive system to create the encrypted message signal. The input secret message is modulated in the chaotic dynamics by insertion rather than being directly inserted into the chaotic signal on the transmission line. On the receiver side, a step-by-step fractional-order chaotic observer subject to unidentified input is suggested as the response system to obtain robust synchronization between the emitter and the receiver. The parameters of the sliding mode observer have been selected optimally using two optimization algorithms: the well-known intelligent optimization Grey-Wolf Optimizer (GWO) and the artificial hummingbird algorithm (AHA). We then made a comparison between both of them in order to achieve the best parameters. After the chaos, synchronization is attained. On the receiver side, the state variable estimation is successfully obtained, which leads to the precise reconstruction of the confidential information containing a signal, plain text, and voice message.

Dedication and acknowledgements

I worked hard on this project. However, it would not have been possible without the generous support and assistance of many people. I'd want to express my heartfelt gratitude to each and every one of them. I am grateful to Dr. Nail Bachir for his advice and regular monitoring, as well as for supplying important project information. I would like to extend my appreciation to Pr.Saadi Slami for his assistance and inspiration in completing this project. My gratitude and appreciation also go to my parents and wife, who have supported and assisted me in many tasks, as well as everyone else who have freely assisted me with their skills.

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General introduction

Chaotic systems are dynamic systems that exhibit complex and unpredictable behavior, even though they are governed by deterministic equations. These systems have been extensively studied in various fields of science and engineering, including physics, biology, economics, and engineering. The concept of chaos was first introduced in the 1960s by the mathematicians Edward Lorenz and Benoit Mandelbrot. Since then, chaotic systems have been studied extensively, and their applications have been explored in various fields, including physics, chemistry, biology, engineering, and economics. One of the key features of chaotic systems is their sensitivity to initial conditions, known as the butterfly effect. This means that even small differences in the initial conditions of the system can lead to significantly different outcomes. This sensitivity to initial conditions makes it difficult to predict the long-term behaviour of chaotic systems. Furthermore, chaotic systems often exhibit a periodic and irregular behaviour, making them difficult to analyze using traditional mathematical techniques. However, chaotic systems can be studied using methods from nonlinear dynamics, which have been developed specifically for this purpose. Some notable works on chaotic systems include: [7, 44, 72, 88, 107]. Chaotic systems continue to be an active area of research in science and engineering, with new applications and insights being discovered regularly. Their unpredictable and complex behavior makes them useful in various areas of research, such as cryptography, signal processing, and control theory.

In the other hand Fractional calculus has been shown to be a powerful tool for modeling and analyzing complex systems with non-integer dynamics, such as chaotic systems. Fractional order differential equations have been used to describe a wide range of chaotic phenomena, including strange attractors, bifurcations, and intermittency. Some notable books on the use of fractional calculus in chaotic systems include:[99, 114].

One of the advantages of using fractional calculus in the study of chaotic systems is its ability to capture memory effects, which are often present in physical systems. Traditional integer-order models do not capture these memory effects, leading to inaccurate predictions and limited understanding of the system dynamics [4, 115] Fractional calculus has been applied to various types of chaotic systems, including discrete-time systems, continuous-time systems, and fractional-order systems. For example, fractional calculus has been used to study chaotic dynamics in systems such as the Lorenz system, Rössler system, and the Chua system, among others. Furthermore, fractional calculus has been used to develop new control strategies for chaotic systems, such as chaos synchronization and chaos control. These strategies have been shown to be effective in stabilizing chaotic systems and suppressing the unpredictable behavior associated with chaos.

Secure communication is essential for protecting sensitive information, and chaotic systems have been shown to be effective in providing a secure communication channel. However, traditional chaotic systems with integer-order dynamics have some limitations, such as being susceptible to attacks and having limited security capabilities. Fractional-order chaotic systems have been proposed as a solution to these limitations.

Fractional order chaotic systems have gained considerable attention in recent years due to their ability to generate complex and unpredictable dynamics. This has led to their application in various fields, including communication, control, and cryptography.

In particular, fractional order chaotic systems have shown promise in the field of security due to their potential use in encryption and decryption algorithms. The chaotic behavior of these systems can provide a high level of randomness and unpredictability, which is essential for secure communication and data protection.

Furthermore, Fractional-order chaotic systems have been applied in various secure communication schemes, such as encryption, decryption, and key distribution. Overall, the use of fractional-order chaotic systems in secure communication has shown great potential in providing a more secure and robust communication channel. The complex dynamics of these systems and their ability to be controlled using fractional calculus make them a valuable tool for designing secure communication protocols.

CHAPTER 1

LITERATURE OF FRACTIONAL-ORDER CHAOTIC SYSTEMS AND ITS APPLICATIONS IN SECURE TRANSMISSION SCHEME

1.1 Introduction

Among the nonlinear dynamical systems, the chaotic system has gained many authors' interest in the last thirty years. Due to its various benefits, for instance, the ability to avoid the hypotheses related to the condition of the system, the sensitivity to initial conditions, and being a deterministic system that exhibits complex and unexpected behaviour, these significant advantages stimulated the researchers to introduce and study several chaotic systems, Some of the most well-known systems are the unified chaotic system [90], Lorenz system [88], Chen system [21], Rössler system [125], and the Lü system [89], the chaotic attitude of these effective systems are helpful in different fields such as business [113], electrical circuits [86], medicine [32], mathematics [87], biology [14], chemical reactors [71], secure communication [155], and etc.
On the other hand, there has been a great deal of interest in studying the fractional calculus, which is the non-integer order; it has received more attention due to its precision compared to the integer one. Fractional calculus investigated that different applications can be modeled elegantly and precisely with the help of fractional derivatives such as the earthquakes, so the Researchers and the physicians depended on the strategy of using the chaos system in fractional-order. The pioneers of this significant phenomena were

Grigorenko and Grigorenko in 2003 [42] then it spread out to several works like Lorenz system [42], Chen system [73], Chua's circuit [48], Duffing system [38], Lü system [91], financial system [143], and so on.
Nowadays, the secure communication branch has become a worldwide aim in encrypting images, audio, and other data, because of the necessity for many functions such as online banking, instant messaging, military and architectural plans, the recording of conferences, and business events. Among the numerous solutions, the fractional-order chaotic system is considered the most effective one. The chaotic system utilizes the synchronization between two similar chaotic systems, "emitter/receiver," with various primary conditions. This achievement made the secret information possible to recover on the receiver side after being encrypted in the emitter. Recently, the synchronization issue has become a challenging subject. therefore, many methods have been proposed by the researchers, some of them are the fractional extended Kalman filter proposed in (2009) [64], Lyapunov based methods in (2014) [78], sliding mode observer in (2013) [33], the impulsive control in (2016) [76] and the adaptive control in (2016) [15]. In the current paper, we exhibit the secure communication system's literature review using the fractional-order chaotic systems.

1.2 Chaotic system and its application in secure communication

Toshimitsu Ushio (1996)[139] suggested two control techniques for synchronising two subsystems, which are referred to as S1 and S2, respectively. S1 refers to the master system, while S2 refers to the slave system. The use of these principles results in the construction of a safe half-duplex communication system, which means both sides are able to speak with one another, but not at the same time; instead, communication can only occur in one direction at a time.

Hector Puebla and Jose Alvarez-Ramirez (2001)[118] showed a global procedure to modulate the information signals using low-dimensional chaotic oscillators. The encrypting/decrypting method [111] is set up on the inverse system masking (ISM) techniques. The secret message is recovered via a feedback approximation to the encoder inverse[6, 117]. Using hyperchaotic signals was the principle thought to inconvenience the data signal retrieving by chaotic attractor rebuilding methods (relayed on the theorem of Taken)[66]. Numerical examples of continuous systems were proposed To represent the fundamental thought.

Z Li, K Li, C Wen, and YC Soh (2003)[81] proposed a digital chaotic secure communication system using the idea of the magnifying glass in order to increase the sensitivity of the cryptosystem. The synchronization of the system is based on the impulsive control strategy [80, 82] for synchronizing two similar chaotic systems established in the encryption and decryption where the extent of impulsive intervals are piecewise fixed, which makes it complicated for an outsider to detect the synchronization installment [50, 144]. The equations of Chua‚Äôs circuit are given as :

$$\begin{cases} \frac{dx_1(t)}{dt} = ka(x_2 - x_1 - f(x_1)) \\ \frac{dx_2(t)}{dt} = k(x_1 - x_2 + x_3) \\ \frac{dx_3(t)}{dt} = k(-\beta x_2 - \gamma x_3) \end{cases} \tag{1.1}$$

f(x) is the nonlinear characteristic of Chua‚Äôs diode in Chua‚Äôs circuit given by :

$$f(x) = m_1 x + \frac{1}{2}(m_0 - m_1)\{| x+1 | - | x-1 |\} \tag{1.2}$$

M Chen, D Li, and A Zhang (2004)[22] presented a universal nonlinear state-observer for a class of universal chaotic systems to recognize a robust strong chaotic synchronization and the followed approach was easy alive and universal [41, 145], and it showed a satisfying result in secure communication [30, 43].

XY Wang and MJ Wang (2009)[142] presented a new scheme for secure communication stand on chaotic modulation method, this technique is depended on observer identification [128] instead of synchronization [40, 58, 132], this observer is designed to recognize the parameter and rebuild the information signal which is illustrated in the parameter of chaotic Liu system. Lyapunov exponents spectrum of chaotic system [121]has been used to analyze the parameter range to guarantee chaos. Some signals are used as examples to exhibit the efficiency and feasibility of the process.

The chaotic system that has been used is Liu‚Äôs System, and the differential equations that represented the system are [84]:

$$\begin{cases} \frac{dx(t)}{dt} = -ax(t) - ey^2(t) \\ \frac{dy(t)}{dt} = by(t) - kx(t)z(t) \\ \frac{dz(t)}{dt} = -cz(t) + mx(t)y(t) \end{cases} \tag{1.3}$$

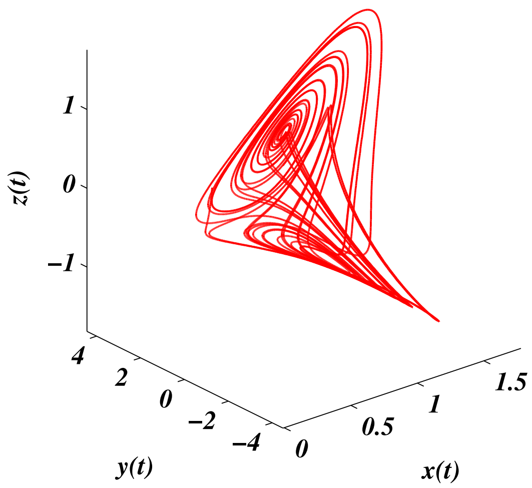

Figure 1.1: The Simulation of Liu's system (1.3).

OM. Kwon, JH. Park and SM. Lee (2011)[69] presented a new synchronization scheme of Lur'e systems [49, 151] for secure communication based on delay feedback control [17]. The encryption method was achieved by the methods of N-shift cipher and public key [156]. A new delay-dependent synchronization criterion [63, 158] is built up using the Lyapunov strategy and linear matrix inequality (LMI) formulation to obtain the stability and reconstruction of the secret message.

Qu Shaocheng, Liu Di, and Wang Li (2011) [119] were the pioneers of the nonlinear controller for hyper-chaotic Lorenz system based on Lyapunov stability theory, which synchronized the emitter and the receiver. The transmitted signal is modulated in the drive system on the transmitter, leading to an improvement in communications security.

The hyper-chaotic Lorenz system is defined as [141]:

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = cx - y - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -kx \end{cases} \tag{1.4}$$

Xiaohui Xu (2011)[148] presented a new communication security scheme depending on generalized function projective synchronization (GFPS) [35, 157], instead of function projective synchronization (FPS) [26, 34, 36, 127]. On the emitter side, the secret message is converted by an invertible function. Toward the chaotic system's parameter, the processed signal was modulated. The parameter and the controllers have been designed to recognize GFPS of uncertain Liu chaotic systems and identify the receiver system's unknown parameter using the Lyapunov stability theory. The secret message can be recovered successfully using the estimated parameter.

Jing Pan, Qun Ding, and Baoxiang Du (2012) [109] presented a progressed scheme for chaotic masking for communication security depending on the Lorenz system. This scheme could defeat the Lorenz system's signal by changing some features to make it more chaotic in both time and frequency domains.
Shutang Liu and Fangfang Zhang (2014) [104] designed a new scheme of communication security complex coupled chaotic systems depending on complex function projective synchronization (CFPS). The complex scaling functions were chosen due to their arbitrariness and unpredictability. The chaotic system that has been used is a coupled complex Lorenz system.

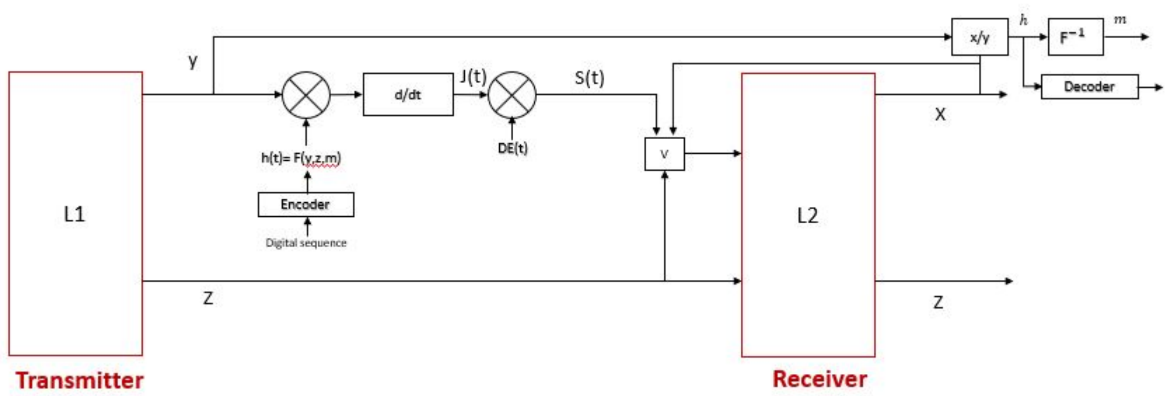

Figure 1.2: The block diagram of the proposed communication scheme [104].

Nie Chun Yan, Tian Hua, and Sun Hai Xin (2014) [105] proposed a new system for secure communication that combines the synchronization theory and the fourth-order hyper-chaotic system. In addition to producing the unexpected pseudo-random sequences with this hyper-chaotic system.

Ammami Sonia, Djemai Mohamed, et al. (2015) [47] investigated the synchronization of a unified chaotic system to encrypt several forms of information signals. Via the Lyapunov theory, they confirmed the asymptotic convergence between the emitter and the receiver.

Naderi Bashir and Kheiri Hossein (2016) [102] presented the application of the exponential synchronization of the chaotic system without linear term in communication security, the convergence time was shorter because of the exponential stability mechanism while the security and synchronization were accomplished through convenient robust controllers.

Israr Ahmad Muhammad Shafiq and M. Mossa Al-Sawalha (2018) [5] proposed a novel strategy and scheme of the global exponential multi switching combination synchronization (GEMSCS) within three unlike chaotic systems, two of them for the emitter system and one for the receiver system. The Lyapunov theorem was applied to achieve the global exponential stability of the synchronization error, which complicated the level of the digital message and made the decryption process faster and more precise.

Adnan Javeed, Tariq Shah, and Attaullah. (2020) [53] They presented a novel technique to achieve secure communication using a chaotic oscillator generated by a second-order differential equation OED. They formulated an algorithm to build a substitution box utilizing this OED. In addition to proposing a lightweight process to encrypt images using the duffing oscillation and the S-box to create confusion and diffusion in the cryptosystem in an advanced way.

YJ Chen, HG Chou, WJ Wang, SH Tsai (2020)[25] presented a new synchronization of multi-scroll Chen chaotic systems based on polynomial fuzzy control [45] for disturbance refusal, in addition to keeping the performance into account and Inputs restraints. This control [138] is performed according to sum-of-squares (SOS) [134, 137] that can also be managed by the polynomial optimization Matlab toolbox SOSOPT.

The Chen‚Äôs system is defined as :

$$\begin{cases} \frac{dx(t)}{dt} = a(y(t) - x(t)) \\ \frac{dy(t)}{dt} = (c-a)x(t) - x(t)z(t) + cy(t) \\ \frac{dz(t)}{dt} = x(t)y(t) - bz(t) \end{cases} \tag{1.5}$$

Van Nam Giap, et al. (2021)[103] proposed a new form of Lorenz chaotic system by changing the original form into a Takagi-Sugeno (T-S) fuzzy model with two sub-linear systems in order to decrease the cost of experimental equipment without affecting the generalizability of master and slave system features. A new adaptive disturbance observer (ADOB) with a fast convergence rate was presented for the synchronisation system; this method was created to remove the perturbation values on both the master and slave sides of the system. Furthermore, the transmitter and receiver systems of a secure communication system are synchronized with the help of adaptive sliding-mode control (ASMC).

Joseph Chang Lun Chan, et al. (2022)[19] proposed a secure communication scheme based on chaotic systems on the transmitter side and a sliding-mode observer (SMO) on the receiver side for purposes of estimating system states and synchronisation. The encryption process is divided into two stages: first, using an N-cipher and encryption key signal, and subsequently injecting the encrypted message into the chaotic dynamics of the transmitter system. Instead of injecting the message directly into the chaotic system on the transmitter side to encrypt the message. For further safety, the encryption key is created separately from the transmitter’s status information. The proposed scheme is developed to isolate the effects of the disturbances from the recovered messages. The secret information was reconstructed using the decryption key signal and the recovered encrypted messages (N-cipher).

Chih‚ÄêHsueh Lin, et al. (2022)[83]. Secure network-based video and audio streaming is proposed using a novel cryptosystem-based, four-dimensional hyper-chaotic Lorenz system. In order to get more and better random numbers, the 4-D hyper-chaotic Lorenz system is used instead of the 3-D system. The SHA3 (Secure hash algorithm 3) method is used to make randomness much better and to make synchronised dynamic key generators possible. With the synchronised dynamic key generators, the AES CFB (Advanced Encryption Standard Cipher Feedback) algorithm is used to encrypt the secret message.

The secret information is sent to the slave system through the public channel, where it will be completely rebuilt using the dynamic random keys that are made at the slave system at the same time. The 4D-LS system that has been used (1.6) is described as follows:

$$\begin{aligned}
\dot{x}_1(t) &= -ax_1(t) + ax_2(t) + \lambda x_3(t) \\
\dot{x}_2(t) &= dx_1(t) + \gamma x_2(t) - x_1(t)x_4(t), \\
\dot{x}_3(t) &= -cx_1(t) - x_3(t) \\
\dot{x}_4(t) &= x_1(t)x_2(t) - bx_4(t)
\end{aligned} \tag{1.6}$$

1.3 Fractional-Order chaotic system and its application in secure communication

A Kiani-B, K Fallahi,et al (2009)[64] investigated for the first time secure communication using chaotic system with Fractional-order in order to increase nonlinearity and complexity of the power spectrum. For data encryption, fractional-order Lorenz system has been used. An extended fractional Kalman filter (EFKF) was used for the synchronization and recovering the original message [126]. The results of fractional Lorenz system were compared by numerical example to the integer one [67, 68].

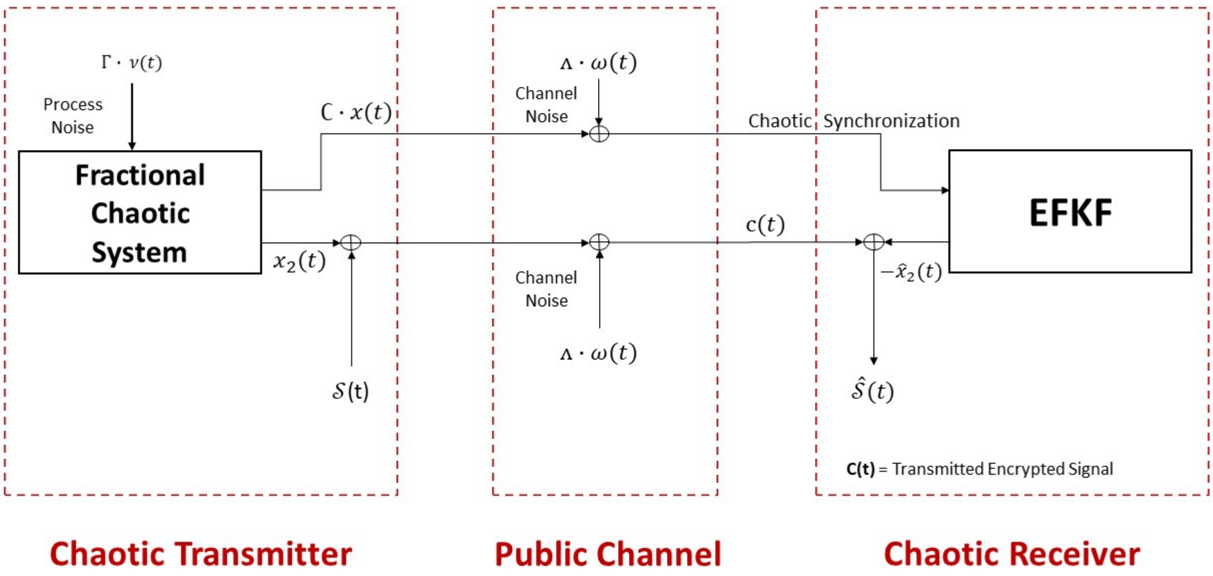

Figure 1.3: The proposed scheme [64].

A Kiani-B, K Fallahi, et al. (2009) [64] investigated for the first time secure communication using the chaotic system with Fractional-order in order to increase nonlinearity and complexity of the power spectrum. For data encryption, a fractional-order Lorenz system has been used. An extended fractional Kalman filter (EFKF) was used to synchronize and recover the original message [126]. The fractional Lorenz system results were compared by numerical example to the integer one [67, 68].

Y Xu, H Wang, et al. (2014)[149] Showed the synchronized of tow fractional-order Lorenz-like chaotic system is established by Pecora and Carroll (PC) control method [112] based on Laplace transformation theory [100]. A new cryptosystem technique for the image was proffered depending on chaos synchronization [16, 110]. As a result, the original image is well encrypted, and decrypted.
The equation of fractional-order Lorenz,Äôs system is defined as [74] :

$$\begin{cases} {}_0D_t^{q1}x(t) = \sigma(y(t) - x(t)) \\ {}_0D_t^{q2}y(t) = x(t)(\rho - z(t)) - y(t) \\ {}_0D_t^{q3}z(t) = x(t)y(t) - \beta z(t) \end{cases} \tag{1.7}$$

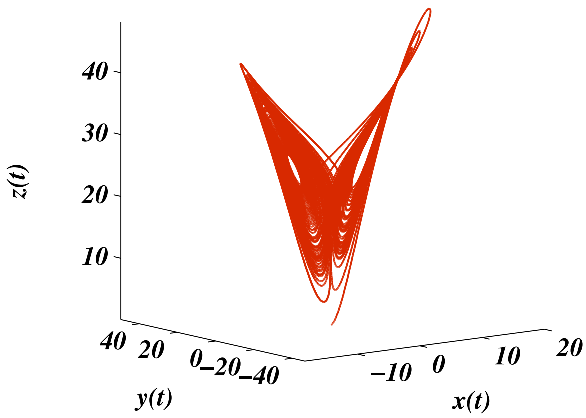

Figure 1.4: The simulation outcome (1.7)

Xia Huang, et al. (2014)[51] investigated a novel encryption algorithm of color image based on a fractional order hyper chaotic system. The secret keys include the parameters and the initial conditions, while the image is encrypted using the XOR and shuffling operations. As a result, this new study can resist the statistical attacks and it achieved a higher security.

Luo chao (2015) [20] investigated A new method instead of synchronization, which is the unsynchronised communication architecture, in order to guarantee better robustness

and make the transmission errors able to be recognized in real-time and self-corrected. He also presented a procedure with a high probability against the lack of a data-dependent error management mechanism. The author used the fractional Order [106, 116] shifting Chaotic system [64, 123] as a transmission signal generator to achieve a higher complication and non-linearity in both fields, time and frequency.
The fractional-order Chen,Äôs system is defined as [92] :

$$\begin{cases} {}_0D_t^{q1}x(t) = a(y(t) - x(t)) \\ {}_0D_t^{q2}y(t) = (c-a)x(t) - x(t)z(t) + cy(t) \\ {}_0D_t^{q3}z(t) = x(t)y(t) - bz(t) \end{cases} \tag{1.8}$$

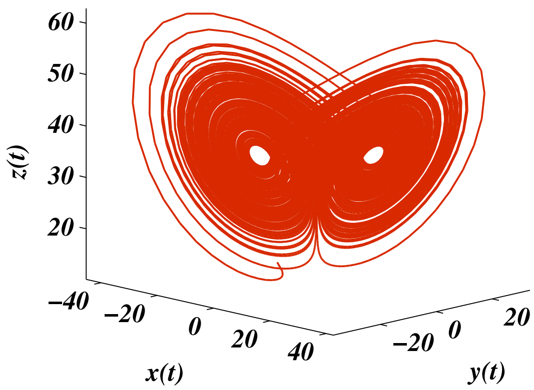

Figure 1.5: (1.8)

Delavari Hadi and Mohadeszadeh Milad (2016) [28] presented a novel adaptive sliding mode control (ASMC) method for finite-time synchronization between two Fractional-order chaotic and hyper-chaotic systems [95], in addition to the presence of foreign disturbances and system uncertainties. Stable synchronization [133] is achieved by adequate conditions of the Lyapunov stability theorem. Relevant adaptive laws and Particle Swarm Optimization (PSO) are used in order to estimate the unknown controller parameters and optimize the parameters of the (ASMC), respectively.

$$\begin{cases} \frac{d^q x}{dt^q} = a(x - yz) \\ \frac{d^q y}{dt^q} = by + xz \\ \frac{d^q z}{dt^q} = z(c - x) + xy \end{cases} \tag{1.9}$$

Mohammadzadeh Ardashir and Ghaemi Sehraneh (2017) [98] proposed a new scheme of robust controller relayed on a novel self-evolving non-singleton type-2 fuzzy neural network (SE-NT2FNN) in order to achieve the synchronization of the undetermined fractional-order hyper-chaotic systems and estimating the unsure functions in the dynamic of the system.[8, 93].

Jia Hongyan, Guo Zhiqiang, et al. (2018) [54] discussed the chaotic behaviors in a four-wing fractional-order system [56] based on frequency-domain [48] and time-domain approach [31]. The chaos synchronization of the fractional-order system is implemented by a novel analog circuit that has been planed. The synchronization is realized depending on the control proposal of the observer in his previous work [55].

Shukla MK and Sharma BB (2018)[131] investigate a backstepping-based feedback control strategy for synchronization of two similar fractional-order Rossler's system in master-slave configuration[131, 146], and also this strategy has been used widely for stabilization[130]. Two signals have been transmitted, one for the message and the other for the synchronization purpose and recover the original message.
The fractional-order Rossler‚Äôs system is defined as [73] :

$$\begin{cases} {}_0D_t^{q1} x(t) = -(y(t) + z(t)) \\ {}_0D_t^{q2} y(t) = x(t) + ay(t) \\ {}_0D_t^{q3} z(t) = b + z(t)(x(t) - c) \end{cases} \tag{1.10}$$

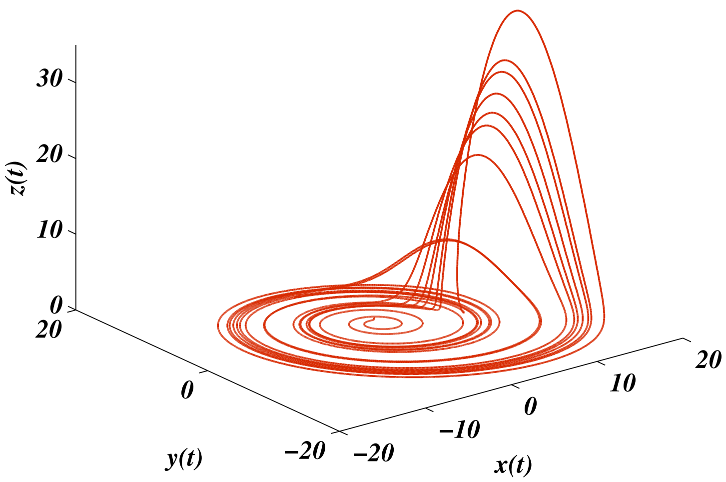

Figure 1.6: The simulation outcome (1.10)

Bettayeb Maamar, Al-Saggaf Ubaid Muhsen, and Djennoune Said (2018)[13] designed a new step-by-step sliding mode observer [10] to obtain robust synchronization of two fractional-order modified Chua‚Äôs systems and to recover the original message infinite time. The encrypted and the synchronization signals are transmitted through one channel.

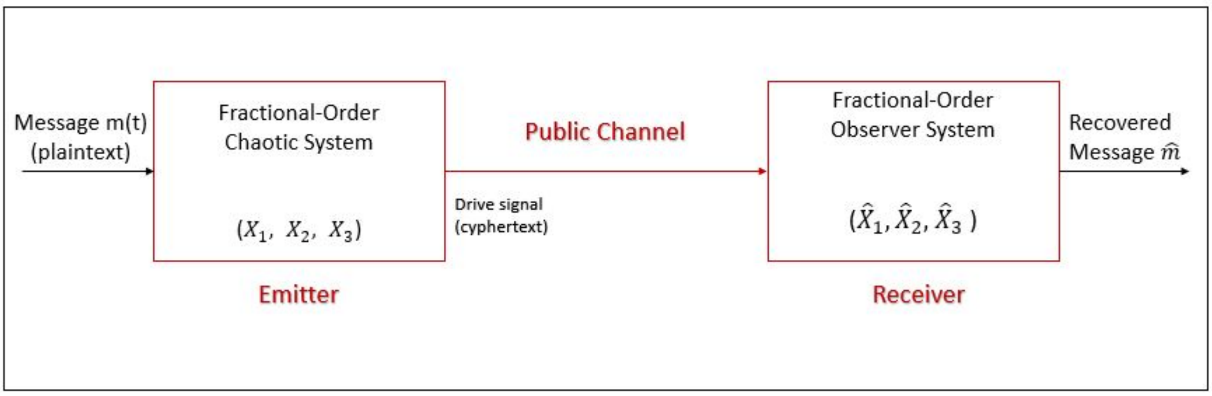

Figure 1.7: The proposed secure communication scheme [13].

D-P Sergio M, M-G Rafael, et al. (2018)[29] proposed a new methodology for a class of non-linear system depending on the fractional algebraic observability and defined the degree of concealment to demonstrate the capacity of hiding a signal into the dynamic of a system. In addition to indicating where the message is settled so that it can be estimated [101, 146], they introduced a closed-loop fractional-order differentiator developed by the sliding mode process in order to obtain secure communication [27], the proposed methodology helps to get the security in the presence of uncertainty.

Zouad Fadia, Kemih Karim, and Hamiche Hamid (2019) [161] presented a new scheme of secure communication using the Chen fractional-order delayed chaotic system under the receiver perturbations, this approach is developed, and the electronics circuit is simulated with Multisim [61]. Using an H-infinity controller. achieved The synchronization and recovering the transmitted signal. The secret message was injected in the Chen fractional order's dynamics that delayed the emitter side's chaotic system.

Liu Jiaxun, Wang Zuoxun, et al. (2019)[85] wanted to make Secure Communication of Fractional Complex Chaotic Systems (FCCS) possible. Hence, they used Fractional Difference Function Synchronization (FDFS) [24] by following a specific method, which is Extending the (DFS) to (FDFS) and investigating the general controller. In order to validate the usefulness and benefits of (FCCS), a new protected communication system (FDFS) was implemented. Four kinds of signals are transmitted, which include analog signal [94], voice signal, and digital, in addition to proposing a new cryptosystem with (FDFS) for image signal [59].
The fractional-order Genesio-Tesi‚Äôs system is defined as [57] :

$$\begin{cases} {}_0D_t^{q1}x(t) = y(t) \\ {}_0D_t^{q2}y(t) = z(t) \\ {}_0D_t^{q3}z(t) = -\beta_1 x(t) - \beta_2 y(t) - \beta_3 z(t) + \beta_4 x^2(t) \end{cases} \tag{1.11}$$

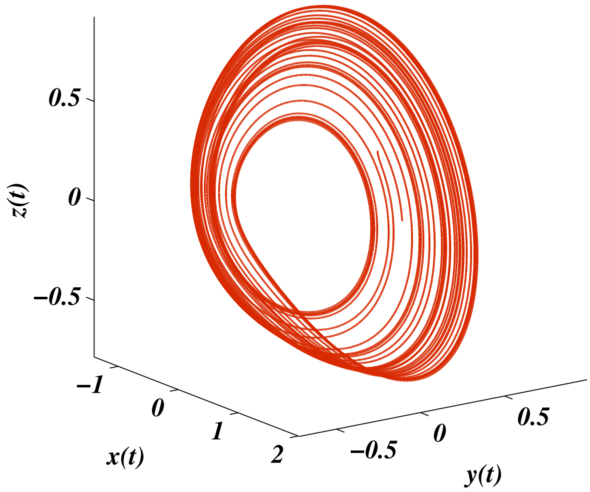

Figure 1.8: The simulation outcome (1.11).

Li Ye, Wang Haoping, and Tian Yang (2019)[79] investigated a fractional-order adaptive non-singular terminal sliding mode control (FONTSMC). The synchronization of two nonlinear fractional-order chaotic systems under the external disturbance was achieved by (FONTSMC) method [23, 97]. In addition to get better performance of the controller, they proposed terminal sliding mode (TSM)[153]. The Lyapunov theorem has been used for analyzing the stability of the system. This method was applied to a dual-channel secure communication system, and the simulation results guarantee the shortness in the time of synchronization, and the secret message can be built successfully [75].

Ghiasi Amir Rikhtegar, Gharamaleki Mona Saber, et al. (2019)[39] presented a new type of optimized time-delayed feedback control method which the Optimizing method was based on particle swarm optimization method (PSO)[12, 60]. The primary purpose of Using this controller is to stabilize [11] the chaotic behavior of fractional order electrical oscillator [154] by adding a control signal to the chaotic system. The proposed method was compared to the general (TDFC) and succeeded.

$$\begin{cases} {}_0D_t^{q1}x(t) = y(t) \\ {}_0D_t^{q2}y(t) = z(t) \\ {}_0D_t^{q3}z(t) = -\beta_1 x(t) - \beta_2 y(t) - \beta_3 z(t) + \beta_4 x^2(t) \end{cases} \tag{1.12}$$

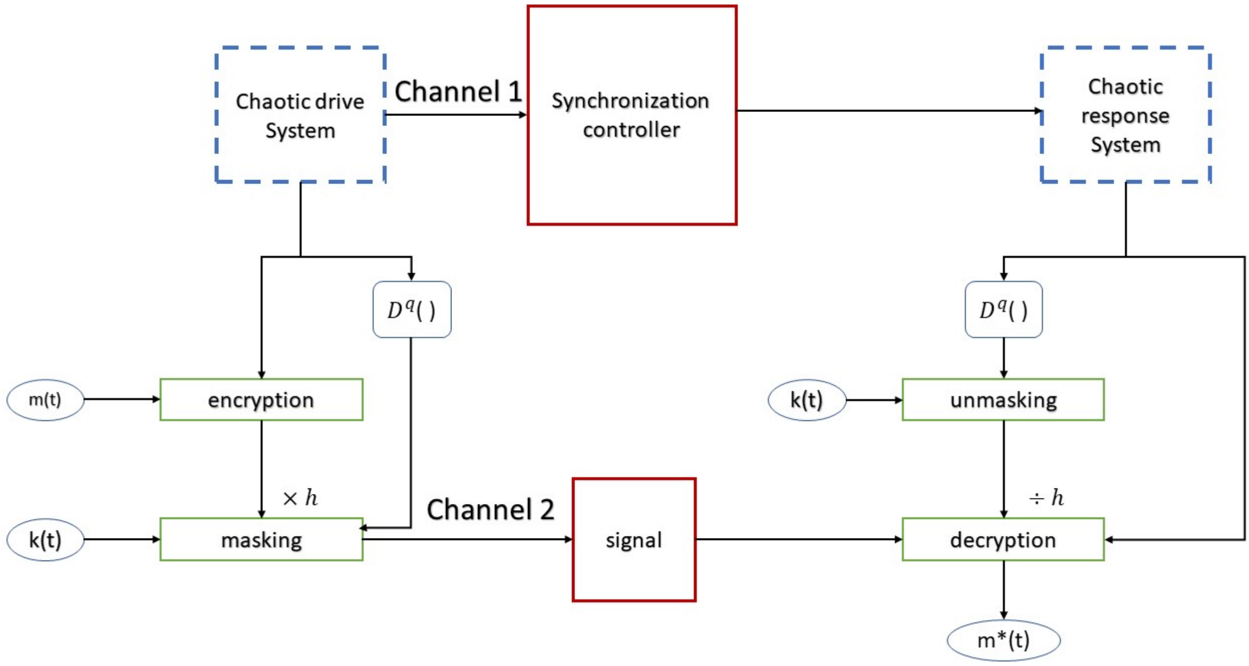

Figure 1.9: the proposed secure communication scheme [79].

Abd El-Maksoud Ahmed J, Hassan, Bahy G, et al. (2019)[1] presented the FPGA implementation of fractional order multi-scrolls [96] Tang, Yalcin[152] and Ozoğuz[108] chaotic systems . After comparing all the fractional-order (Yalcin, Ozoğuz, Tang) chaotic systems on FPGA [129] they found that Yalcin is the highest throughput. Therefore, is used as the chaotic and the chaotic generator in a speech encryption algorithm, this algorithm was introduced using stream ciphering blocks and its robustness was shown by analysis technique.

Lahdir Mourad, Hamiche Hamid, et al. (2019) [70] investigated a new cryptosystem for image which is compression and encryption using SPIHT coding-decoding [46, 147] and fractional-order chaotic respectively. considering that it is a fast technique for image compression and more complexity to the cryptosystem. The synchronization was achieved by the basic Parallel processing BPP method. The results show that the encryption and decryption of the image were successfully achieved.

Karthikeyan Rajagopal, Ali Durdu, et al. (2019) [120] the Adams-Bashforth-Moulton algorithm was used to derive a new scheme of synchronization for the fractional-order multi scroll system. So they proposed a multi scroll attractor and were able to control the number of scrolls by proper choice of the nonlinear function. The Lyapunov exponents of the proposed scheme are obtained to ensure the chaotic behavior of the system. This system has many real-time applications like image and voice encryption, random number generators, and so on.

Li Rui-Guo and Wu Huai-Ning (2019)[77] Investigated a synchronization based on fractional-order chaotic systems in parameter uncertainty and disturbance. This process went by designing a feedback controller using the adaptive control method and the fractional calculus theory. In addition to providing a policy of optimization to the controller, then they improved the quantum particle swarm optimization (QPSO) algorithm by introducing an interval estimation strategy to it then applying it to optimize the controller parameters. As a final step, they offered a feedback mechanism with the encrypted signal in secret communication.

Ismail Samar M, Said Lobna A, et al. (2020)[52] investigated a novel image encryption algorithm without any data loss based on chaotic maps and edge maps to add extra degrees of freedom to the system and offer fewer correlation coefficients, respectively [18]. Using fractional-order edge detection filters [160] in order to get better noise performance over the conventional integer filter. The system's flexibility can be used with different edge detectors and suitable for medical imaging security. This algorithm has been compared with other existing cryptosystems.

Ayub Khan, Lone Seth Jahanzaib, and Pushali Trikha (2020) [62] introduced a novel 4-D fractional-order chaotic system that has been synchronized with its parallel systems in dislocated phase and anti-phase synchronization by building non-linear control functions [150]. Consider the novel fractional-order chaotic system as (1.13):

$$\begin{cases} D^q z_1 = az_2 - az_1 + z_3 z_4 \\ D^q z_2 = z_1(a - z_3) - z_2 + z_1 \\ D^q z_3 = z_1 z_2 - bz_3 + |z_3| \\ D^q z_4 = z_2 z_3 - cz_4 \end{cases} \tag{1.13}$$

1.4 Conclusion

In this chapter, the literature review on the secure communication system using the chaotic system and the fractional-order chaotic system has been illustrated due to the numerous researchers that have been trying to develop the application of the chaotic system in this field (communication security). In addition to the fractional calculus that received more attention, especially with the chaotic system, due to its precision and sensitivity, this review explains that most of the studies achieved recently in secure communication are depending on the fractional-order chaotic system.

CHAPTER 2

FUNDAMENTALS OF DYNAMICAL SYSTEMS AND CHAOS BASED FRACTIONAL-ORDER

2.1 Introduction

Dynamic systems and chaos theory are fascinating fields that have revolutionized the way we think about complex systems in the natural world. These disciplines use mathematical models to analyze and describe the behavior of systems that evolve over time, such as populations of animals, weather patterns, or the motion of celestial bodies. By understanding the underlying dynamics of these systems, scientists can make predictions about how they will behave in the future and develop strategies for controlling or manipulating them.

Dynamic systems theory is a broad field that encompasses many different types of systems, from linear to nonlinear, deterministic to stochastic, and continuous to discrete. One of the central concepts in dynamic systems theory is the idea of a state space, which is a mathematical representation of all possible states of a system. By analyzing the state space, scientists can gain insights into the long-term behavior of the system and identify critical points or attractors, which are stable states that the system tends to move towards.

Chaos theory, on the other hand, is concerned with systems that are extremely sensitive to small changes in initial conditions. These systems are often nonlinear and may exhibit seemingly random or chaotic behavior, even though they are entirely deterministic. Chaos theory explores the underlying mechanisms that give rise to this behavior and provides tools for predicting and controlling it. One of the key concepts in chaos theory is the idea of a strange attractor, which is a fractal object that describes the long-term behavior of a chaotic system.

The study of dynamic systems and chaos theory has many practical applications, from modeling and predicting the behavior of complex systems in biology, ecology, and physics, to developing new technologies in fields like engineering, finance, and computer science. These fields provide a deep understanding of the underlying principles that govern complex systems and offer powerful tools for predicting and controlling their behavior. By mastering the fundamentals of dynamic systems and chaos, we can unlock a deeper understanding of the natural world and the complex systems that govern it.

Fractional calculus is a branch of mathematical analysis that extends the traditional notions of differentiation and integration to non-integer orders. In recent years, it has gained increasing interest in the field of dynamic systems, which studies the evolution of complex systems over time. Fractional calculus provides a powerful tool for modeling and analyzing systems that exhibit non-locality, long memory, and complex dynamics. It has been applied to a wide range of fields, including physics, engineering, finance, and biology, and has led to new insights into the behavior of complex systems. In this introduction, we will explore the fundamentals of fractional calculus and its application to dynamic systems.

2.2 Dynamical systems

Dynamical systems refer to mathematical models that describe how a system changes over time. These systems can be used to represent a wide range of phenomena, from the motion of planets to the behavior of populations or the dynamics of economic systems. The behavior of dynamical systems is determined by the interaction of different variables or parameters that influence the system's evolution. Dynamical systems can be classified as either continuous or discrete, depending on whether time is treated as a continuous or a discrete variable. In continuous dynamical systems, the variables evolve continuously over time, while in discrete systems, the variables change in discrete steps at specific time intervals. One of the key concepts in dynamical systems theory is the notion of attractors, which are states to which the system tends to evolve over time. Attractors can be fixed points, limit cycles, or strange attractors, depending on the nature of the system. Dynamical systems theory is used in many fields, including physics, engineering, biology, economics, and ecology, among others. It provides a powerful framework for analyzing and understanding the behavior of complex systems and can help identify patterns, predict future behavior, and control the evolution of the system.

2.3 Linear and nonlinear dynamical systems

Linear and nonlinear dynamical systems are two types of dynamic systems that differ in how they respond to inputs and how they evolve over time. Linear dynamical systems are systems that satisfy the principle of superposition, meaning that the output of the system is proportional to the input. In other words, if the input is doubled, the output is also doubled. Linear systems are characterized by a set of linear differential equations, and their behavior can be analyzed using techniques such as Laplace transforms and matrix algebra. Nonlinear dynamical systems, on the other hand, are systems that do not satisfy the principle of superposition.

This means that the output of the system is not proportional to the input, and the behavior of the system is often more complex and difficult to predict. Nonlinear systems can be described by nonlinear differential equations, and their behavior can be analyzed using techniques such as numerical simulation, bifurcation analysis, and chaos theory. One important difference between linear and nonlinear systems is their stability properties. Linear systems are generally more stable than nonlinear systems, and their behavior can be more easily predicted and controlled. Nonlinear systems, on the other hand, can exhibit a wide range of behaviors, including chaos, bifurcations, and multiple steady states, making them more challenging to analyze and control. Overall, both linear and nonlinear dynamical systems have their own unique characteristics and applications. Linear systems are often used in control systems and signal processing, while nonlinear systems are used in areas such as chaos theory, biology, and economics. Understanding the behavior of both types of systems is crucial for advancing our understanding of the natural world and developing new technologies.

2.4 Continuous time and discrete time systems

Continuous-time and discrete-time systems are two types of dynamical systems that differ in the way they model the evolution of a system over time. A continuous-time system represents a system in which the state changes continuously over time. The system is modeled by a set of differential equations that describe the rates of change of the system's state variables with respect to time. Examples of continuous-time systems include physical systems such as the motion of a pendulum, electrical circuits, and chemical reactions. On the other hand, a discrete-time system represents a system in which the state changes only at specific time instances. The system is modeled by a set of difference equations that describe how the state of the system changes from one time step to the next. Examples of discrete-time systems include digital filters, digital signal processing, and computer algorithms. Both continuous-time and discrete-time systems can be analyzed using tools from dynamical systems theory, such as stability analysis, bifurcation analysis, and Lyapunov stability analysis. However, the analysis techniques used for these systems differ, with continuous-time systems requiring the use of calculus and differential equations, while discrete-time systems require the use of difference equations and discrete mathematics. The choice between continuous-time and discrete-time modeling depends on the nature of the system being studied and the available data. Continuous-time modeling is often used when the system evolves continuously over time and data is available at a high sampling rate, while discrete-time modeling is used when data is only available at specific time instances or when the system is inherently discrete, such as in digital signal processing.

2.5 Autonomous and non-autonomous dynamical systems

Autonomous and non-autonomous dynamical systems are two types of mathematical models used to describe the behavior of systems over time.

An autonomous dynamical system is a system that does not explicitly depend on time. In other words, its behavior is determined solely by the current state of the system, without any explicit reference to the time variable. Autonomous systems are often represented by systems of ordinary differential equations of the form:

$$\frac{dx}{dt} = f(x) \tag{2.1}$$

where x is a vector of state variables and $f(x)$ is a vector function that describes the system's behavior. Autonomous systems are also sometimes called time-invariant systems because their behavior does not change over time.

On the other hand, a non-autonomous dynamical system is a system that explicitly depends on time. In other words, its behavior changes over time, and this change is explicitly described by a function of time. Non-autonomous systems are often represented by systems of ordinary or partial differential equations of the form:

$$\frac{dx}{dt} = f(x,t) \tag{2.2}$$

where x is a vector of state variables and $f(x,t)$ is a vector function that describes the system's behavior as a function of both the state variables and time.

The distinction between autonomous and non-autonomous systems is important because it affects the mathematical techniques used to analyze and control the system. For example, autonomous systems have certain mathematical properties, such as the existence of invariant sets and Lyapunov stability, that are not present in non-autonomous systems. Non-autonomous systems can also exhibit time-varying behavior, such as periodic or chaotic oscillations, that are not possible in autonomous systems.

In practice, many real-world systems can be modeled as either autonomous or non-autonomous systems, depending on whether they exhibit time-invariant or time-varying behavior. Engineers and scientists use a variety of mathematical tools and techniques to analyze and control both types of systems, such as phase portraits, stability analysis, bifurcation theory, and numerical simulations.

2.6 The behaviour of dynamical systems

The behavior of dynamical systems refers to how their states evolve over time, and how they respond to changes in their inputs or initial conditions. The behavior of dynamical systems can be classified into different types, depending on their stability, periodicity, or complexity.

One common classification of dynamical behavior is based on the system's equilibrium points or fixed points. An equilibrium point is a state where the system's behavior does not change over time, and it is characterized by a state vector where the system's derivative or rate of change is zero. Depending on the stability of the equilibrium point, the system can exhibit the following behaviors:

1. Stable equilibrium: If the system's behavior converges to the equilibrium point when it is perturbed, the equilibrium point is said to be stable. This means that the system returns to its original state after a disturbance or a small change in the initial conditions.

2. Unstable equilibrium: If the system's behavior diverges from the equilibrium point when it is perturbed, the equilibrium point is said to be unstable. This means that the system moves away from its original state after a disturbance or a small change in the initial conditions.

3. Semi-stable equilibrium: If the system's behavior converges to the equilibrium point along some trajectories and diverges along others, the equilibrium point is said to be semi-stable.

Another important classification of dynamical behavior is based on the system's periodicity or oscillations. A periodic or oscillatory system exhibits behavior that repeats over time, with a fixed or variable period. Depending on the amplitude and stability of the oscillations, the system can exhibit the following behaviors:

1. Limit cycle: If the system's oscillations are stable and converge to a fixed periodic orbit, the system is said to exhibit a limit cycle.

2. Chaotic behavior: If the system's behavior is sensitive to initial conditions and exhibits complex, unpredictable, and non-repeating patterns over time, the system is said to exhibit chaotic behavior.

3. Quasi-periodic behavior: If the system's behavior is not periodic but can be decomposed into a combination of two or more periodic oscillations with incommensurate frequencies, the system is said to exhibit quasi-periodic behavior.

Overall, the behavior of dynamical systems can be very rich and complex, and it can exhibit a wide range of patterns, including steady states, oscillations, bifurcations, chaos, and other nonlinear phenomena. Understanding and predicting the behavior of dynamical systems is an important task in many fields, including physics, engineering, biology, economics, and social sciences.

2.7 Attractors

In dynamical systems, an attractor is a subset of the system's state space that the system tends to approach or "attract" over time, regardless of its initial conditions or perturbations. An attractor can be a point, a curve, a surface, or a higher-dimensional object that represents a stable state or a pattern of behavior in the system.

The behavior of a system near an attractor can be characterized by its stability and its basin of attraction. The stability of an attractor refers to the tendency of the system's trajectories to converge to or remain in the attractor's vicinity, while the basin of attraction is the set of initial conditions that lead the system to the attractor.

Attractors can be classified into different types, depending on their geometry and the system's dynamics. Some common types of attractors are shown in Fig 2.1 and include:

Fixed point or point attractor: A fixed point is an attractor that corresponds to a stable equilibrium point in the system's state space. The system tends to approach the fixed point and remain there indefinitely.

Limit cycle or periodic attractor: A limit cycle is an attractor that corresponds to a stable periodic orbit in the system's state space. The system tends to oscillate around the limit cycle with a fixed or variable period.

Strange attractor: A strange attractor is a fractal or chaotic attractor that exhibits complex and non-repeating behavior in the system's state space. The system's trajectories converge to the attractor, but the attractor has a complex and irregular geometry that cannot be described by a simple equation.

Torus attractor: A torus attractor is a type of strange attractor that exhibits complex, non-repeating behavior in the state space of a dynamical system. It has a torus-like geometry, with two or more independent frequencies of oscillation that are not necessarily harmonically related.

The concept of attractors is widely used in many fields of science and engineering, including physics, biology, ecology, economics, and computer science. Attractors can provide insight into the long-term behavior and stability of dynamical systems, and they can be used to predict the system's future behavior, control its dynamics, or optimize its performance.

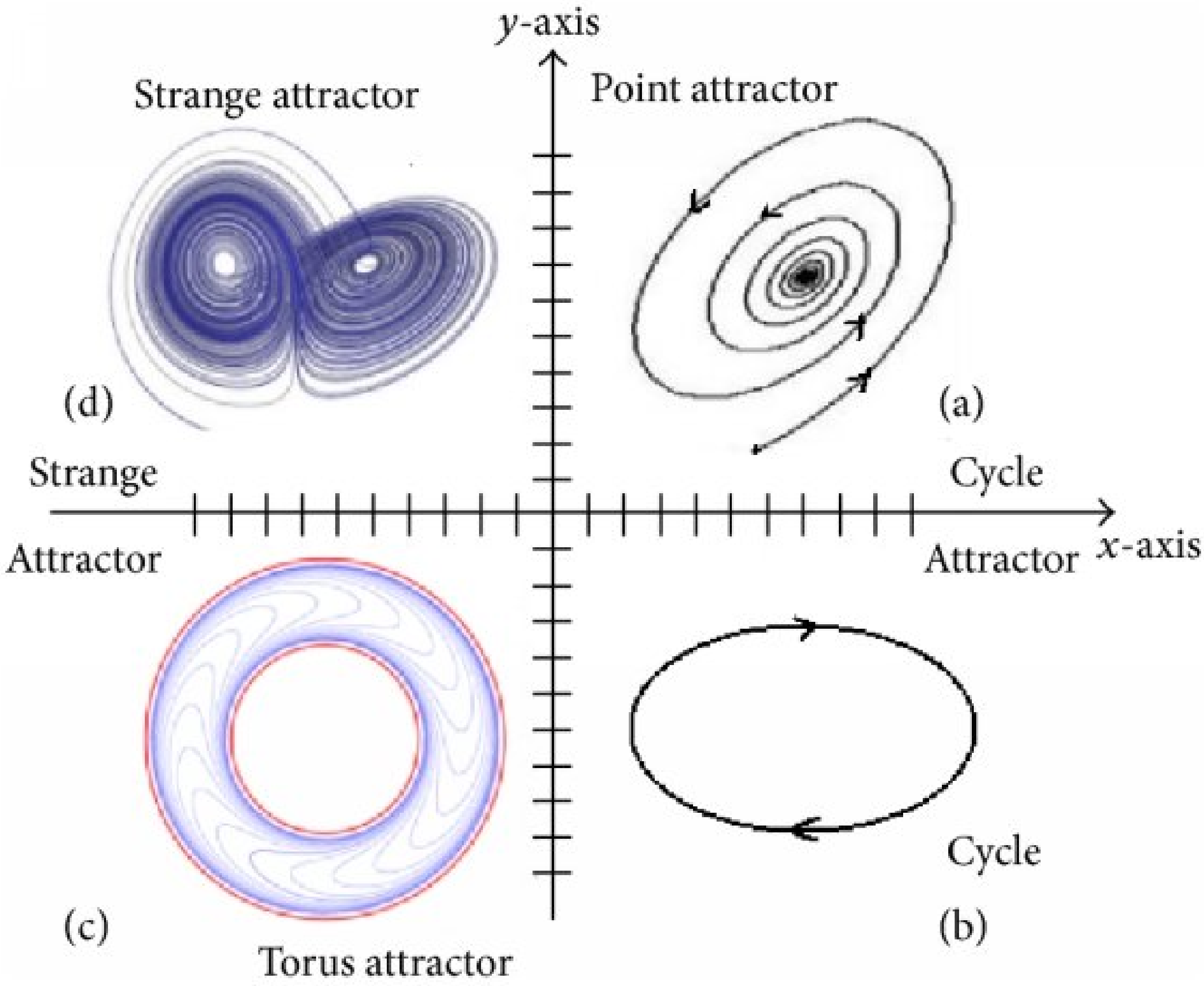

Figure 2.1: Different types of attractors constructed [2].

2.8 stability of dynamical systems

Stability of dynamical systems refers to the behavior of a system over time and how it responds to small changes or perturbations in its initial conditions. A system is considered stable if it remains close to its initial state after a small perturbation, whereas an unstable system diverges rapidly from its initial state.

There are several types of stability that can be used to describe the behavior of a dynamical system:

1. Lyapunov stability: A system is Lyapunov stable if it remains bounded in a small neighborhood around its equilibrium point over time, even if it is subject to small perturbations. This type of stability is named after Russian mathematician Aleksandr Lyapunov, who developed a criterion for determining the stability of a dynamical system based on the sign of the derivative of a Lyapunov function.

2. Asymptotic stability: A system is asymptotically stable if it returns to its steady-state or equilibrium point over time, regardless of its initial conditions. In other words, if the system is disturbed from its equilibrium, it will eventually converge back to the equilibrium point as time goes to infinity.

3. Exponential stability: A system is exponentially stable if the perturbations decay exponentially fast to zero as time goes to infinity. This type of stability is commonly used in the analysis of linear time-invariant systems.

4. BIBO stability: A system is BIBO (bounded input-bounded output) stable if, for any bounded input signal, the output of the system remains bounded. This type of stability is commonly used in the analysis of linear time-invariant systems.

5. Input-output stability: A system is input-output stable if its output remains bounded for any bounded input signal. This type of stability is commonly used in the analysis of nonlinear systems.

6. Conditional stability: A system is conditionally stable if its stability depends on the values of its parameters or initial conditions. For example, a system may be asymptotically stable for some values of its parameters but unstable for others.

7. Unstable: A system is unstable if it does not return to its equilibrium point over time or if it diverges from its initial conditions. An unstable system can exhibit chaotic or explosive behavior, which can be difficult to predict or control.

Stability is an important concept in many fields, including control theory, signal processing, and physics. The study of stability is an important topic in dynamical systems, as it allows us to predict the long-term behavior of a system and design control strategies to stabilize or manipulate the system's dynamics.

2.8.1 Stability in the sense of Lyapunov

Lyapunov stability is a concept used in the analysis of dynamic systems, which describes how small perturbations to the system evolve over time. A system is considered Lyapunov stable if small perturbations to its initial conditions result in a trajectory that stays close to the original trajectory for all time. In this sense, Lyapunov stability provides a measure of the system's robustness to small disturbances.

One classic example of Lyapunov stability is a simple pendulum. A pendulum is said to be in a stable equilibrium when it is hanging vertically and undisturbed, and an unstable equilibrium when it is balanced horizontally at the top of its swing. By analyzing the potential energy of the pendulum as a Lyapunov function, we can show that the stable equilibrium is Lyapunov stable and the unstable equilibrium is Lyapunov unstable.

Another example is the Lorenz system, which is a set of three coupled ordinary differential equations that describe the behavior of a simplified atmospheric convection model. The Lorenz system exhibits chaotic behavior, with trajectories that are sensitive to initial conditions. However, by analyzing the Lyapunov exponents of the system, we can show that it is still Lyapunov stable in a statistical sense. That is, although individual trajectories may exhibit chaotic behavior, the overall behavior of the system is still predictable and stable over the long-term.

Lyapunov stability can also be used to analyze feedback control systems, where the goal is to design a controller that stabilizes the system in the presence of disturbances. By analyzing the Lyapunov function associated with the system and the controller, we can design a controller that ensures Lyapunov stability of the closed-loop system, and thus guarantee the robustness of the system to small perturbations.

Overall, Lyapunov stability is a fundamental concept in the analysis of dynamic systems, and has wide-ranging applications in physics, engineering, and other fields. It provides a powerful tool for understanding the behavior of complex systems, and designing controllers that ensure stability and robustness in the face of disturbances.

Consider the following dynamical system:

$$\dot{x} = f(x,t) \tag{2.3}$$

Where f is the nonlinear function.

Definition: The equilibrium point x^* of system 2.3 is said to be:

- Stable if:

$$\forall \varepsilon > 0, \exists \delta > 0 : \left\| x(t_0) - x^* \right\| < \delta \Rightarrow \left\| x(t, x(t_0)) - x^* \right\| < \varepsilon, \forall t > t_0 \tag{2.4}$$

- Asymptotically stable if:

$$\exists \delta > 0 : \left\| x(t_0) - x^* \right\| < \delta \Rightarrow \lim_{t \to \infty} \left\| x\left(t, x(t_0) - x^*\right) \right\| = 0 \tag{2.5}$$

- exponentially stable if:

(2.6) $\forall \varepsilon > 0, \exists \delta > 0 : \|x(t_0) - x^*\| < \delta \Rightarrow \|x(t, x(t_0)) - x^2\| < a\,\|x(t_0) - x^*\| \exp(-bt), \forall t > t_0$

- Unstable if: equation 2.4 is not satisfied.

2.9 Chaotic systems

Chaotic systems are a class of dynamic systems that exhibit complex and unpredictable behavior, even though they may have relatively simple underlying rules. Chaotic systems are characterized by their sensitivity to initial conditions and their non periodic, aperiodic, and seemingly random behavior.

In chaotic systems, small differences in initial conditions can lead to vastly different outcomes over time, making long-term prediction and control of the system difficult or impossible. This sensitivity to initial conditions is known as the butterfly effect, named after the idea that a butterfly flapping its wings in one location can potentially set off a chain of events that leads to a tornado in another location.

Chaotic systems can arise in a wide range of contexts, from weather patterns and fluid dynamics to economics and population dynamics. One of the most famous examples of a chaotic system is the Lorenz system, which was introduced by meteorologist Edward Lorenz in the 1960s as a simplified model of atmospheric convection.

The Lorenz system is a set of three nonlinear differential equations that describe the behavior of a simplified model of the atmosphere:

$$\begin{cases} dx/dt = \sigma(y - x) \\ dy/dt = x(\rho - z) - y \\ dz/dt = xy - \beta z \end{cases} \tag{2.7}$$

where x, y, and z are variables that represent the state of the system, and σ, ρ, and β are parameters that control the behavior of the system.

The behavior of the Lorenz system is highly sensitive to initial conditions, and small changes in the initial state can lead to wildly different trajectories over time. The system also exhibits strange attractors, which are complex, non periodic patterns in the system's behavior that can be visualized using techniques like phase-space plots.

Chaotic systems like the Lorenz system are difficult to predict and control, but they also have important applications in a range of fields, including weather forecasting, cryptography, and chaos-based communication systems. They also provide a rich source of mathematical and computational challenges for researchers to explore.

2.9.1 Chaotic system characteristics

Chaotic systems are characterized by a number of distinctive properties that distinguish them from other types of dynamic systems. Here are some of the most important characteristics of chaotic systems:

1. Sensitivity to initial conditions: Chaotic systems are highly sensitive to their initial conditions, meaning that small changes in the starting state of the system can lead to vastly different outcomes over time. This is often referred to as the butterfly effect.

2. Nonperiodicity: Chaotic systems do not exhibit periodic behavior, meaning that their trajectories do not repeat themselves over time. Instead, chaotic systems may exhibit aperiodic or seemingly random behavior.

3. Boundedness: Chaotic systems are typically bounded, meaning that they do not grow infinitely over time. However, the bounds of a chaotic system may be difficult to predict or control.

4. Mixing: Chaotic systems are often said to exhibit mixing, meaning that the different parts of the system become increasingly intertwined over time. This can lead to a complex, tangled pattern of behavior that is difficult to analyze or predict.

5. Strange attractors: Chaotic systems often exhibit strange attractors, which are complex, nonperiodic patterns in the system's behavior that can be visualized using techniques like phase-space plots. Strange attractors are often fractal in nature and can have a rich, intricate structure.

6. Nonlinearity: Chaotic systems are typically nonlinear, meaning that the relationship between the system's inputs and outputs is not a simple, linear one. This nonlinearity is what gives rise to the complex, unpredictable behavior of chaotic systems.

Together, these characteristics make chaotic systems a fascinating and challenging area of study, with important applications in fields ranging from physics and engineering to economics and finance. While chaotic systems can be difficult to analyze and predict, they also offer rich opportunities for exploration and discovery

2.10 Conclusion

In conclusion, the use of chaotic systems in secure communication has shown promising results due to the unique properties of chaos, such as unpredictability, sensitivity to initial conditions, and high complexity. The fundamental principles of chaos theory, such as the butterfly effect and the strange attractors, provide a basis for understanding the dynamics of chaotic systems and their potential applications in secure communication. Various techniques, such as chaos shift keying, chaos masking, chaos synchronization, chaotic modulation, and chaotic encryption, have been proposed and tested for chaos-based secure communication. However, the effectiveness of these techniques depends on several factors, such as the selection of appropriate chaotic systems, synchronization between the transmitter and receiver, and the level of noise and interference in the communication channel. Overall, the integration of chaos theory into secure communication provides a promising area for further research and development of robust and secure communication systems.

CHAPTER 3

THE TRANSMITTER AND RECEIVER SYSTEMS CONFIGURATION

3.1 Introduction

Fractional calculus is an area of mathematical analysis that has attracted the attention of several academics due to its intriguing potential as an unique modelling tool in a variety of scientific and engineering disciplines. Because it was found that modelling experimental dynamics by fractional differential equations offers better performance and greater results, which allows you to make powerful controllers. Among the many benefits of this field, model generalisation and long-term memory stand out, and fractional order modelling is superior when trying to establish a link between any two points. Data for the case of interest can be obtained at any moment from a fractional order model. But with an integer order model, we can only get information that applies to this situation. Choosing the best operator for your model is based on the issue and the data that has been collected. One of the most common definitions of a fractional derivative in use today is the one suggested by Caputo. as well as studying several possibilities besides the anomalous behaviour of dynamical systems in multiple fields like electrochemistry, viscoelasticity, physics, and biology. Numerous useful applications of fractional calculus can be seen in modern theory ([3, 37, 122]). Chaos theory is a difficult subject of mathematics characterised by unexpected and complex behavior, which concentrates on deterministic laws and underlying patterns that are hyper-sensitive to the initial conditions. Chaotic systems have become a topic of interest in the last three decades, such as the Lorenz system, the Unified chaotic system, the Rössler system, the Lü system, and the Chen system ([88–90, 124]. There are many applications for these highly effective systems that benefit from their chaotic, unpredictable behavior [140], including commerce, electricity, healthcare, math, chemical, and biological reactors, and security ([3, 14, 32, 71, 113, 155]), and they have been applied successfully in a great number of control fields, among them secure communication. Whereas the researchers found that combining the fractional calculus with the chaos theory would give better results in secure communication because the FOCS display much more complicated behavior. In this study, we used a powerful tool, the optimal SBS-SMO; the use of the observer has unique characteristics, such as generating a sliding movement on the synchronization error, which guarantees that a sliding mode observer generates a collection of states estimates that are precisely proportional to the transmitter's actual output. Therefore, it is obvious that a lot of effort was and is still dedicated to the development of the synchronization of chaotic systems with fractional order using a sliding mode observer, and several papers have been published as an outcome like ([28, 135]). This study proposed a novel method for the security of communication employing two fractional-order Chua oscillators and an ideal SBS-SMO to achieve synchronization between them. The message is included in the chaotic system's third-state derivative of fractional order (in the emitter section). Because the receiving process is thought to be broken, a SBS-SMO is used to check if the master and slave are in sync and get the secret information.

3.2 The secure communication system design

Chaotic systems have been used in the past to enhance secure communication systems due to their inherent properties of unpredictability, sensitivity to initial conditions, and nonlinearity. The idea is to use a chaotic system as the basis for generating a secret key, which can then be used for encryption and decryption of messages. One way to use a chaotic system for secure communication is through a process called chaos synchronization. In this process, two chaotic systems are coupled together in such a way that they synchronize their chaotic behavior. One of the chaotic systems is held by the sender, and the other is held by the receiver. By synchronizing the two systems, a shared secret key can be generated that can be used for encryption and decryption of messages. Another way to use a chaotic system for secure communication is through a process called chaotic masking. In this process, the message to be transmitted is combined with a chaotic signal generated by a chaotic system. The resulting signal is then transmitted over the communication channel. At the receiver end, the chaotic signal is removed from the transmitted signal, revealing the original message. Both chaos synchronization and chaotic masking can be used to enhance the security of a communication system. However, it is important to note that the security of the system depends on the complexity and randomness of the chaotic system used. Therefore, it is important to carefully choose a chaotic system that meets the necessary security requirements. Additionally, it is important to ensure that the communication channel used is also secure and cannot be easily intercepted by an attacker.

The suggested system is composed of two components, the transmitter where the encryption process is done, and the receiver where the decryption process happens. The three state variables $x(t), y(t)$ and $z(t)$ are used in the transmitter part and the hidden message $m(t)$ inserted through the third state variable $z(t)$ of the dynamic system, the first state variable is the driving signal $y(t)$, which will be delivered from the master to the slave $Y(t) = x(t)$. At the slave side, the synchronization must be achieved between the emitter and the receiver in order to rebuild the secret information. An attractive approach to chaos synchronization based on the concept of observer design has been introduced by [**?**]. In this theses, a novel plan for synchronizing two fractional-order chaotic system based on an optimal step-by-step sliding mode observer in order to provide a safe communication process. the original information $m(t)$ is recovered successfully in the receiver section by the estimate information $\widehat{m}(t)$ in finite period of time. Fig 1 shows the general idea for the secure communication scheme, and the proposed secure communication scheme in 3.2.

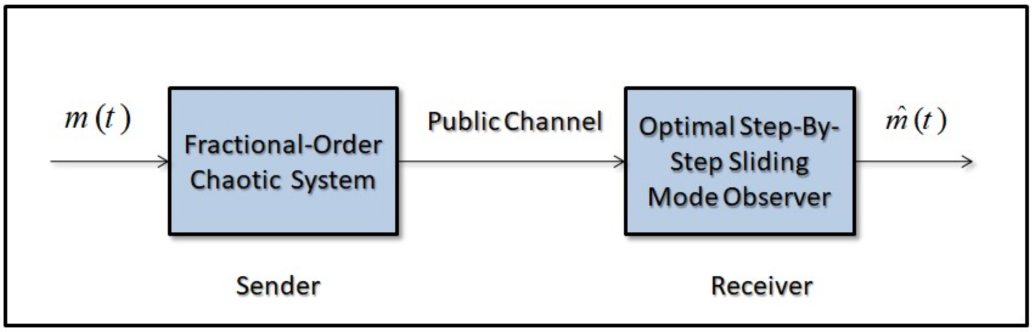

Figure 3.1: The secure communication scheme.

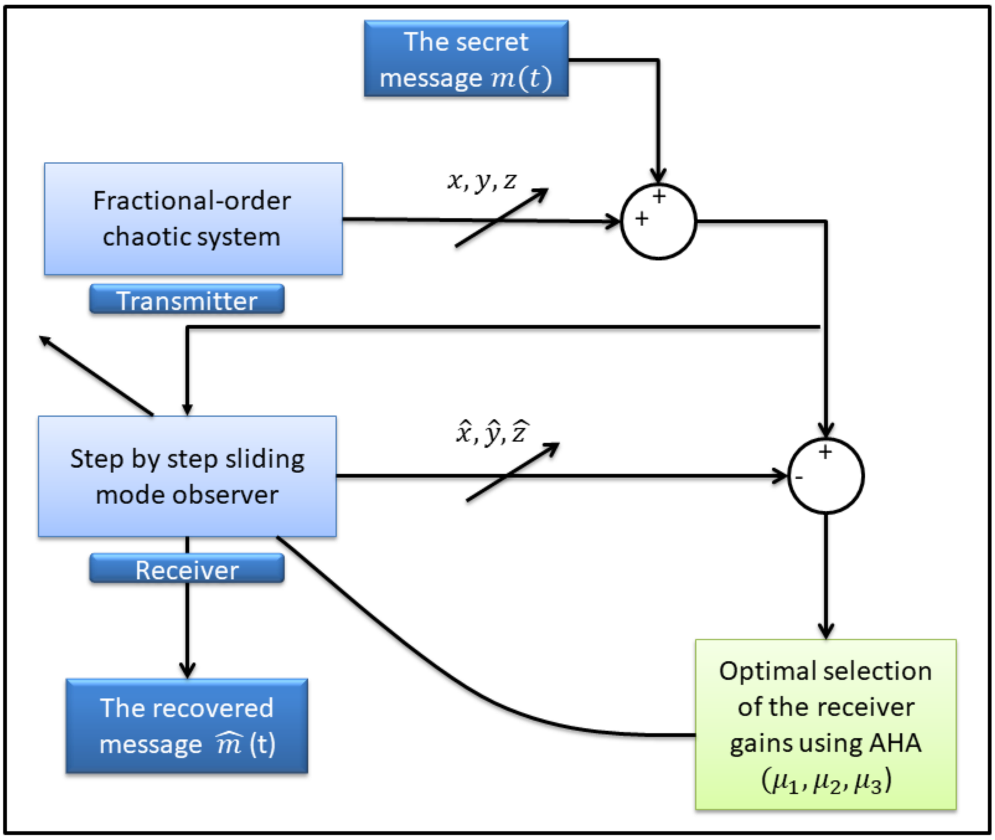

Figure 3.2: The proposed secure communication scheme.

3.3 The fractional-order Chua's chaotic oscillator

Chua's oscillator is a well-known chaotic system that was first introduced by Leon O. Chua in 1986. The system is composed of a set of nonlinear differential equations that exhibit chaotic behavior. The fractional-order Chua's chaotic oscillator is an extension of the original Chua's oscillator, where the order of differentiation is not limited to integer values. Chua‚Äôs oscillator is an extremely famous system due to its simple structure and chaotic attitude, and many other chaotic systems have been enlarged with non-integer order derivative [13]. Fig 2 represents circuit of the fractional-order Chua's system that was proposed by [136], The piecewise-linear nonlinearity is replaced with an appropriate cubic nonlinearity in this circuit, which differs from the usual one. resulting in remarkably similar behavior. The three factors represented by an inductor L which is the real coil and two super-capacitors C1 and C3, R is a constant resistor, i_0 is a biasing constant current source, I controlled current source, and NR is the nonlinear resistor.

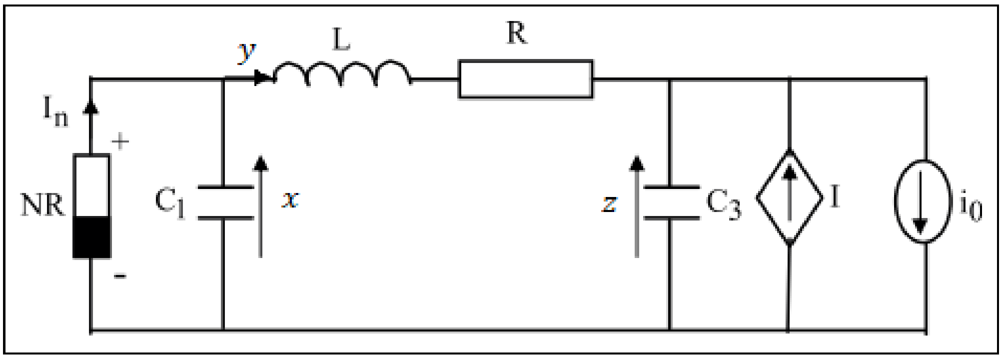

Figure 3.3: Circuit of the fractional-order Chua's system.

Definition 01. Caputo presented a novel definition of the derivative of fractional order, which is used here and described as follow [65]:

$$D^{\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\pi)^{-\alpha} \frac{df(\tau)}{d\tau} d\tau \tag{3.1}$$

The following equation indicates the mathematical description of the circuit.

$$\begin{cases} C_1 D^{q_1} x(t) = -y(t) + \alpha x^2(t) + \beta x^3(t) \\ L D^{q_2} y(t) = x(t) - R y(t) - z(t) \\ C_3 D^{q_3} z(t) = i_0 - \gamma y(t) \end{cases} \tag{3.2}$$

where the fractional order of the system is $q_i, 0 < q_i < 1$ and $q_i \in \Re_+, i = 1, 2, \ldots, n$. After division, we obtain:

$$\begin{cases} D^{q_1}x(t) = \frac{1}{C_1}[-y(t) + \alpha x^2(t) + \beta x^3(t)] \\ D^{q_2}y(t) = \frac{1}{L}[x(t) - Ry(t) - z(t)] \\ D^{q_3}z(t) = \frac{1}{C_3}[i_0 - \gamma y(t)] \end{cases} \tag{3.3}$$

3.4 The transmitter system configuration

Fractional calculus can be applied to a variety of electric and magnetic phenomena. Chua's oscillator is an extremely famous system due to its simple structure and chaotic attitude, and many other chaotic systems have been enlarged with non-integer order derivatives. The transmitter system is described as follow:

$$\begin{cases} D^{q_1}x(t) = \delta_1\left[-y(t) + \alpha x^2(t) + \beta x^3(t)\right] \\ D^{q_2}y(t) = \delta_2[x(t) - Ry(t) - z(t)] \\ D^{q_3}z(t) = \delta_3\left[i_0 - \gamma y(t)\right] \end{cases} \tag{3.4}$$

With $\delta_1 = \frac{1}{C_1}, \delta_2 = \frac{1}{L}, \delta_3 = \frac{1}{C_3}$.
$D^{[q]} = [D^{q_1}D^{q_2}\ldots D^{q_n}]^T$ represents the order fractional differentiation vector operator $q_i \in \Re_+, i = 1,2,\ldots,n$. If all the orders of the derivative q_i are equal, which is $q_i = q, i = 1,2,\ldots,n$, system is said to be of equivalent fractional order (commensurate) if and only if the fractional orders are equal. take the example of commensurate fractional-order system which is defined as:

$$\begin{aligned} D^q x(t) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \tag{3.5}$$

The table 3.1 provides a representation of the parameters that have been used in the transmitter system, while the state variables' initial conditions (x,y,z) are set at zero. Figure (**??**) illustrated the states time responses of (x, y, z) and (3.4) the three-dimensional space of the chaotic attractor in (x,y,z) respectively.

Table 3.1: Fractional-order chaotic system values.

Parameters	$q_1 = q_2 = q_3$	α	β	γ	δ_1	δ_2	δ_3	R	i_0
Value	0.9	1.5	-1	0.0035	100	1	1	0.1	0.0005

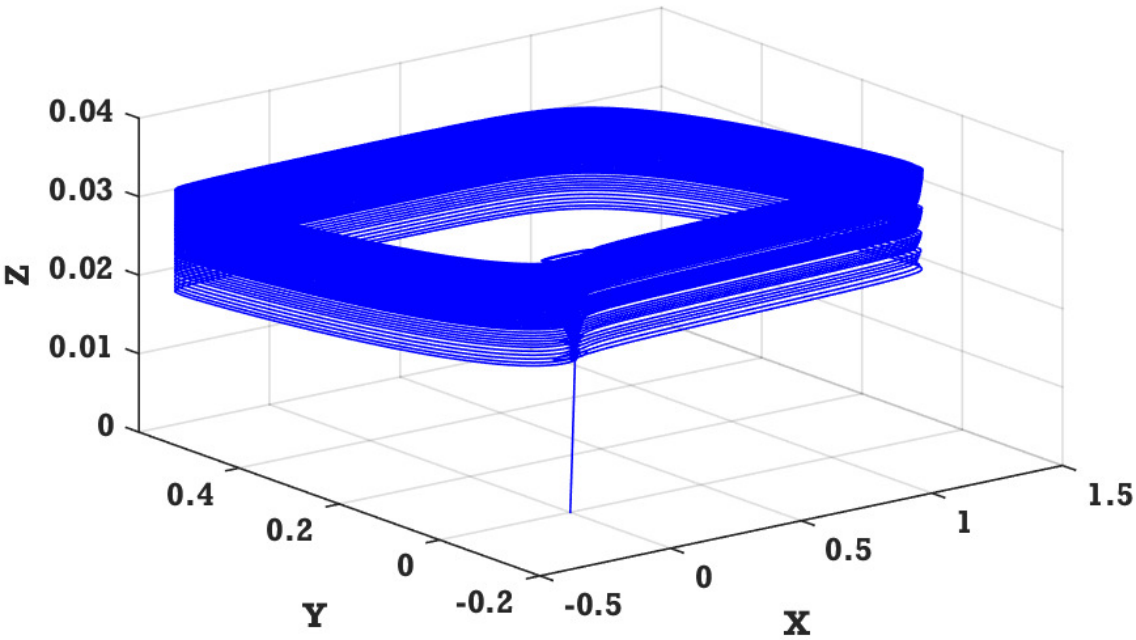

Figure 3.4:

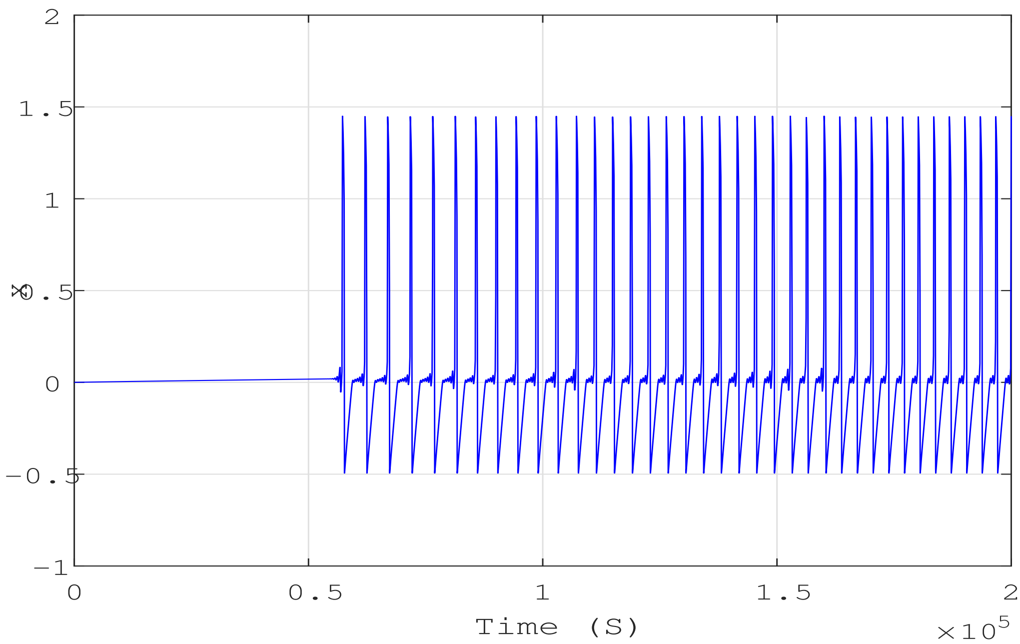

Figure 3.5: a

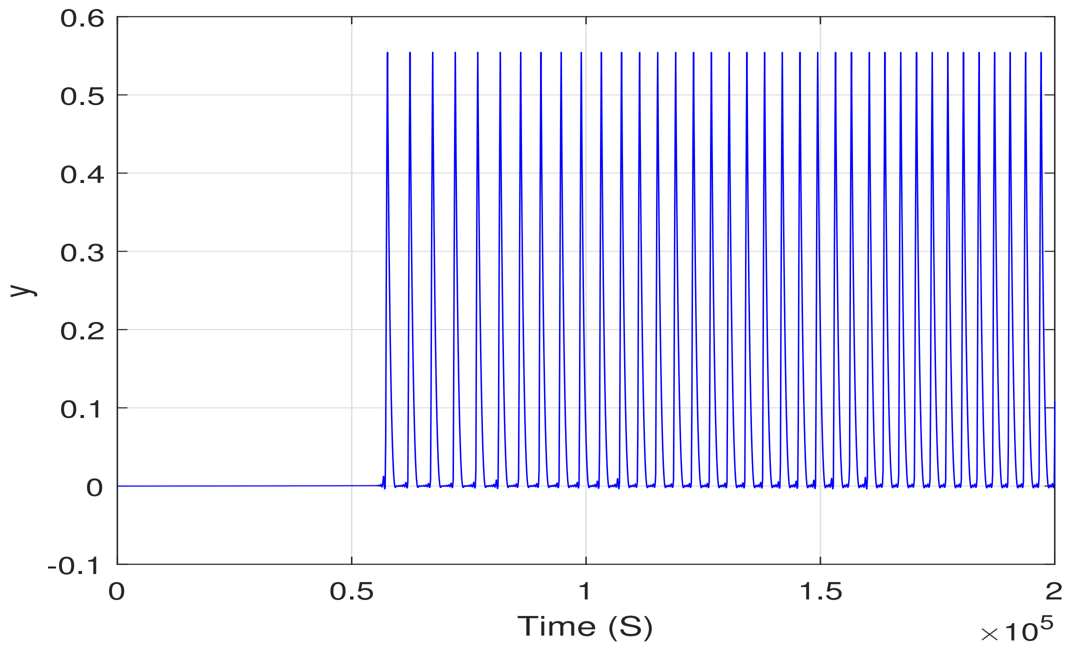

Figure 3.6: b

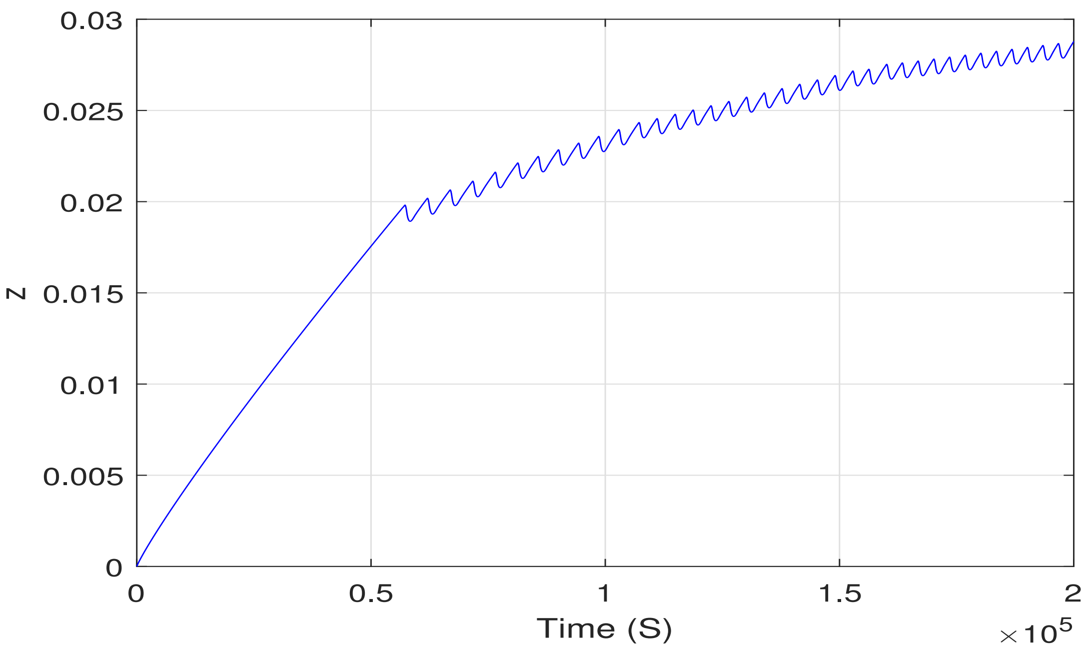

Figure 3.7: c

The secret information $m(t)$ is injected into the third derivative $z(t)$ rather than $m(t)$

in order to enhance safety of the communication system, and it will be sent as a bias current i_0 in the dynamic of the state. So, the master system model will be written as.

$$\left\{\begin{array}{c} D^{q_1}x(t) = \delta_1\left[-y(t)+\alpha x^2(t)+\beta x^3(t)\right] \\ D^{q_2}y(t) = \delta_2[x(t)-Ry(t)-z(t)] \\ D^{q_3}z(t) = \delta_3\left[i_0+m(t)-\gamma y(t)\right] \end{array}\right. \tag{3.6}$$

Remark 1: The secret message $m(t)$ is considered as an unknown in order to be estimated and can be considered as a signal, voice, or image; in the next chapter, interactive examples have been investigated.

When this parameter is available, with $q_1 = q_2 = q_3 = q$, the system will be described as:

$$\left\{\begin{array}{c} D^{q}(x,y,z)(t) = f(x,y,z)+g(x,y,z)m(t) \\ Y(x,y,z) = h(x,y,z) \end{array}\right. \tag{3.7}$$

With

$$g(x,y,z) = \begin{pmatrix} 0 \\ 0 \\ \delta_3 \end{pmatrix} \tag{3.8}$$

3.4.1 The Observability of the fractional-order chaotic system

The observability matrix is given as follows:

$$OM = \begin{pmatrix} dh \\ d_{\mathrm{f}}h \\ \vdots \\ d_{\mathrm{f}}^{n-1}h \end{pmatrix} \tag{3.9}$$

$L_f h$ indicated by:

$$L_f h = \frac{\partial f(x,y,z)}{\partial(x,y,z)} h(x,y,z) \tag{3.10}$$

ŸÑÿ•he nonlinear fractional-order system is giving as follows:

$$\begin{cases} D^q(x,y,z)(t) = f(x,y,z)(\mathrm{t}) \\ Y(x,y,z) = h(x,y,z)(\mathrm{t}) \end{cases} \tag{3.11}$$

where

$$f(x,y,z) = \begin{pmatrix} \delta_1\left[-y+\alpha x^2+\beta x^3\right] \\ \delta_2[\mathrm{x}-\mathrm{Ry}-\mathrm{z}] \\ \delta_3\left[\mathrm{i}_0-\gamma \mathrm{y}\right] \end{pmatrix}, \quad h(x,y,z) = x$$

Then, the observability matrix will be:

$$OM = \begin{pmatrix} 1 & 0 & 0 \\ a & -\delta_1 & 0 \\ b & c & \delta_2 \end{pmatrix}$$

Where

$$a = \delta_1 \mathrm{x}(2\alpha+3\beta)$$

$$b = -15\beta^2\delta_1 x^4 + 8\alpha\beta\delta_1 x^3 + 9\beta\alpha\delta_1 x^2 + \left(4\alpha\delta_1 + 3\beta\delta_1 y\right)x + (\delta_2 + 2\alpha\delta_1 y)x$$

$$c = 3\beta\delta_1 x^2 + 2\alpha\delta_1 x + \delta_2 R$$

The determinant of the observability matrix is

$$|OM| = \begin{vmatrix} -\delta_1 & 0 \\ c & \delta_2 \end{vmatrix}$$

Since $\delta_1 \neq 0$ and $\delta_2 \neq 0$ we find rank$(OM) = 3$ $(Fullrank)$

The system is completely observable.

3.5 Step-By-Step Sliding Mode Oobserver model

SBS-SMO design for integer-order nonlinear systems typically follows a step-by-step approach based on the injection of successive virtual inputs derived by employing (SBS-SMO) of iterative first order ([9]). Our goal is the estimation of the secret message $m(t)$ and $x_i, i = \overline{1,n}$. Similar to what was done with integer-order systems, the following equations (3.12) explain how the measured output signal is used by the sliding mode observer.

$$\begin{aligned} \frac{d^q(\hat{x}_{j-1},\hat{y}_{j-1},\hat{z}_{j-1})(t)}{dt} &= \rho_{j-1}(\hat{x}_j,\hat{y}_j,\hat{z}_j)(t) + \psi_{j-1}\left(Y(t),(\tilde{x}_j,\tilde{y}_j,\tilde{z}_j)(t)\right) \\ &+ E_{j-2}\mu_{j-1}\,\mathrm{sign}\left(\tilde{e}_{j-1}\right) \\ \frac{d^q(\hat{x}_j,\hat{y}_j,\hat{z}_j)(t)}{dt} &= E_{j-1}\mu_j\,\mathrm{sign}\left(\tilde{e}_j\right) \end{aligned} \tag{3.12}$$

Where $j = \overline{2...(n-1)}$

(3.13) is a triangular representation of the commensurate fractional-order nonlinear system. $y(t) \in \mathbb{R}$ is the output that was measured, $m(t) \in \mathbb{R}$ is the secret message, nonlinear functions are denoted by the symbols $a(x)$, $b(x)$ and $\Psi_i, i = 1,2,\dots,n-1$,. The parameters $\rho_i \in \mathbb{R}^+$ are all non-zero constants. Consider that the system (3.13) is Bounded Input Bounded State and $b(x)$, $a(x)$, and $m(t)$ are bounded, i.e.

Assumption

1.$|x_i(t)| \le d_i, i = \overline{1,n}, \quad |m(t)| \le M_s$.

2.$|a(x)| \le A_s, \quad |b(x)| \le B_s$ and $b(x) \neq 0$ for all $x \in \mathscr{D} \subset \mathbb{R}^n$.

$$\begin{aligned} \frac{d^q(x_{j-1},y_{j-1},z_{j-1})(t)}{dt} &= \rho_{j-1}(x_j,y_j,z_j)(t) + \psi_{j-1}(x_j,y_j,z_j)(t)) \\ \frac{d^q(x_j,y_j,z_j)(t)}{dt} &= a(x,y,z) + b(x,y,z)m(t) \\ Y(t) &= x(t) \end{aligned} \tag{3.13}$$

The terms μ_i are parameters that are only positive and developed to guarantee the observer's convergence. The secret message estimation is defined as follow:

$$\widehat{m}(t) = b(\tilde{x},\tilde{y},\tilde{z})^{-1}\left(-a(\tilde{x},\tilde{y},\tilde{z}) + E_{j-1}\mu_j\,\mathrm{sign}\left(\tilde{e}_j\right)\right. \tag{3.14}$$

The following are the definitions of the auxiliary variables $(\tilde{x}_i,\tilde{y}_i,\tilde{z}_i)$:

$$(\tilde{x},\tilde{y},\tilde{z})(t) = (\hat{x},\hat{y},\hat{z}) + \frac{1}{\rho_i} E_{i-1}\mu_{i-1}\,\mathrm{sign}(\tilde{e}_{i-1}), i = \overline{2,n} \tag{3.15}$$

The intermediate estimation errors $\tilde{e}_i(t)$ are provided by

$$\tilde{e}_i(t) = (\tilde{x}_i,\tilde{y}_i,\tilde{z}_i)(t) - (\hat{x}_i,\hat{y}_i,\hat{z}_i)(t), \quad i = \overline{1,n} \tag{3.16}$$

With $\tilde{x}(t) = x(t) = Y(t)$ by means of construction. While the estimation errors are characterized as

$$e_i(t) = (x_i,y_i,z_i)(t) - (\hat{x}_i,\hat{y}_i,\hat{z}_i)(t), \quad i = \overline{1,n} \tag{3.17}$$

Remark 2: To obtain the convergence condition, the rule of the switching logic E_i must achieve:

$$E_i, i = \overline{1,n-1} \begin{cases} \|\tilde{x}_i(t) - \hat{x}_i(t)\| \le \varepsilon, & \varepsilon \ll 0 \\ E_i = 0 & \text{other} \end{cases} \tag{3.18}$$

In the following theorem, the observer converges is confirmed.

Theorem . Assume that (assumption) is accurate. with the existence of positive parameters $\mu_i, i = 1,2,\ldots,n$, in this situation the observer states $(\hat{x},\hat{y},\hat{z})$ and the estimated input $\hat{m}(t)$ converges to (x,y,z) and to the secret message $m(t)$, respectively, in a finite period of time [13].

3.6 The Receiver system configuration

To simplify the design of the receiver that is based on a Step By Step Sliding Mode Oobserver (SBS-SMO), which makes the synchronization obtained successfully, Eqn (3.6) is reformulated here :

$$\begin{cases} D^{q_1}x(t) = \rho_1\hat{y}(t) + \psi_1(x(t)) \\ D^{q_2}y(t) = \rho_2\hat{z}(t) + \psi_2(x(t),y(t)) \\ D^{q_3}z(t) = a(x) + b(x)m(t) \\ Y(t) = x(t) \end{cases} \tag{3.19}$$

Where

$$
\begin{aligned}
\psi_1(x(t)) &= \alpha x^2(t) + \beta x^3(t) \\
\psi_2(x(t), y(t)) &= \delta_2(x(t) - R y(t)) \\
\rho_1 &= -\delta_1 \\
\rho_2 &= -\delta_2 \\
a(x) &= \delta_3 \left[i_0 - \gamma y(t) \right] \\
b(x) &= \delta_3
\end{aligned}
$$

The sliding mode observer, shown below, describes the receiver.

$$
\left\{
\begin{array}{c}
D^{q_1} x(t) = \rho_1 \hat{y}(t) + \psi_1(Y(t)) + \mu_1 \operatorname{sign}(x(t) - \hat{x}(t)) \\
D^{q_2} y(t) = \rho_2 \hat{z}(t) + \psi_2(x(t), \tilde{y}(t)) + E_1 \mu_2 \operatorname{sign}(\tilde{y}(t) - \hat{y}(t)) \\
D^{q_3} z(t) = E_2 \mu_3 \operatorname{sign}(\tilde{z}(t) - \hat{z}(t))
\end{array}
\right.
\tag{3.20}
$$

The auxiliary variables $\tilde{y}(t), \tilde{z}(t)$ are denoted by

$$
\begin{aligned}
\tilde{y}(t) &= \hat{y}(t) + \frac{1}{\rho_1} E_1 \mu_1 \operatorname{sign}(x(t) - \hat{x}(t)) \\
\tilde{z}(t) &= \hat{z}(t) + \frac{1}{\rho_2} E_2 \mu_2 \operatorname{sign}(\tilde{y}(t) - \hat{y}(t))
\end{aligned}
$$

The recovered message $m(t)$ is given by

$$
\widehat{m}(t) = \gamma \tilde{y} - i_0 + \frac{1}{\delta_3} E_2 \mu_3 \operatorname{sign}(\tilde{z}(t) - \hat{z}(t))
$$

3.7 Optimal selection of a SBS-SMO parameters

3.7.1 Comparison between the Hummingbird Optimizer and Grey Wolf Optimizer algorithms:

The Hummingbird Optimizer and Grey Wolf Optimizer algorithms are both nature-inspired optimization algorithms used for solving optimization problems. However, there are several differences between these algorithms, which are discussed below:

Inspiration: The Hummingbird Optimizer is inspired by the behavior of hummingbirds in nature, while the Grey Wolf Optimizer is inspired by the social behavior and hunting strategies of grey wolves.

Search Strategy: The Hummingbird Optimizer algorithm utilizes a local search strategy, where solutions are found by exploring and exploiting the neighborhood of the current solution. In contrast, the Grey Wolf Optimizer algorithm uses a global search strategy, where solutions are searched across the entire solution space by simulating the social hierarchy and hunting behavior of wolves.

Parameterization: The Hummingbird Optimizer uses only one parameter, which is the number of iterations, to control the optimization process. On the other hand, the Grey Wolf Optimizer uses three parameters, namely the population size, the number of iterations, and the alpha parameter that controls the rate of convergence.

Performance: Both algorithms have been shown to perform well on a variety of optimization problems, but their performance may vary depending on the problem characteristics. In general, the Grey Wolf Optimizer algorithm has been shown to perform better for high-dimensional optimization problems, while the Hummingbird Optimizer algorithm may perform better for low-dimensional problems.

Convergence rate: The Grey Wolf Optimizer algorithm is generally faster in terms of convergence rate compared to the Hummingbird Optimizer algorithm.

Complexity: The Hummingbird Optimizer algorithm is generally simpler and more straightforward to implement compared to the Grey Wolf Optimizer algorithm.

In summary, both the Hummingbird Optimizer and Grey Wolf Optimizer are powerful optimization algorithms with their unique strengths and weaknesses. The choice between these algorithms ultimately depends on the specific optimization problem being solved and the preferences of the user.

In summary, the numerical simulation has been used to illustrate the efficiency of the suggested strategy. Using certain parameters for the sliding mode is crucial in retrieving information in a short amount of time with ensuring high quality, especially in real time. To choose the appropriate parameters for the sliding mode observer, comparison experiments were performed between two optimization methods, the artificial hummingbird algorithm (AHA) and the grey wolf optimizer (GWO). And we concluded that (AHA) is faster than (GWO) so the parameters $\{\mu_1 \quad \mu_2 \quad \mu_3\}$ are set by AHA algorithm as $(123.9434, 28.8376, 59.1636)$, respectively.

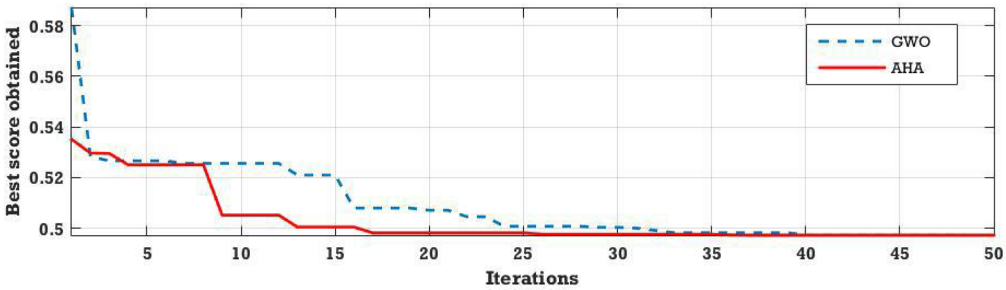

Figure 3.8: Best score of GOW and AHA during iteration.

Table 3.2: Parameters and values that obtained from the comparison.

	AHA	GWO
Search Agents	15	15
Max iteration	50	50
Best score	0.4972	40.4972
Convergence time	**26**	**40**
Parameters [u1 u2 u3]	[**123.9434**, **28.8376**, **59.1636**]	[123.8894, 28.8187, 59.1849]

3.7.2 Hummingbird Optimizer algorithms

The Hummingbird Optimizer algorithm is a nature-inspired optimization algorithm that is based on the behavior of hummingbirds. Hummingbirds are known for their exceptional ability to hover in mid-air and navigate through complex environments with great agility and precision. The Hummingbird Optimizer algorithm mimics the behavior of these birds to solve optimization problems.

The algorithm utilizes a local search strategy, where solutions are found by exploring and exploiting the neighborhood of the current solution. The exploration is done by randomly selecting a new solution from the neighborhood of the current solution, while the exploitation is done by selecting the best solution from the neighborhood. This search strategy allows the algorithm to quickly converge to a near-optimal solution.

The Hummingbird Optimizer algorithm requires only one parameter, which is the maximum number of iterations. The algorithm has been shown to perform well on a variety of optimization problems, especially on low-dimensional problems.

In summary, the purpose of this section is to select the suitable parameters of the receiver system that achieve the synchronisation by using the new artificial hummingbird algorithm (AHA). The AHA is chosen among several optimization techniques because it is a simple yet effective that is inspired by the behavior of hummingbirds in nature. It utilizes a local search strategy to efficiently explore and exploit the search space, with only one parameter to control the optimization process, in addition to the high performance that it provides in more than 50 benchmark optimization problems, compared to the other meta-heuristic algorithms. AHA was introduced by ([159]). The general structure of AHA is demonstrated in Figure (3.9). The AHA algorithm solves the convex optimization problem, which is written as an objective function (3.21) that attempts to minimise the mean square error (MSE) between the emitter and receiver states.

$$[H]J = \min \sum \begin{pmatrix} x(t) - \hat{x}(t) \\ y(t) - \hat{y}(t) \\ \underbrace{z(t) - \hat{z}(t)}_{e} \end{pmatrix}^2 \tag{3.21}$$

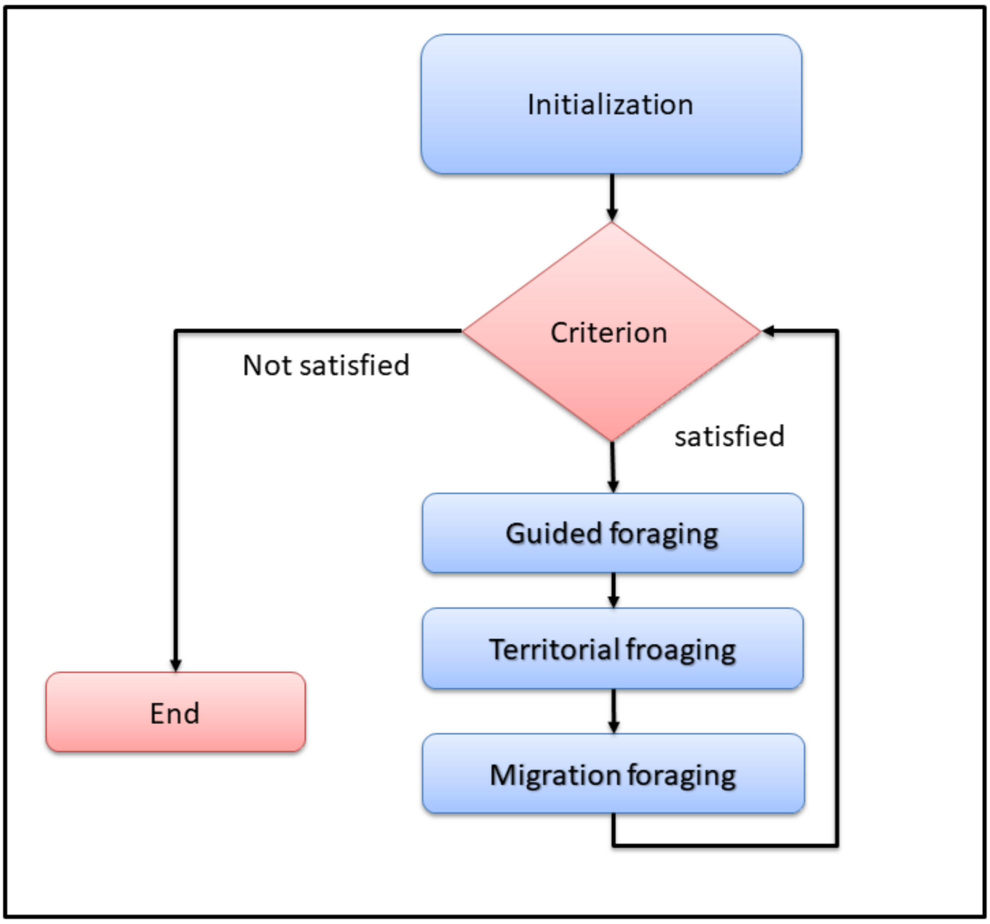

Figure 3.9: structure of AHA.

This part illustrates the efficiency of the suggested strategy. Using certain parameters for the sliding mode is crucial in retrieving information in a short amount of time while ensuring high quality. However, in particular works of literature, the parameters are randomly selected. Figure (3.12) show how the appropriate parameters for the SBS-SMO are obtained. Table (3.3) represents the parameters and values used in the simulation.

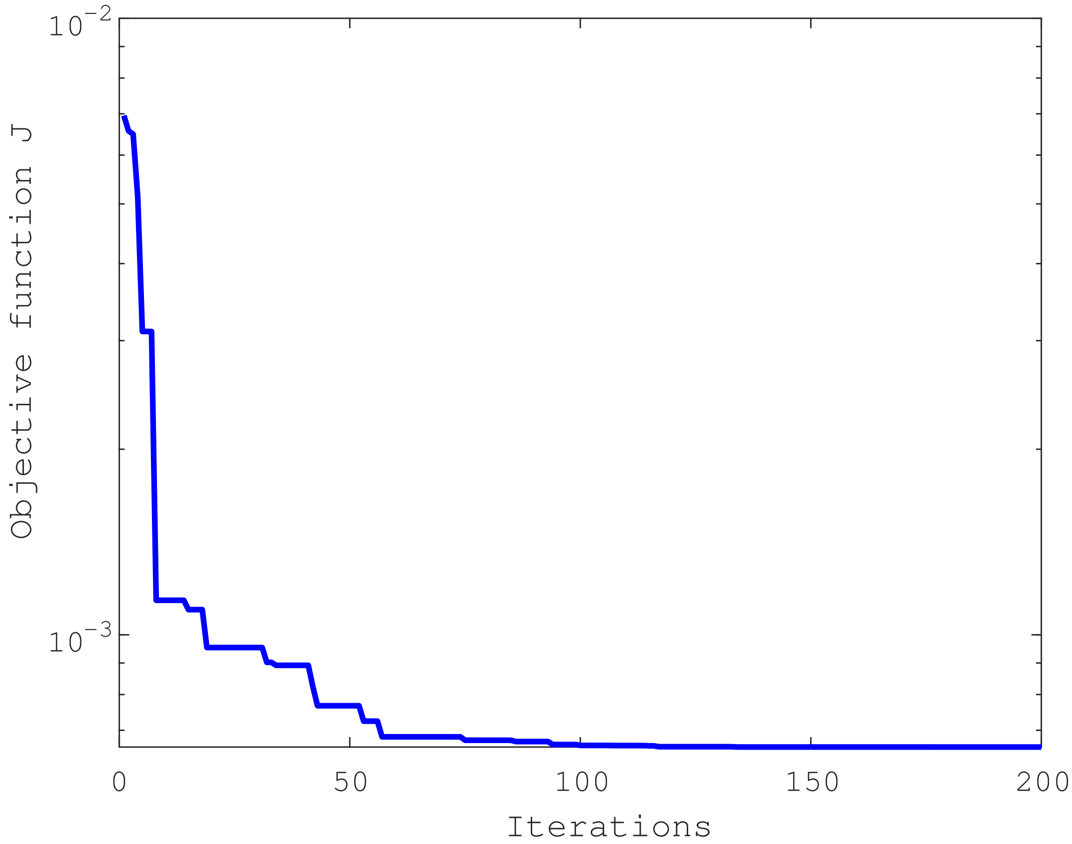

Figure 3.10: The greatest possible AHA score attained during iterations.

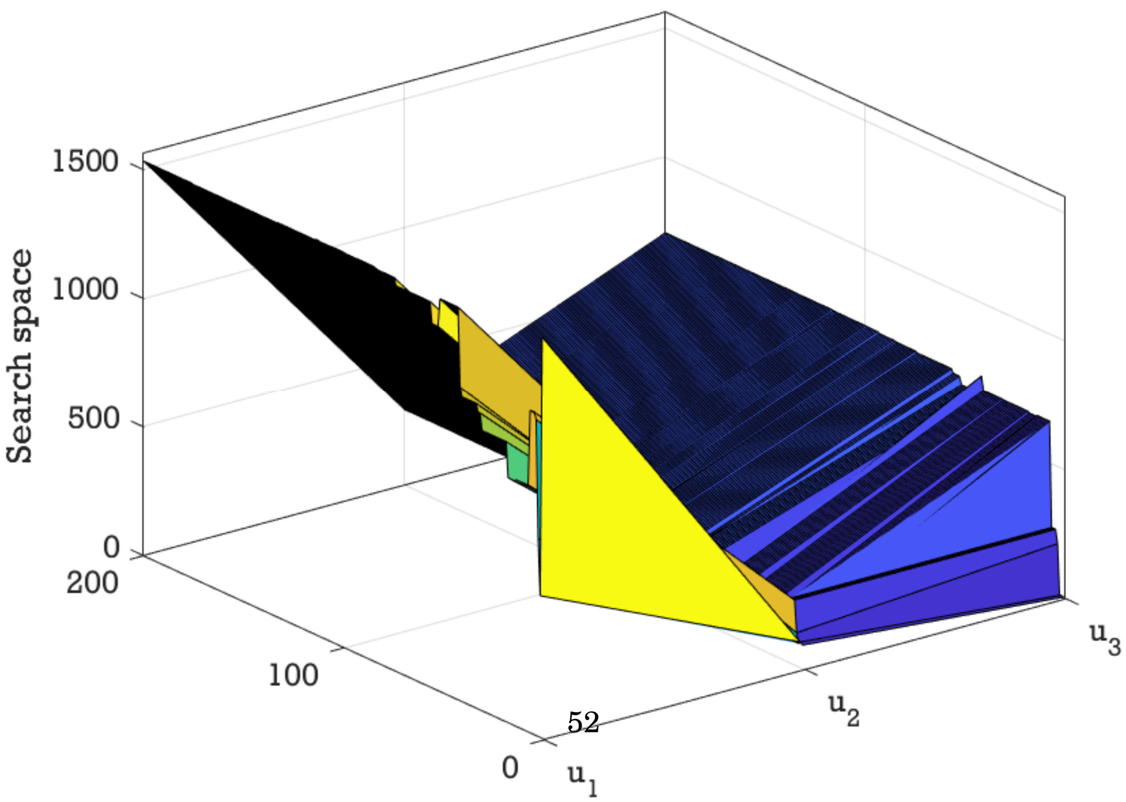

Figure 3.11: The optimization vector of the AHA algorithm and its dynamic changes.

Table 3.3: Parameters and values used in simulation.

Parameters	Values
Search Agents	15
Max iteration	200
Parameters [u1 u2 u3]	[1494.3, 283, 712.2]

3.8 Conclusion

This study proposes a new approach to improve the safety of encrypted communications by using an optimization algorithm to enhance the performance of an observer. An optimal Step-By-Step Sliding Mode Observer (SBS-SMO) is set up on the receiver's side to ensure synchronization between the sender and receiver. Based on the results obtained, the study concludes that outsiders can only retrieve secret information if they have access to certain details, including the type of chaotic system (such as Lorenz, Rössler, Chua's, and Chen's), the dynamic parameters of the system, the order and initial conditions of the system, the state in which the message was included, the mechanism of the synchronization process (such as controller or observer), and the nature and behavior of the encrypted/decrypted data (text, image, voice, and video). However, obtaining all of this information can be extremely difficult.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

Fractional calculus is a branch of mathematical analysis that deals with integrals and derivatives of non-integer orders. It extends the traditional calculus, which deals with integrals and derivatives of integer orders, to include integrals and derivatives of fractional orders. In fractional calculus, the concept of a fractional derivative is used, which is a generalization of the traditional derivative to non-integer orders. It represents the rate of change of a function with a fractional order, such as 1/2, 3/4, or any other non-integer value. Fractional calculus has many applications in various fields of science and engineering, such as physics, signal processing, control systems, finance, and medicine. It has been used to model complex systems that cannot be accurately represented using traditional calculus, such as fractional Brownian motion, fractional differential equations, and fractional-order control systems. Overall, fractional calculus is an important tool for modeling and analyzing complex systems that exhibit non-integer behaviors. Chua's oscillator is a simple nonlinear circuit that was introduced by Leon Chua in 1983. It is a three-dimensional autonomous system that exhibits chaotic behavior, which means that its output signal is unpredictable and appears random. The Chua's oscillator consists of three basic components: a resistor, an inductor, and a capacitor, and it can be realized using electronic components. The Chua's oscillator is one of the most well-known chaotic systems, and it has been widely used in various applications, such as secure communication, image encryption, and random number generation. The dynamics of the Chua's oscillator are characterized by the presence of a double-scroll attractor, which is a strange attractor with two scroll-like structures. Overall, Chua's oscillator is an important system in the field of chaos theory and has many practical applications in engineering and science. Its simple structure and complex dynamics make it an attractive candidate for various applications, especially in the area of secure communication and encryption.

This thesis focuses on developing a secure communication system using fractional-order chaos. However, identifying the order of the derivative and the identifiability of fractional-order derivatives are still unresolved issues. To address this, a fractional-order chaotic system is used to generate the encrypted message signal on the transmitter side, where the secret message is modulated within the chaotic dynamics. On the receiver side, a step-by-step fractional-order chaotic observer is proposed to achieve robust synchronization between the transmitter and receiver, and the original message is recovered once chaos synchronization is achieved and the state variables are estimated.

4.2 The simulation results

4.2.1 result of the states

Figure (**??**) depict the responses of the states (x, y, z) and their estimated $(\hat{x}, \hat{y}, \hat{z})$. (4.4) shows the three state variable estimation errors altogether. It proves that the two FOCS can be synchronized optimally in finite time. The initial conditions are assumed to be (x(0) = y(0) = z(0) = 0), whereas the responding system's initial conditions are all set to zero $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0)$.

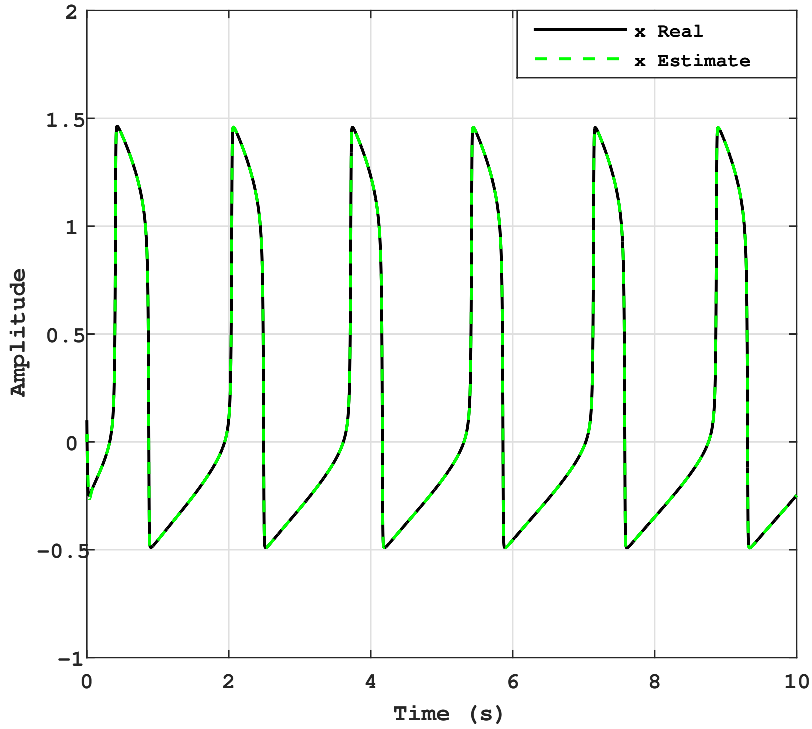

Figure 4.1: dataset synchronization of $x - \hat{x}$.

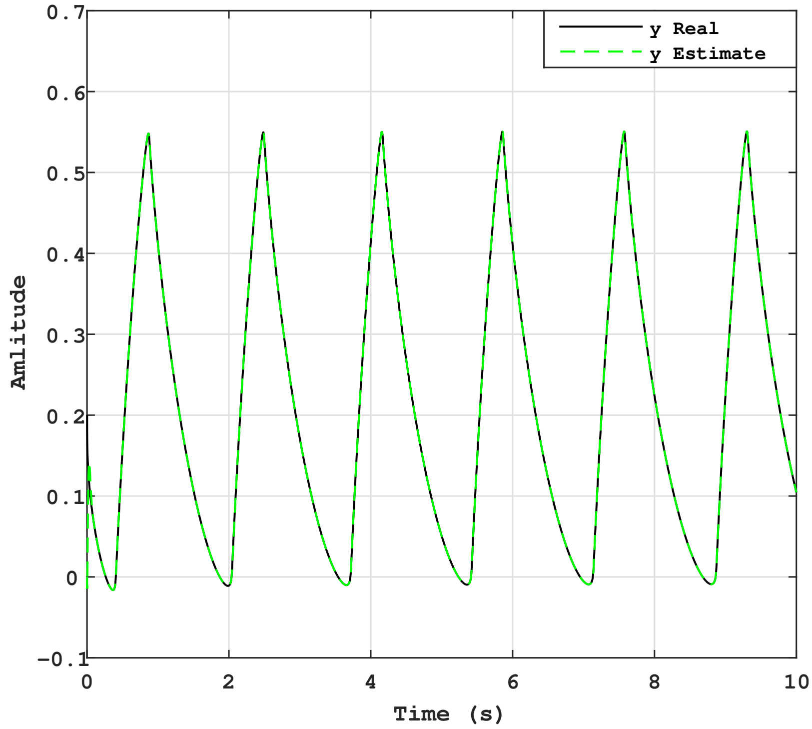

Figure 4.2: dataset synchronization of $y - \hat{y}$.

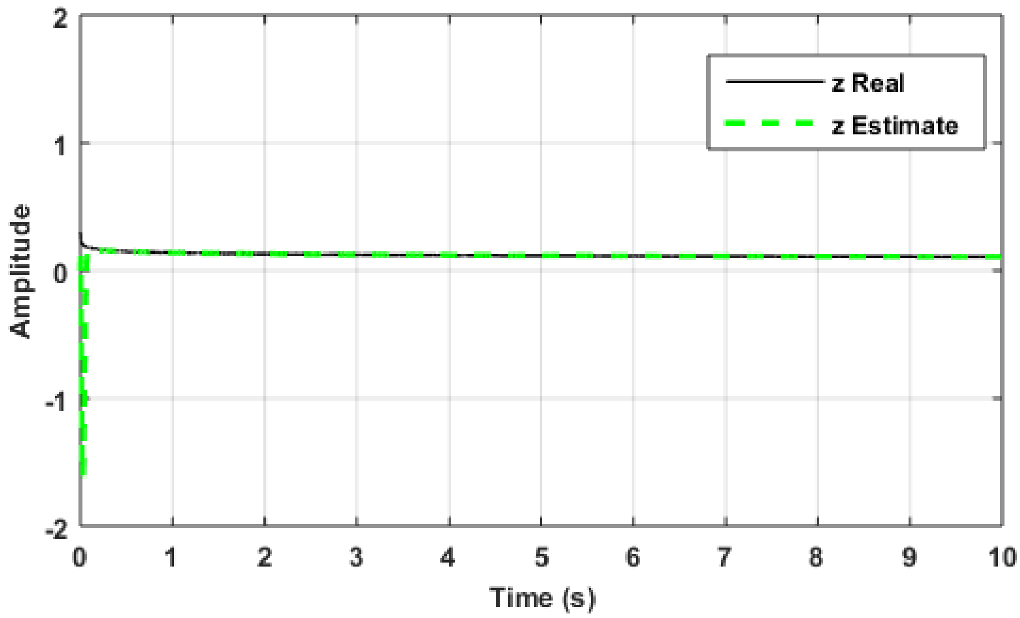

Figure 4.3: dataset synchronization of $z - \hat{z}$.

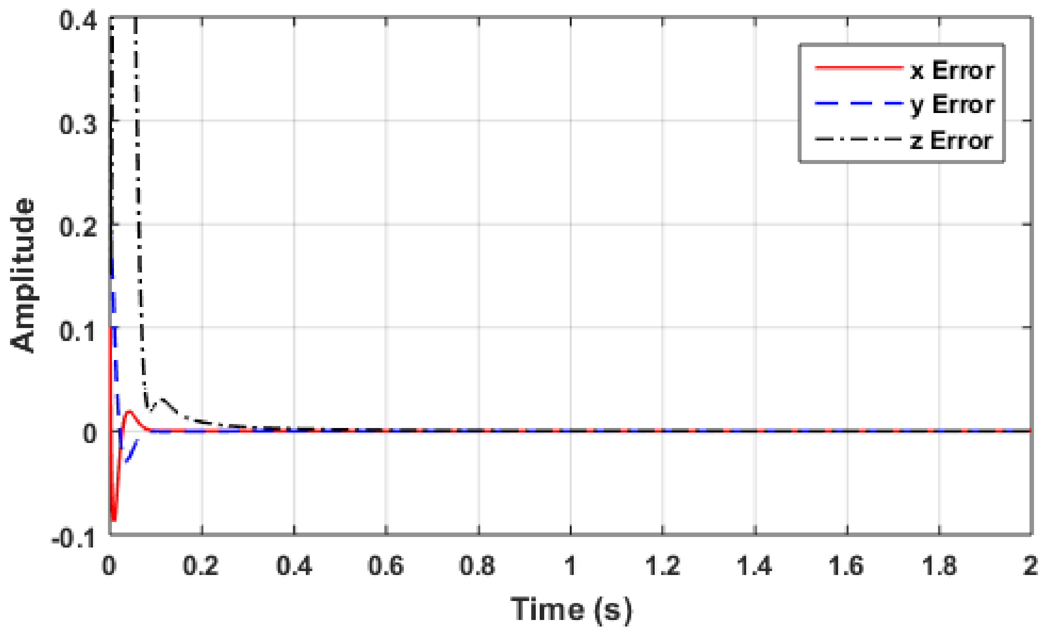

Figure 4.4: The estimation errors: x error, y error and z error.

4.2.2 Security level study

The Fig (4.5,4.6,4.7) depict the responses of the state (x, y, z) and their estimations $(\hat{x}, \hat{y}, \hat{z})$. and Fig 4.8 demonstrates the three state variables' estimation errors all together. It proves that the two fractional-order chaotic systems can be synchronized. The initial conditions are assumed to be $(x(0) = y(0) = z(0) = 0.1)$, whereas the responding system's initial conditions are all set to zero $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0.02)$.

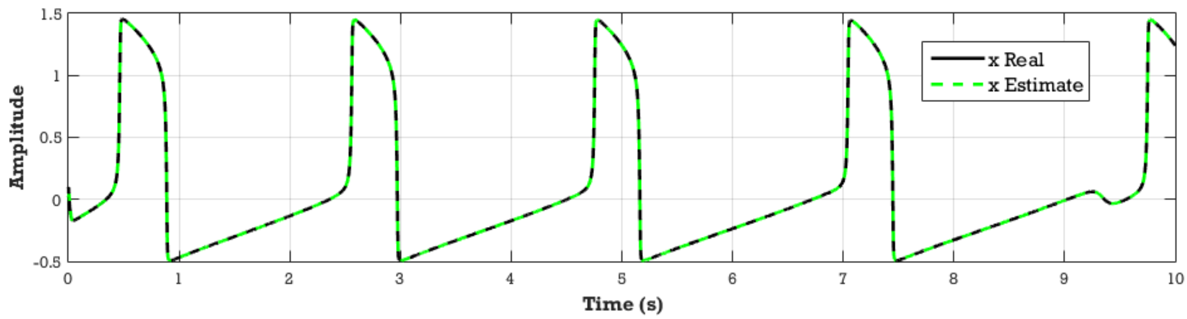

Figure 4.5: dataset synchronization of $x - \hat{x}$.

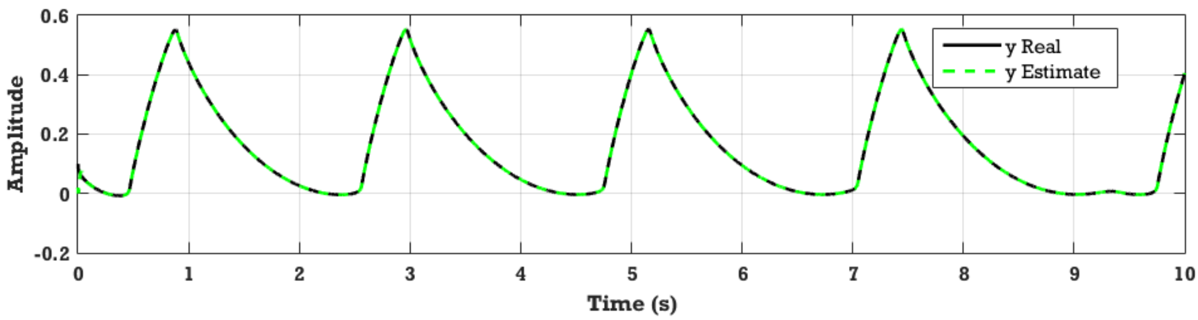

Figure 4.6: dataset synchronization of $y - \hat{y}$.

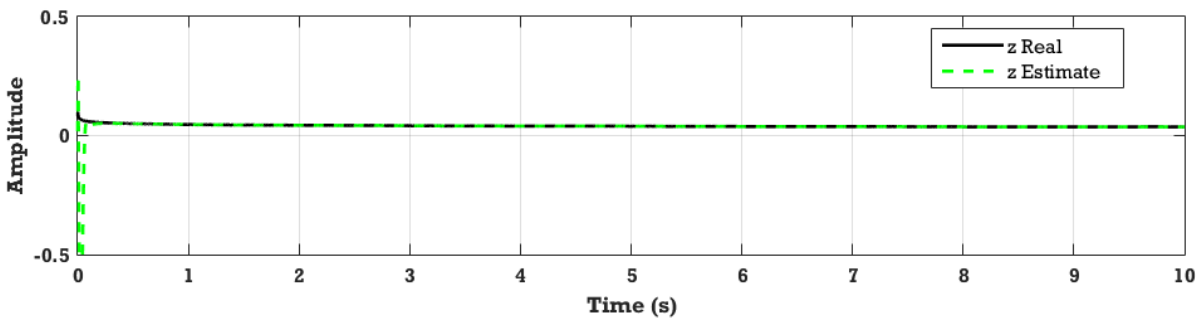

Figure 4.7: dataset synchronization of $z - \hat{z}$.

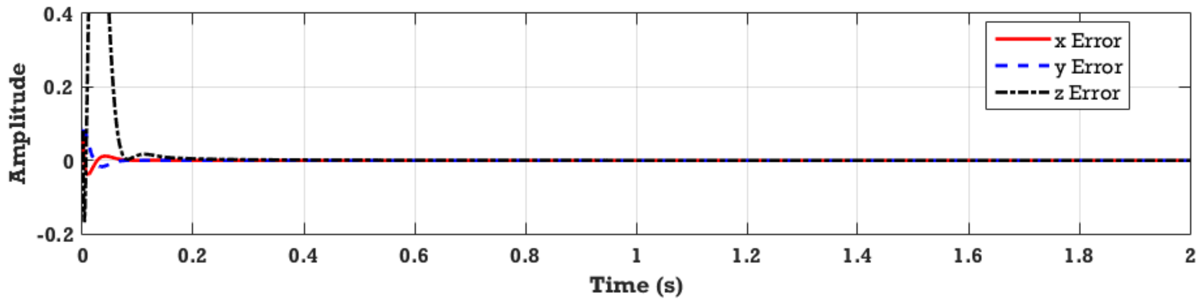

Figure 4.8: The estimation errors: x error, y error and z error..

Fig (4.9,4.10,4.11) plots the three state responses (x, y, z) and their estimations $(\hat{x}, \hat{y}, \hat{z})$ and Fig 4.12 is the synchronization errors with the following values, the fractional order of the drive system is = 0.9801, the Fractional order of the responding system = 0.98, and initial conditions of the transmitter are all set as: $(x(0) = y(0) = z(0) = 0.1)$ and the reciver system are: $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0.02)$. In order to ensure the convergence and the synchronization.

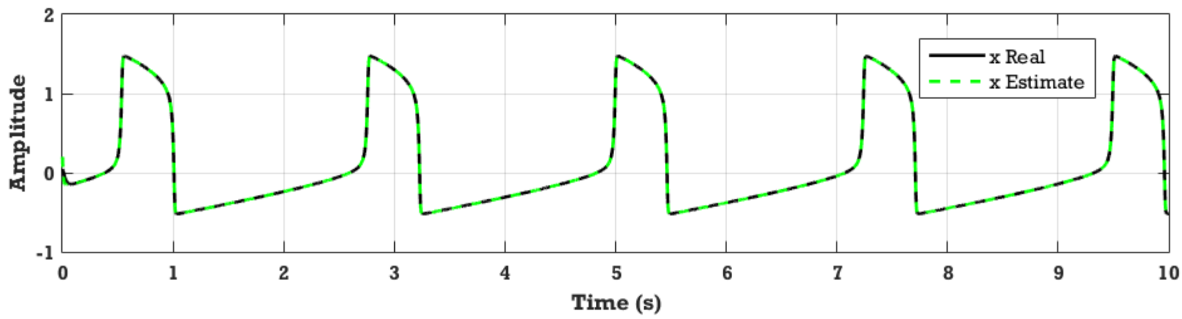

Figure 4.9: dataset synchronization of $x - \hat{x}$.

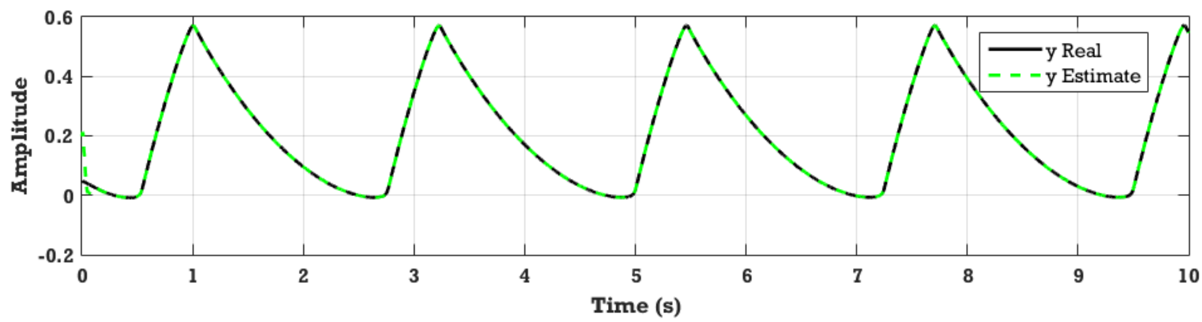

Figure 4.10: dataset synchronization of $y - \hat{y}$.

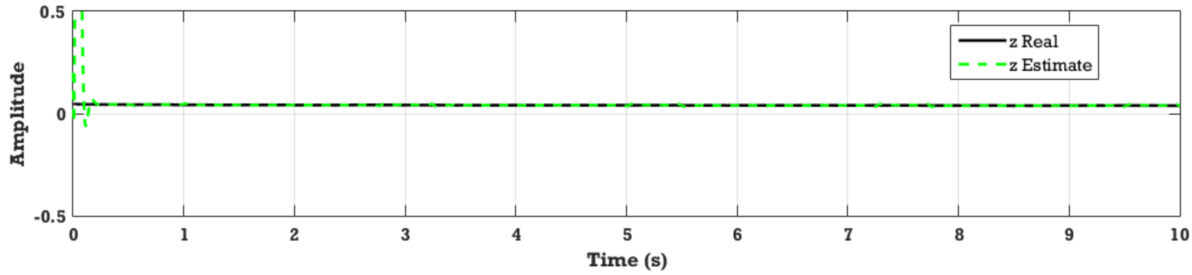

Figure 4.11: dataset synchronization of $z - \hat{z}$.

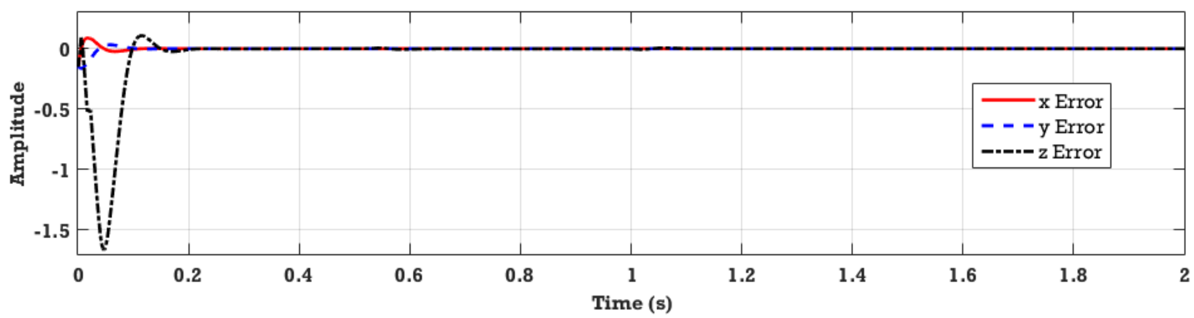

Figure 4.12: The estimation errors: x error, y error and z error..

Here in Fig (4.13,4.14,4.15 and 4.16) we kept the fractional order of the drive system and the responding at 0.9, the but the initial conditions for the drive system are: $(x(0) = y(0) = z(0) = 0.02)$, and the responding system kept at zero $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0)$.

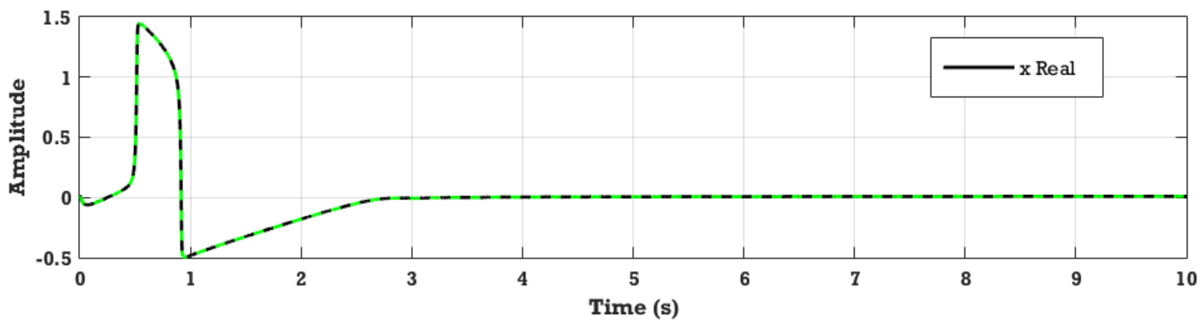

Figure 4.13: dataset synchronization of $x - \hat{x}$.

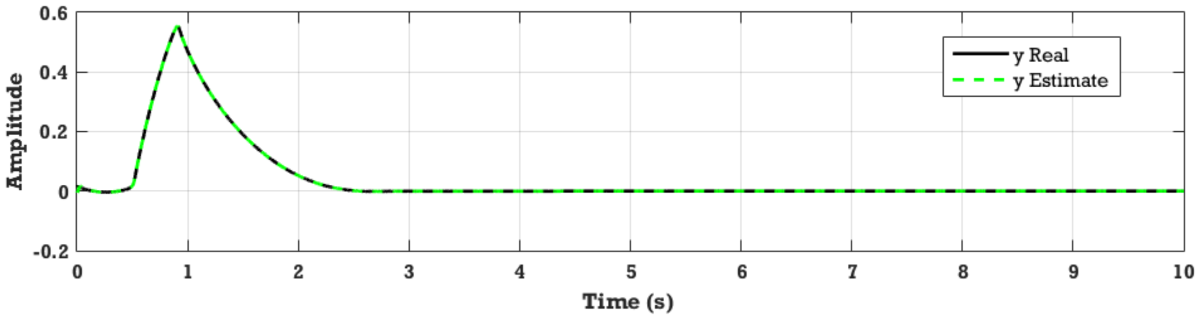

Figure 4.14: dataset synchronization of $y - \hat{y}$.

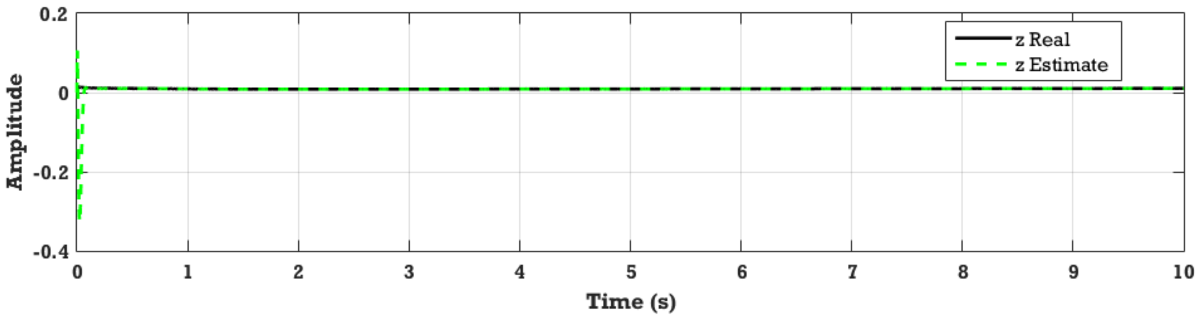

Figure 4.15: dataset synchronization of $z - \hat{z}$.

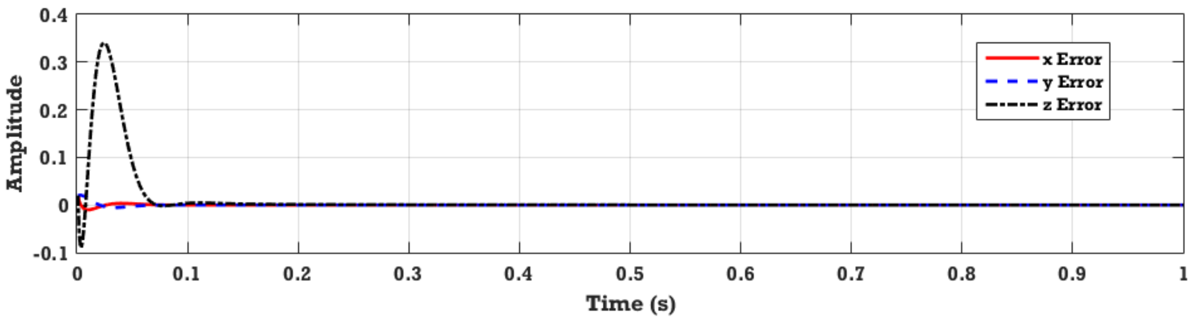

Figure 4.16: The estimation errors: x error, y error and z error..

4.2.3 Robustness study

To prove the strength of the system, we changed the values of the fractional order of the drive system = 0.95, the Fractional order of the responding system = 0.95005, and initial conditions of the transmitter and the receiver ($x(0) = y(0) = z(0) = 0.025$) ($\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0.01$) respectively. as shown in fig **??**

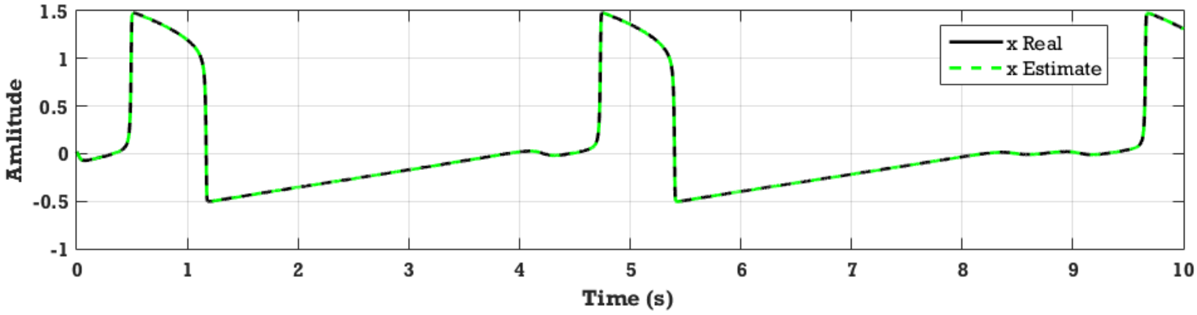

Figure 4.17: dataset synchronization of $x - \hat{x}$.

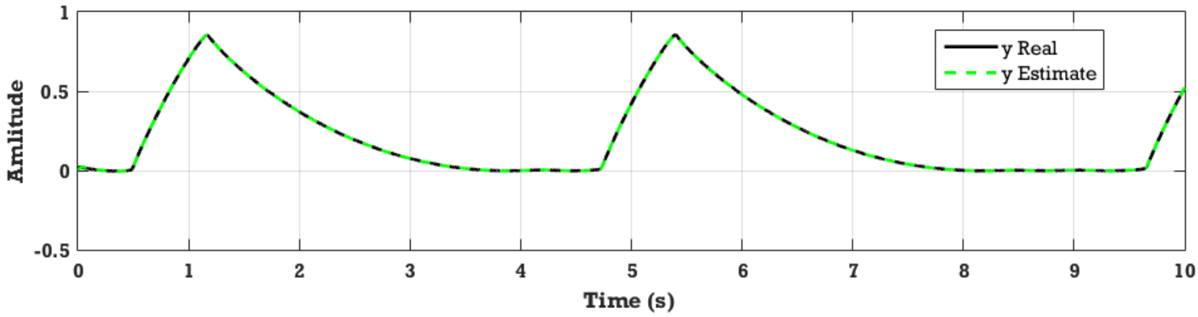

Figure 4.18: dataset synchronization of $y - \hat{y}$.

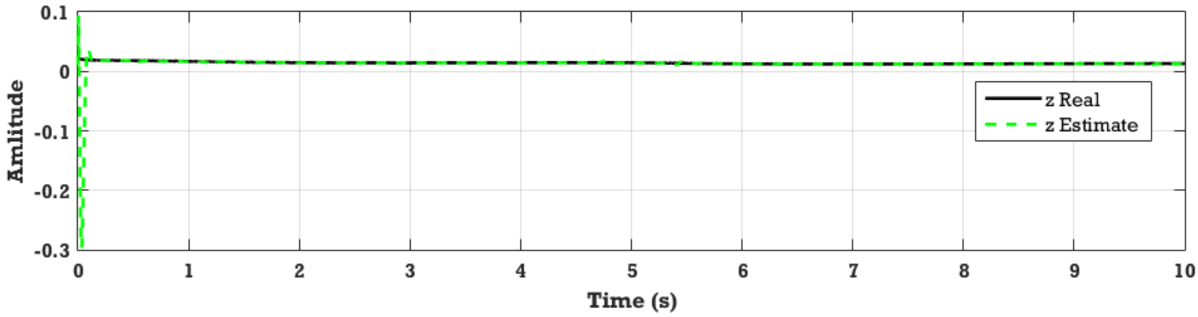

Figure 4.19: dataset synchronization of $z - \hat{z}$.

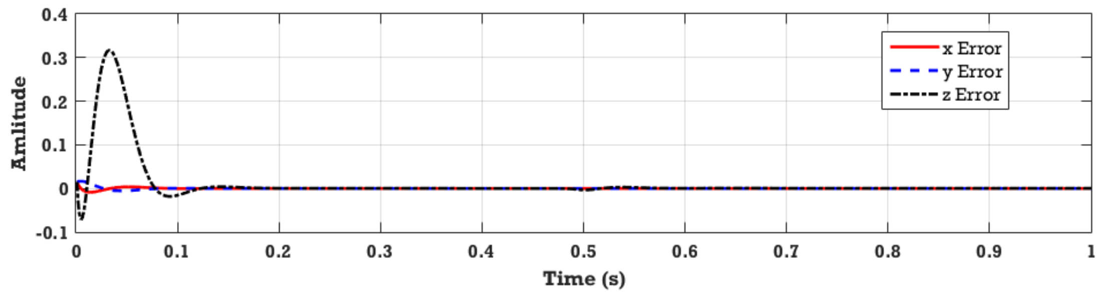

Figure 4.20: The estimation errors: x error, y error and z error..

4.2.4 Retrieving the secret message

In this part, we try to exploit the features and characteristics of the SBS-SMO (synchronization and estimating inputs that are unknown) to harness it in the information encryption. First, we include the secret information, which we consider as: sin, square, and voice functions, given by $m(t) = 0.1sin(2t), m(t) = 0.1square(2t)and, m(t) = voice$, respectively. As shown in Figures (**??**), the recovered messages in the receiver system are similar to the original ones, and Table (4.1) illustrates the compression of our work to the work of ([13]) in four different error messages (sin, square, sawtooth, and voive) The MSE errors in our work are about zero in all cases, taking into consideration that ([13]) did not achieve the voice message, but we tried it in order to compare it with our voice signal.

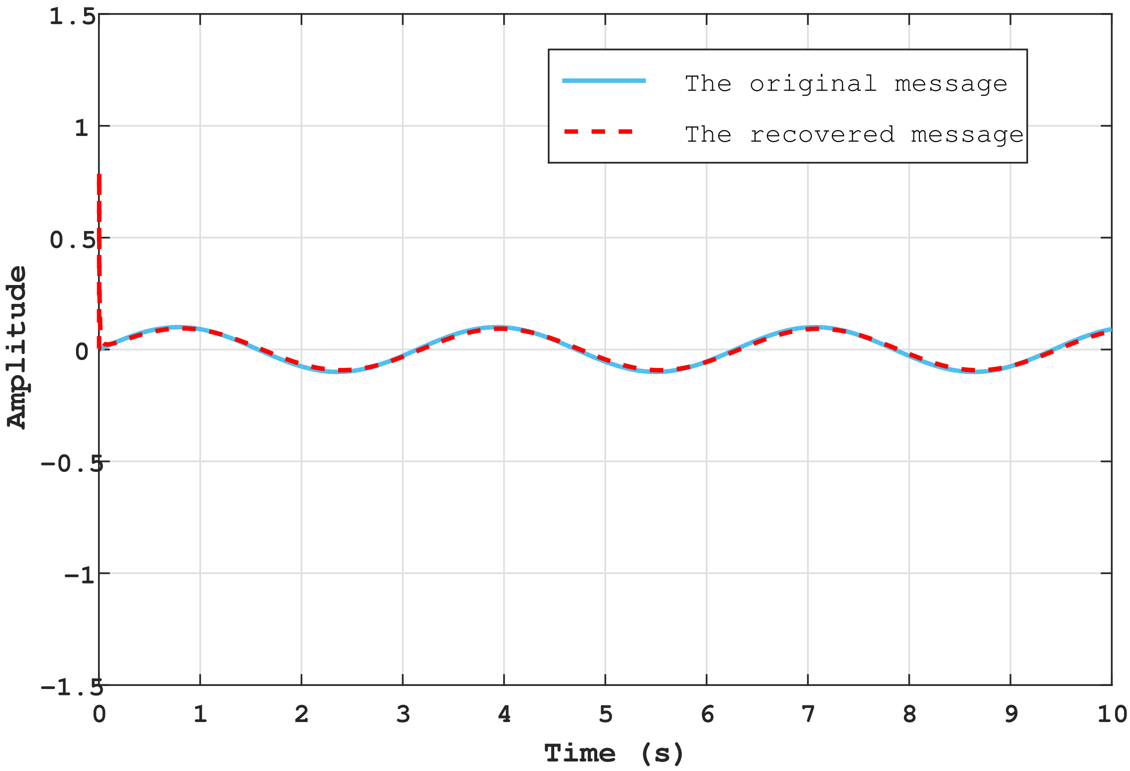

Figure 4.21: The original message (sinusoidal wave $m(t) = \sin(t)$) and the recovered $\hat{m}(t)$.

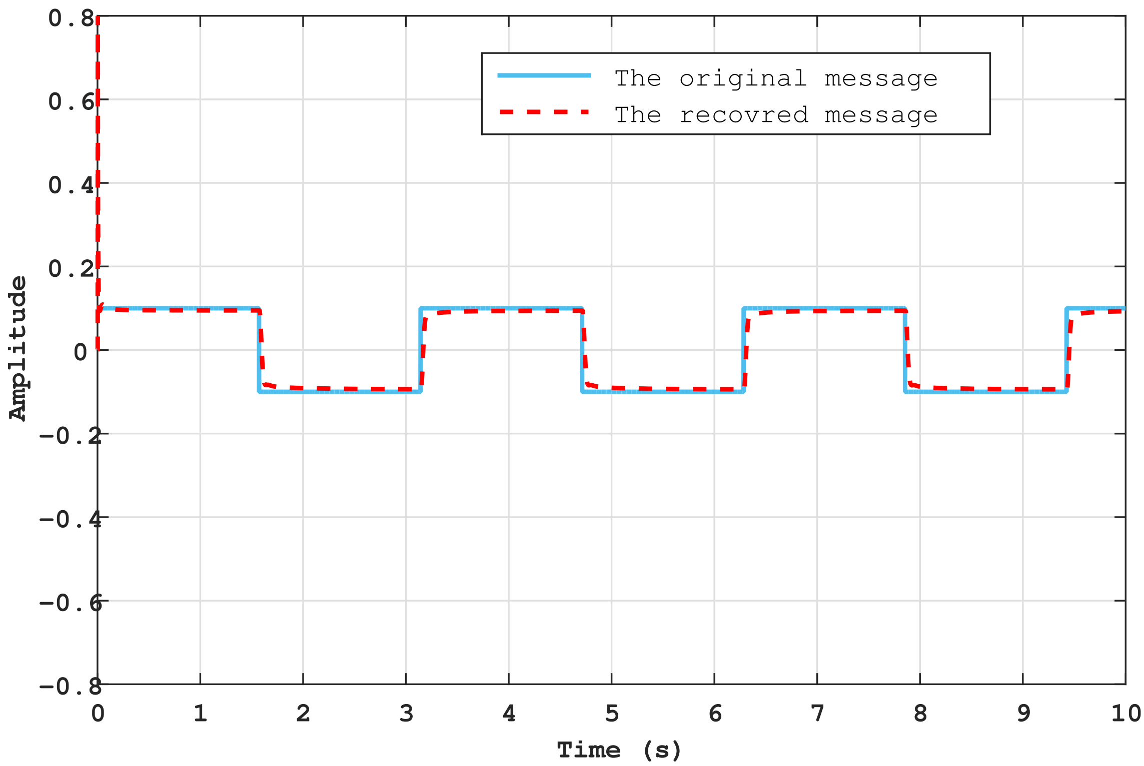

Figure 4.22: The original message $m(t) = \text{square(t)}$ and the recovered $\hat{m}(t)$.

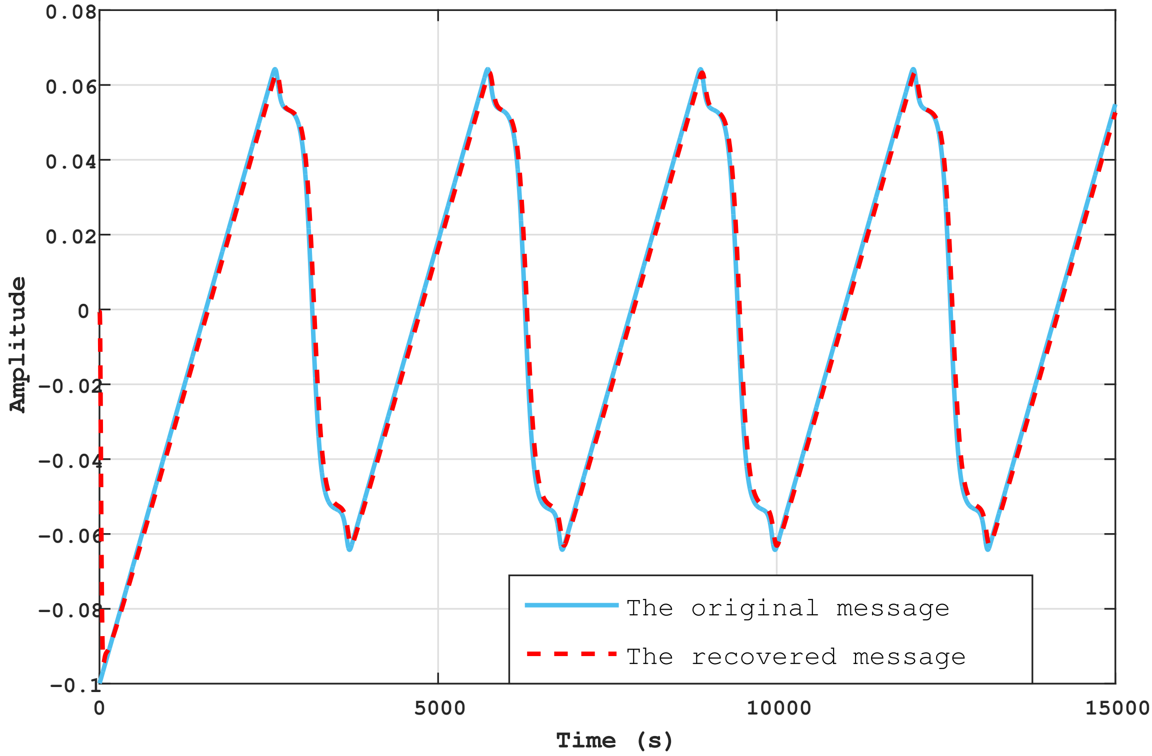

Figure 4.23: The original message $m(t) = \text{sawtooth(t)}$ and the recovered $\hat{m}(t)$.

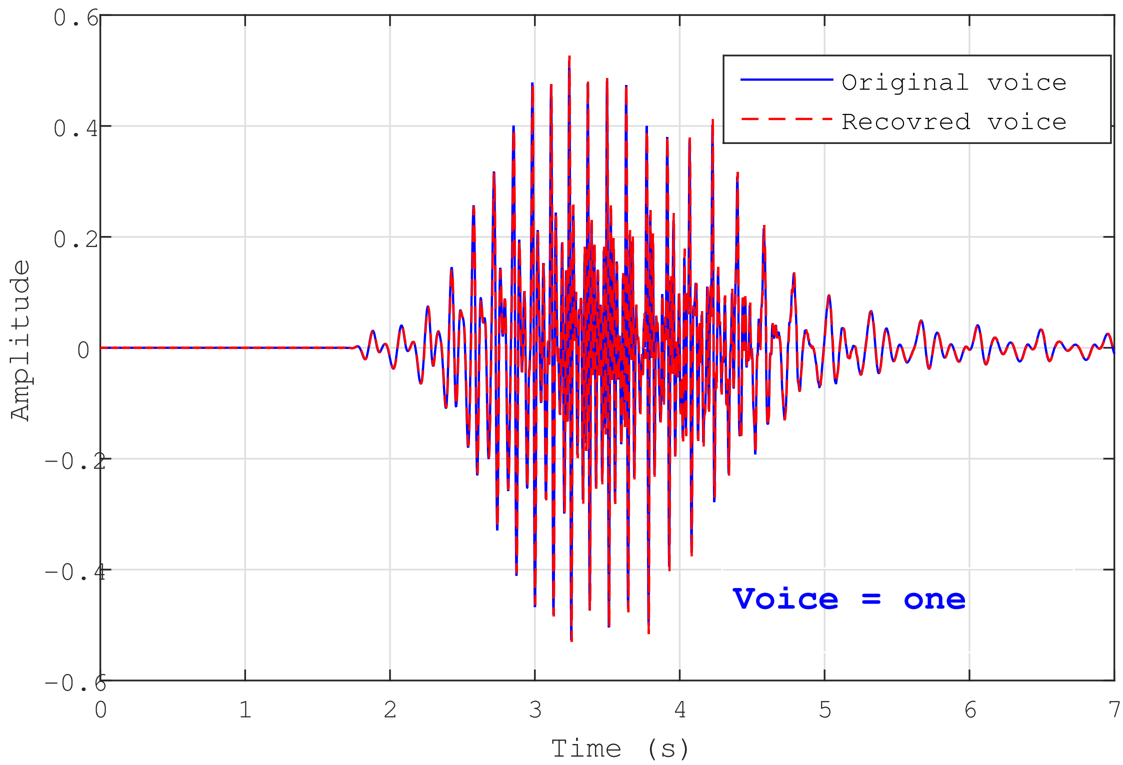

Figure 4.24: The original message $m(t) = \text{voice}$ and the recovered $\hat{m}(t)$.

Table 4.1: Comparative study in several kinds of messages.

Methods	Sin message	Square message	Sawtooth message	Voice message
Proposed	7.8278×10^{-05}	2.2965×10^{-04}	2.1986×10^{-05}	6.5561×10^{-04}
[13]	8.2397×10^{-04}	0.0024	0.0022	0.0069

4.2.5 Security analysis of the proposed scheme

In order to evaluate the amount of security provided by the suggested secure transmission scheme, two principal modifications have been made; the initial condition and the order of the transmitter or the receiver. The original voice message was picked among the previous messages due to its importance. In Figure (4.25). We only made a tiny modification in the initial conditions of the transmitter system's first state ($x(0) = 10^{-}6¬†, y(0) = z(0) = 0$). Based on our analysis, the receiver side completely lost the included message. The second check involved a small modification in the first state order of the transmitter system. $q_1 = 0.905, q_2 = 0.9, q_3 = 0.9$, as shown in Figure (4.26). And we discovered that the embedded message is absolutely lost as well.

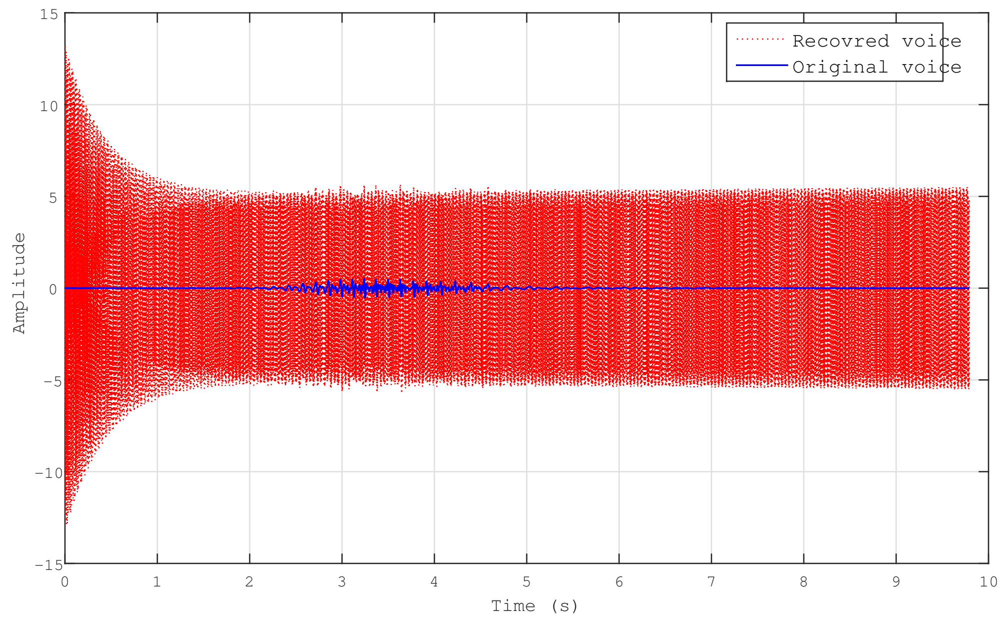

Figure 4.25: The original message $m = voice$ and the recovered $\hat{m}(t).x(0) = 10^{-6}, y(0) = z(0) = 0$. and $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0)$. And both order of transmitter and receiver are $q_1 = q_2 = q_3 = 0.9$.

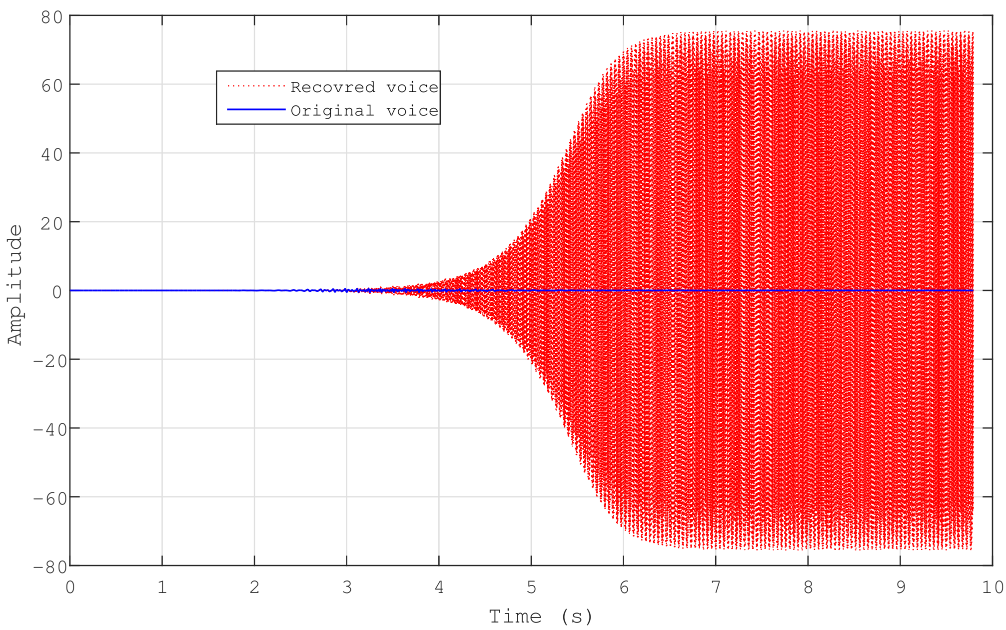

Figure 4.26: The original message $m = voice$ and the recovered $\hat{m}(t)$. $(x(0) = y(0) = z(0) = 0)$ and $(\hat{x}(0) = \hat{y}(0) = \hat{z}(0) = 0)$. And the order of transmitter are $q_1 = 0.905, q_2 = q_3 = 0.9$. while of the receiver are set all 0.9.

4.3 Conclusion

In this work, a new way to improve the safety of encrypted communications is suggested by using a new optimization algorithm to make the observer's performance as good as possible. We set up an optimal SBS-SMO on the receiver side to make sure that the emitter and receiver are synchronized. Based on the obtained results, we conclude that the secret information can be retrieved by outsiders only when the following items are provided:

- Chaotic system type such as Lorenz, Rössler, Chua's and the Chen....etc.
- Dynamic Parameters of the chaotic system.
- The order and the initial conditions of the system and in which state was the message included.
- The mechanism of the synchronization process such as controller or an observer and which type of them.
- What is the nature and the behavior of the encrypted/decrypted data like (text, image, voice and video But it is extremely difficult to get all this equipment together.

General Conclusion

In this thesis, we have designed and implemented a fractional-order chaotic safe communication system, which has demonstrated the potential to provide a high level of security for transmitting sensitive information. The system consists of a transmitter and a receiver, with the transmitter utilizing a fractional-order chaotic system to create an encrypted message signal, and the receiver employing an optimal Step-By-Step Sliding Mode Observer to ensure synchronization between the emitter and receiver.

Our research has shown that retrieving secret information from the system requires knowledge of several key components, including the type and dynamic parameters of the chaotic system, the order and initial conditions of the system, the state in which the message was included, and the mechanism of the synchronization process. This suggests that the system has a high level of security, as it would be difficult for outsiders to access the encrypted data without possessing this knowledge.

Furthermore, our study has highlighted the importance of designing a secure communication system that can protect against potential security threats. While the fractional-order chaotic system has shown promise as a means of achieving this goal, continued research and development will be necessary to ensure its continued effectiveness in the face of evolving security threats.

In conclusion, our work has demonstrated the potential of fractional-order chaotic systems as a means of providing secure communication, and has highlighted the importance of designing and improving such systems to ensure the highest level of security possible. We believe that our research will contribute to the ongoing efforts to develop secure communication systems that can protect against potential security threats, and we hope that it will inspire further research in this important field.

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ملخص:

توفر هذه الأطروحة نظام اتصال آمنًا يعتمد على الوضع الانزلاقي الأمثل للمراقبين خطوة بخطوة وأنظمة تشوا الفوضوية الجزئية. على جانب الباعث ، يتم استخدام نظام فوضوي من أجل كسري كنظام محرك الأقراص لإنشاء إشارة الرسالة المشفرة. يتم تعديل رسالة الإدخال السري في الديناميكيات الفوضوية عن طريق الإدخال بدلاً من إدخالها مباشرة في الإشارة الفوضوية على خط الإرسال. على جانب جهاز الاستقبال ، يُقترح مراقب فوضوي من الدرجة الجزئية خطوة بخطوة يخضع لمدخلات غير محددة كنظام استجابة للحصول على مزامنة قوية بين الباعث وجهاز الاستقبال, تم اختيار معاملات مراقب الوضع المنزلق خطوة بخطوة على النحو الأمثل باستخدام خوارزميتين للتحسين: التحسين الذكي المعروف محسن الذئب الرمادي وخوارزمية الطائر الطنان الاصطناعي . بعد تحقيق التزامن على جانب جهاز الاستقبال ، يتم الحصول على تقدير متغير الحالة بنجاح ، مما يؤدي إلى إعادة البناء الدقيقة للمعلومات السرية التي تحتوي على إشارة ونص عادي ورسالة صوتية.

Résumé :

Cette thèse fournit un système de communication sûr basé sur des observateurs pas à pas optimaux en mode coulissant et des systèmes chaotiques à ordre fractionnaire de Chua. Côté émetteur, un système chaotique à ordre fractionné est utilisé comme système de lecteur pour créer le signal de message crypté. Le message secret d'entrée est modulé dans la dynamique chaotique par insertion plutôt que d'être directement inséré dans le signal chaotique sur la ligne de transmission. Côté récepteur, un observateur chaotique étape par étape d'ordre fractionnaire soumis à une entrée non identifiée est suggéré comme système de réponse pour obtenir une synchronisation robuste entre l'émetteur et le récepteur. Les paramètres de l'observateur pas à pas en mode coulissant ont été sélectionnés de manière optimale à l'aide de deux algorithmes d'optimisation: l'optimisation intelligente bien connue Optimizer loup gris et l'algorithme de colibri artificiel. Nous avons ensuite fait une comparaison entre les deux afin d'atteindre les meilleurs paramètres. Après la synchronisation est atteinte. Côté récepteur, l'estimation de la variable d'état est obtenue avec succès, ce qui conduit à la reconstruction précise des informations confidentielles contenant un signal, un texte brut et un message vocal.