

CHAPTER I: DC-DC Converter Modeling

I.1. Introduction

Electrical energy, between its initial production and its final use, must often go through multiple conversions to suit the needs of consumers. Given its low cost and high efficiency, power electronics becomes more and more the optimal solution to condition the electrical energy. One of the branches of electrical engineering which had a major technological change is the static converters. It has seen a significant improvement in their performance.

Static converters generate the electrical energy in industrial processes increasingly complex and varied. Also, their structures are diversifying to satisfy the highest requirements. They also use the same principle: from a given source of energy, it must obtain the desired energy, through the controller elements a series of switches.

DC-DC converters are widely used in industrial environment. With their higher performance and reduced cost, they occupied an important place in the power sources of laptops, mobile phones, home appliances.

A power converter may be characterized as a periodic, non-linear and varies over time due to the change of the topology of their circuit according to the ON states and blocked switches and diodes. These circuits are typically controlled by pulse width modulation PWM (Pulse Width Modulation) or similar techniques to adjust the voltage or current supplied to the loads. The controller determines that one passes from one configuration to another by making the transitions occur cyclically or in discrete time.

The most common DC-DC converters types are: the Buck, Boost and Buck-Boost. Buck and Boost decreases increases the output voltage by relative to an input voltage.

A DC-DC converter can operate in two modes:

- Continuous Conduction Mode (CCM)
- Discontinuous Conduction Mode (DCM)

The modeling of converters is designed to analyze the dynamic behavior, and to synthesize the necessary control laws that make it possible to achieve the desired performance. The major difficulty comes from these converters, they are non-linear and having a number of separate electrical configurations during the switching period. Modeling the dynamic behavior must serve to characterize the operation of DC-DC converter in both conduction mode (continuous and discontinuous).

We will restrict our study to the DC-DC Boost converter, this choice is motivated by its wide range of use.

I.2. The Boost Converter

A boost converter can be represented by the circuit of Figure 1-1, where V_g represents power supply voltage, i_L the current through the inductance L , sw an electronic switch, VD a diode, v_c the voltage on the capacitor C and $u_0(t)$ voltage output across the resistive load R . Also it is noted that the switching is instant considered and the threshold voltage of the diode VD is zero.

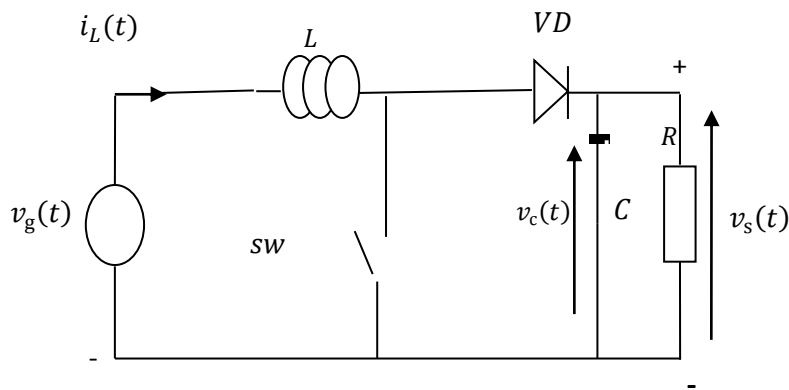


Fig 1- 1 Boost converter .

The operation of this type of converter can be summarized as follows:

The output voltage is obtained by acting on the switch sw which is controlled opening and closing. Closing sw , inductance L is loaded and is opened to sw transferring the stored energy to the load. Over a fixed period T , the ratio is defined cyclic converter as the ratio between the closing time $t_{closing}$ and period T :

$$d = \frac{t_{closing}}{T}$$

I.3. Dc-Dc Converter Operation Mode

The DC/DC boost converter has two fundamentally different modes of operation: Continuous- Conduction Mode (CCM) and Discontinuous-Conduction Mode (DCM).

I.3.1. Continuous Conduction Mode (CCM)

The CCM is the mode in which the inductor current (i_L) is always continuous and exists in all time intervals, and the operation mode consists of two phases:

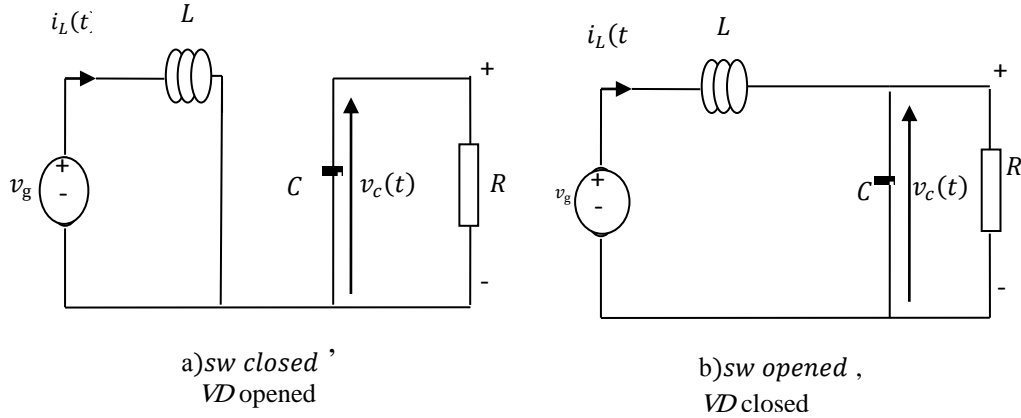


Fig 1- 2 Boost converter configurations in CCM.

In MCC, the first configuration (Fig 1- 2-a) can be described analytically by:

$$L \frac{d}{dt} i_L(t) = V_g(t) \quad (1-1)$$

$$\frac{d}{dt} v_c(t) = -\frac{1}{CR} v_c(t)$$

And the second (Fig 2-b.) By:

$$L \frac{d}{dt} i_L(t) = v_g(t) - v_c(t)$$

$$\frac{d}{dt} v_c(t) = \frac{1}{CR} (R i_L(t) - v_c(t))$$

I.3.2. Discontinuous Conduction Mode (DCM)

The DCM is the mode in which the inductor current is zero in one interval and is not zero in the following interval.

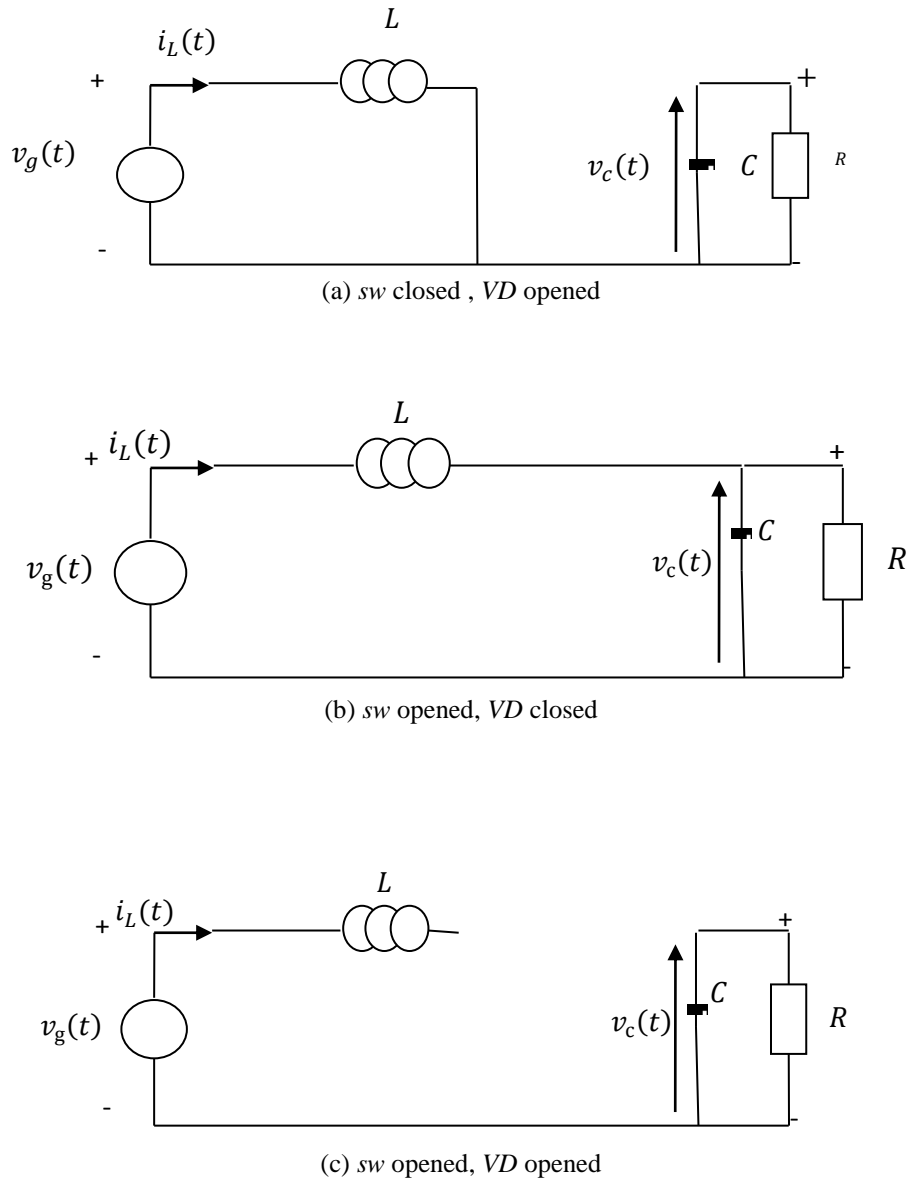


Fig 1- 3 Mode de conduction discontinue

In MCC, the first configuration (Fig.1-3-a) can be described analytically by:

$$L \frac{d}{dt} i_L(t) = V_g(t) \quad (1-3)$$

$$\frac{d}{dt} v_c(t) = -\frac{1}{CR} v_c(t)$$

And the second (Fig 3-b.) By:

$$L \frac{d}{dt} i_L(t) = v_g(t) - v_c(t) \quad (1-4)$$

$$\frac{d}{dt} v_c(t) = \frac{1}{CR} (R i_L(t) - v_c(t))$$

In the case where the MCD is activated, the system goes to the third configuration (Fig 1-3-C.) That can be defined by:

$$L \frac{d}{dt} i_L(t) = 0 \quad (1-5)$$

$$\frac{d}{dt} v_c(t) = -\frac{1}{CR} v_c(t)$$

I.3.3. Passing the Conduction Continuous to Discontinuous Conduction

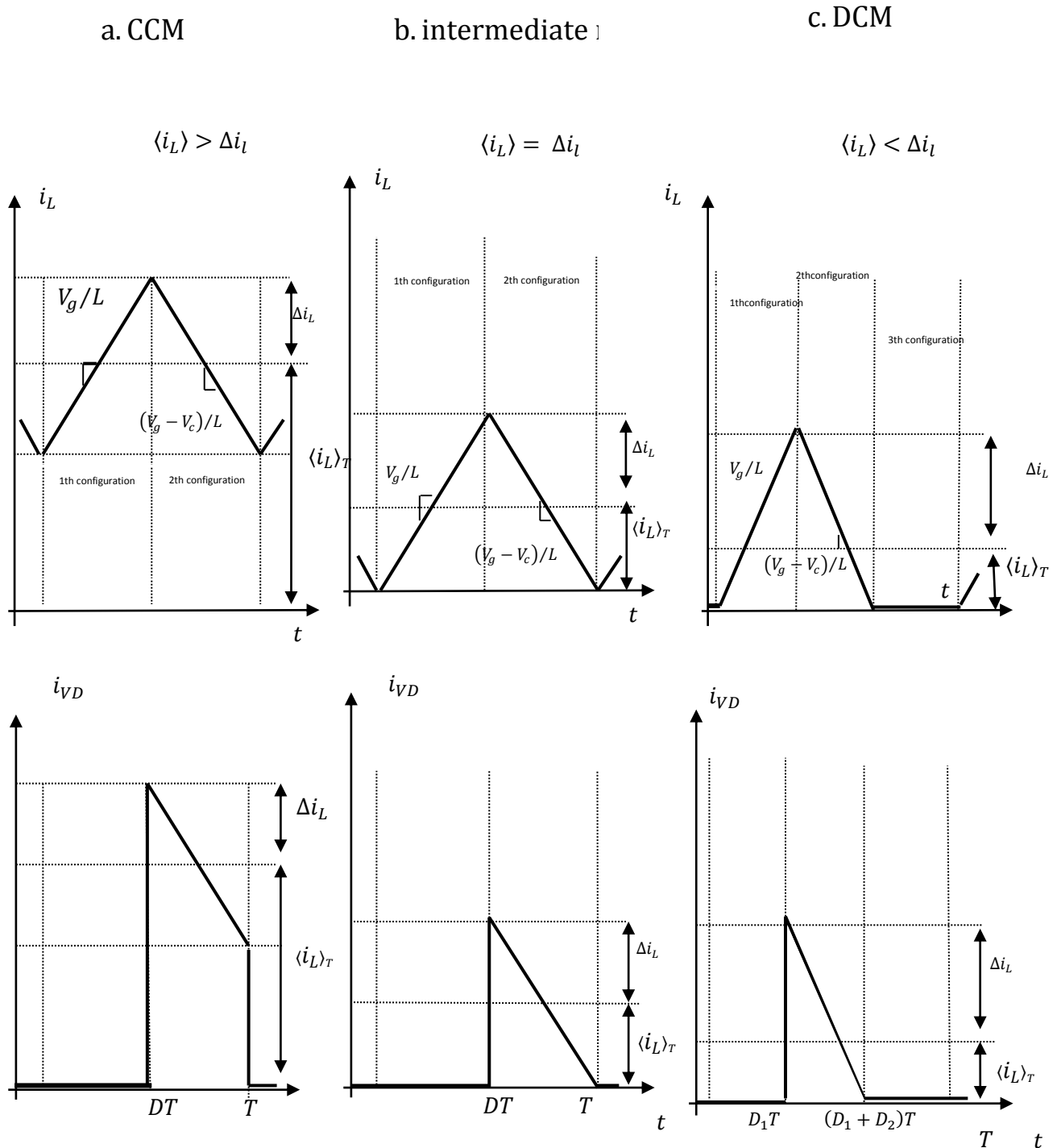


Fig 1- 4 Passing the conduction continuous to discontinuous conduction(inductor current i_L and diode current i_{VD} ,)

The DC-DC converter can operate in both continuous conduction mode (CCM) and discontinuous conduction mode (DCM). We assume that the converter operates in continuous conduction mode (CCM) and that the inductor current is always larger than zero Figure 1-5

shows the schematic circuit diagram of a DC-DC boost converter and the relevant control signals.

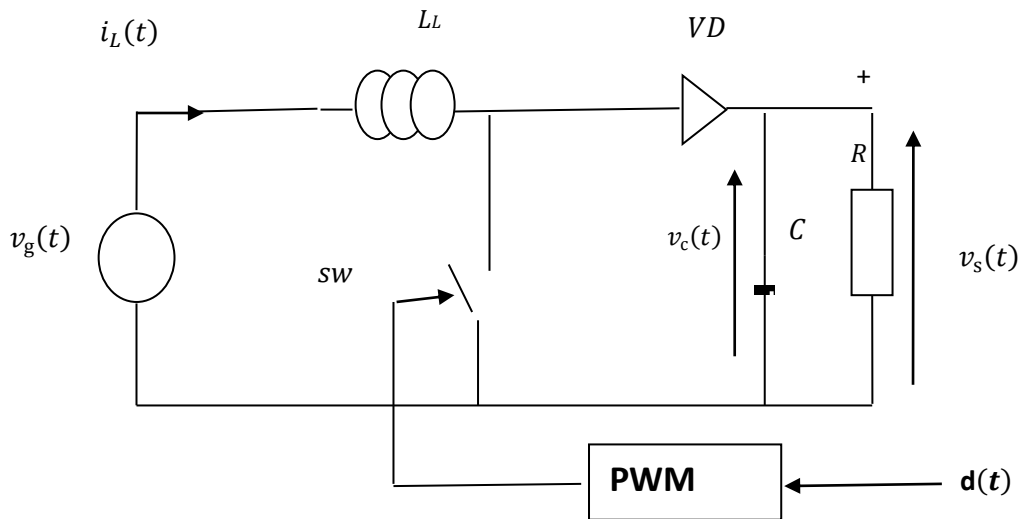


Fig 1- 5 Schematic of the boost converter and the relevant control signals.

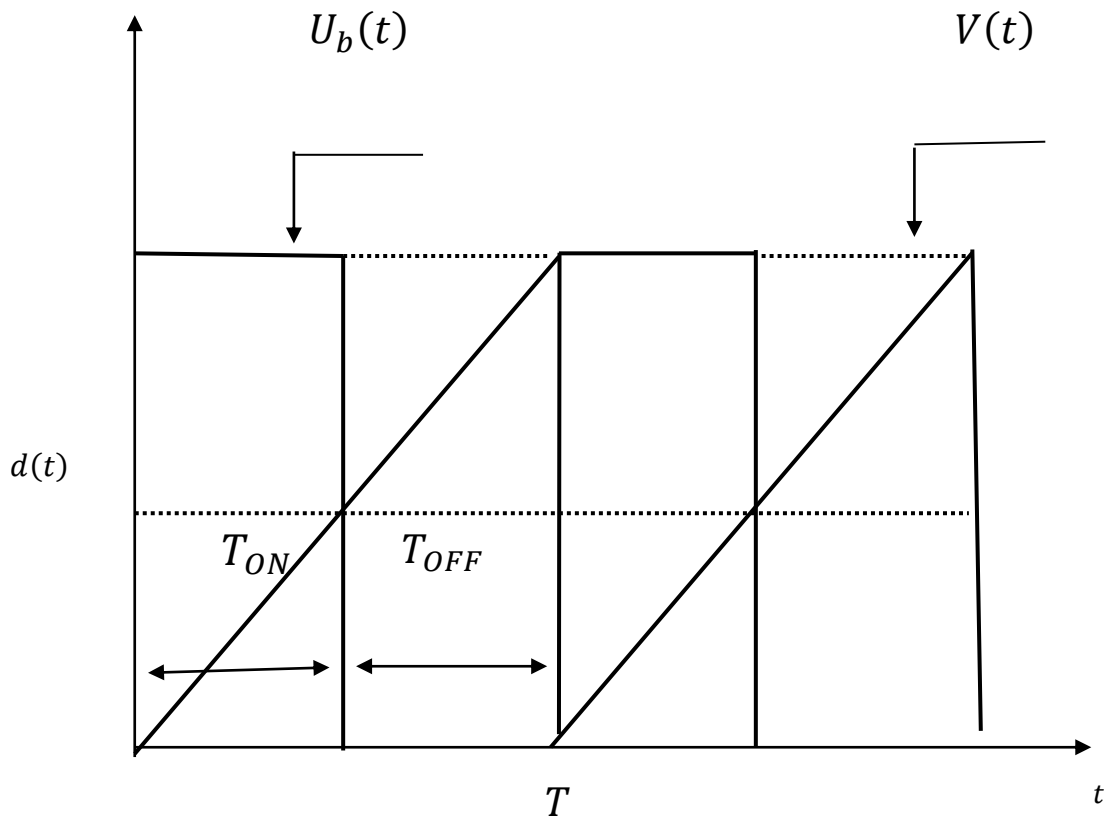


Fig 1- 6 Waveforms of the PWM process

In Fig 1-5, v_s is the output voltage and v_g the line voltage. The output voltage must be kept at a given constant v_{ref} value, the diode D is on inverse polarization, R models the converter load, while C and L represent, respectively, capacitor and inductor values, the switch sw was a power transistor controlled by a binary signal u_b .

The binary signal that triggers on and off the switches is controlled by a fixed frequency pulse width modulation (PWM) circuit (Fig 1- 6). The constant switching frequency is $1/T_s$ with T_s the switching period is given by the sum of T_{on} (when $u_b = 1$) and T_{off} (when $u_b = 0$) and the ratio $T_{on}/(T_{on} + T_{off})$ is the duty cycle $d(t)$. Duty cycle is compared with a sawtooth signal $v(t)$ of amplitude equals to 1. Consequently $0 \leq d \leq 1$

I.4. The State-Space Average Modeling

Middlebrook and Cuk [26] pioneered this field in the late seventies using state space averaging. [27, 28].

A continuous-time linear system can be described by using the state space representation in the following form:

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) \\ y = Cx(t) \end{cases} \quad (1-6)$$

Where $x(t)$ is the state variables vector, $u(t)$ is the sources vector, $y(t)$ is the outputs vector and A, B, C are the coefficient matrices.

A DC/DC converter can be analyzed by a state space representation considering it as a succession of two linear circuits, one corresponding to on state and the other to off state.

To show how this works assume the topology of interest has two modes of operation, and each mode can be described with the following State-Space equations,

Mode I

$$\begin{cases} \dot{x} = A_1x + B_1u \\ y = C_1x \end{cases} \quad (1-7)$$

and Mode II

$$\begin{cases} \dot{x} = A_2x + B_2u \\ y = C_2x \end{cases} \quad (1 - 8)$$

and the duration of Mode I is dT_s and of Mode II is $\dot{d}T_s$ when d is the Duty cycle at the particular operating point and $\dot{d} = 1 - d$. Then [26] proves that the power stage can be approximated by the following continuous State-Space model.

$$\begin{cases} \dot{x} = A_{avg}x + B_{avg}u \\ y = C_{avg}x \end{cases} \quad (1 - 9)$$

Equations (1 - 9) represent a continuous average model.

Where

$$\begin{cases} A_{avg} = dA_1 + \dot{d}A_2 \\ B_{avg} = dB_1 + \dot{d}B_2 \\ C_{avg} = dC_1 + \dot{d}C_2 \end{cases} \quad (1 - 10)$$

Note that the duty cycle d becomes the input of the system report. This new entry is also the order of the system that is used to control the output y . The validity of this model is only guaranteed if the system bandwidth is much lower than the commutation frequency.

Equations (1 - 9) represent a continuous average model, but it is still not linear with respect to the duty cycle as input. The way to achieve the linear model which by nature is small signal is to apply small signal perturbations to all the variables in (1 - 9), and then to group the DC and small signal parts separately. Doing so gives,

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B\hat{V}_{in} + [(A_1 - A_2)X_e + (B_1 - B_2)V_{in}]\hat{d} \\ \hat{y} &= C\hat{x} + (C_1 - C_2)X_e\hat{d} \end{aligned} \quad (1 - 11)$$

With $x = X_e + \hat{x}, y = Y_e + \hat{y}, v_{in} = V_{in} + \hat{v}_{in}, d = D + \hat{d}$

$$X_e = -A^{-1}BV_{in}$$

$$Y_e = -CA^{-1}BV_{in}$$

Equations (1 – 11) is the desired linear small signal model useful for designing an appropriate compensator

I.5. State-Space Averaged Model Of The Boost Converter

The Boost converter scheme of figure 7, with r_L , r_{sw} , r_{VD} , and r_C denoting the resistors of inductor L , switch sw , diode VD and capacitor C , respectively. R is the load, $v_g(t)$ is input voltage, and $u_0(t)$ the output voltage and $i_L(t)$ the inductor current.

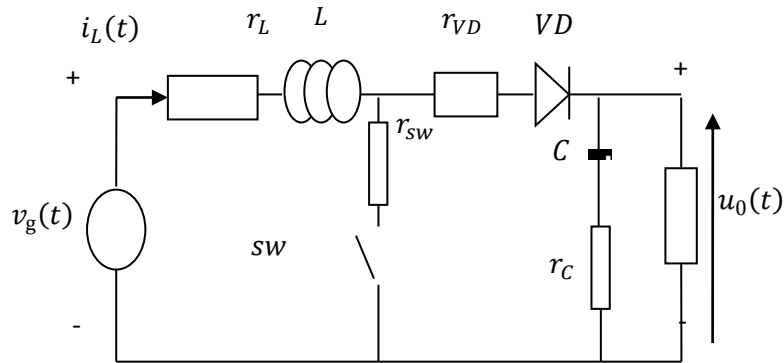


Fig 1- 7 The Boost converter scheme

In the continuous conduction mode (CCM) i.e., $i_L(t) > 0$ we have two topologies depending on the position of switch sw , as shown in fig- 8.

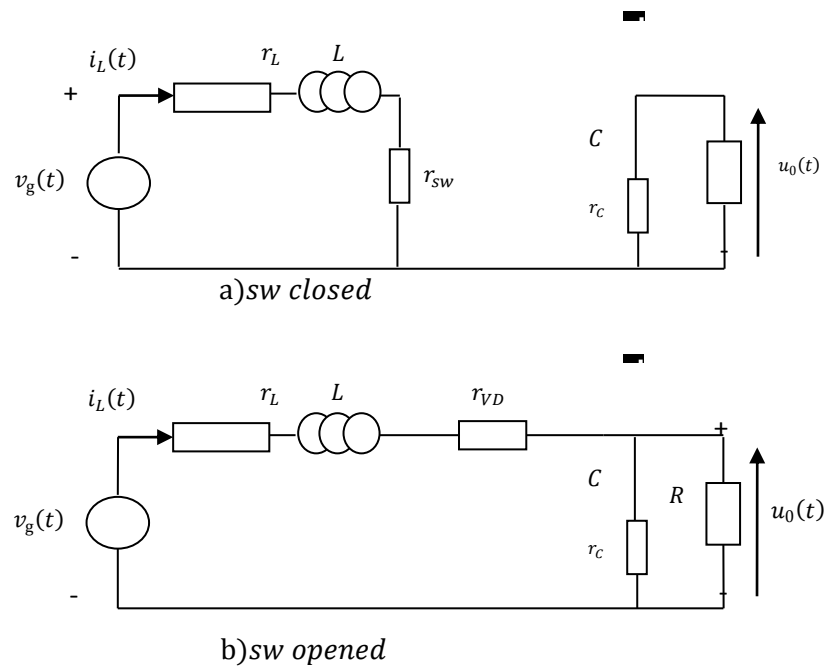


Fig 1- 8 The Boost CCM topologies

In the first topology (Fig 1- 8-a), the system can be presented by the following equations:

$$L \frac{d}{dt} i_L(t) = V_g(t) - (r_L + r_{SW})i_L(t) \quad (1-$$

$$\frac{d}{dt} v_c(t) = -\frac{1}{C(R + r_c)} v_c(t)$$

$$u_0(t) = \frac{R}{R + r_c} v_c(t)$$

The second topology (Figure 1-8-b) gives:

$$L \frac{d}{dt} i_L(t) = v_g(t) - \left(r_L + r_{VD} + \frac{R r_c}{R + r_c} \right) i_L(t) - \frac{R}{R + r_c} v_c(t)$$

$$\frac{d}{dt} v_c(t) = \frac{1}{C(R + r_c)} (R i_L(t) - v_c(t)) \quad (1-13)$$

$$u_0(t) = \frac{R}{R + r_c} (v_c(t) + r_c i_L(t))$$

The state equation of the boost DC-DC converter can be stated as :[29]

$$\begin{aligned} \dot{x} &= A_i x + B_i V_g(t) \\ u_0 &= C_i x \end{aligned} \quad (1-14)$$

Where $x = [i_L v_c]^T$,

and A_i , B_i and C_i are system Matrices of the constituent linear circuits.

The system matrices represent different operating modes (subscript 1 stands for transistor ON, and subscript 2 stands for transistor OFF) of the converter circuit.

The system matrices can be obtained for different operating modes as:

$$B_1 = B_2 = \begin{bmatrix} 1 \\ L \end{bmatrix}^T .$$

$$C_1 = \begin{bmatrix} 0 & \frac{R}{R+r_c} \end{bmatrix} , \quad C_2 = \begin{bmatrix} \frac{Rr_c}{R+r_c} & 0 \end{bmatrix}. \quad (1-15)$$

$$A_1 = \begin{bmatrix} \frac{-(r_L+r_{sw})}{L} & 0 \\ 0 & -\frac{1}{c(R+r_c)} \end{bmatrix} , \quad \text{and}$$

$$A_2 = \begin{bmatrix} -\frac{1}{L} \left(-r_{sw} + r_{VD} + \frac{Rr_c}{R+r_c} \right) & -\frac{R}{L(R+r_c)} \\ \frac{R}{C(R+r_c)} & 0 \end{bmatrix}.$$

The state-space averaging method is very useful for analyzing the low-frequency, small-signal Performance of switch circuits [30,31] .It is applicable when the converter switching period is short while comparing to the response time of the output voltage. By the state-space averaging method, the state equation can be obtained as

$$\begin{aligned} \dot{x} &= Ax + BV_g(t) \\ u_0 &= Cx \end{aligned} \quad (1-16)$$

Where :A , B and C are system Matrices.

$$A = dA_1 + (1-d)A_2, \quad B = dB_1 + (1-d)B_2 \quad (1-17)$$

$$C = dC_1 + (1-d)C_2$$

and d is the switching duty cycle.

I.6. Chapter Conclusion

In this chapter, we presented the topology of DC-DC boost converters and we chose State-Space Averaged Model for the Boost Converter. The DC-DC converter can operate in both continuous conduction mode (CCM) and discontinuous conduction mode (DCM). We assume that the converter operates in continuous conduction mode (CCM) and that the inductor current is always larger than zero.

CHAPTER II : Type-2 Fuzzy Logic Controller

II.1. Introduction

Initially Prof. Zadeh proposed as an extension of type-1 fuzzy set the concept of type-2 fuzzy set [15]. To solve the control problem, some researchers are interested in type-2 fuzzy logic controller (T2FLC) [18, 19]. Type-2 fuzzy logic has been used in a lot of different applications such as control of electrical machines [32, 33], speed control of diesel engines [34], power system stabilizers [35] or transformer [36].

Similar to type-1 fuzzy systems, the type-2 fuzzy systems include fuzzifier, rule base, fuzzy inference engine, and output processor. The output processor includes type-reducer and defuzzifier ; it generates a type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier). Type-reduction methods are extended versions of type-1 defuzzification methods. Type reduction captures more information about rule uncertainties than the defuzzified value (a crisp number) does [16].

The type-2 fuzzy system is characterized by a fuzzy membership function, i.e., the membership value (or membership grade) for each element of this set is a fuzzy set in $[0,1]$, unlike a type-1 fuzzy set where the membership grade is a crisp number in $[0,1]$. Such sets are very useful in circumstances where they are difficult to determine an exact membership function for a fuzzy set; hence, they are useful for incorporating uncertainties [17]. Recently, some researches are interested in type-2 fuzzy logic controller (T2FLC) to solve the control problem [18, 19]. It shows that the T2FLC can handle rule uncertainties when the operation is extremely uncertain and/or the engineers cannot exactly determine the membership grades.

The computational complexity of general T2 FLSs has until recently hindered their widespread use in practical applications. As a restricted class of T2 FLSs, Interval T2 FLSs (IT2 FLSs) [37] were introduced to avoid massive computational requirement (mainly due to type reduction stage). IT2 FLSs are characterized by secondary MFs that only take the value of 1 over their domain. Such a restriction greatly reduces the computational burden of performing inference compared with general T2 FLSs. In the last few years, T2 FLSs, and in particular, to IT2 FLSs have drawn a great deal of attention from both academia and industry.

II.2. Type-1 Fuzzy Sets

A type-1 fuzzy set, A, which is in terms of a single variable $x \in X$ may be represented as:

$$A = \left\{ (x, \mu_A(x)) \mid \forall x \in X \right\} \quad (2-1)$$

A can also be defined as:

$$A = \int_{x \in X} \mu_A(x) / x \quad (2-2)$$

Where \int denotes union over all admissible x

II.3. Type-2 Fuzzy Sets

A type-2 fuzzy set, \tilde{A} may be represented as [38]:

$$\tilde{A} = \left\{ ((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall x \in X \quad \forall u \in J_x \subseteq [0,1] \right\} \quad (2-3)$$

Where $\mu_{\tilde{A}}(x, u)$ is the type-2 fuzzy membership function in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$

\tilde{A} can also be defined as :

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad J_x \subseteq [0,1] \quad (2-4)$$

Where \iint denotes union over all admissible x and u

II.4. Interval T2 fuzzy set

When all $\mu_{\tilde{A}}(x, u)$ are equal to 1, then \tilde{A} is an interval T2FLS. The special case of Equation 2-4, An Interval T2 fuzzy set \tilde{A} is described as in the following definition[39], where all $\mu_{\tilde{A}}(x, u) = 1$, $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$.

$$\tilde{A} = \int_{x \in X} \int_{u \in J_X} 1/(x, u) J_X \subseteq [0, 1] \quad (2 - 5)$$

Where \int denotes the union of all admissible x and u .

An IT2 FS is represented by a bounded region limited by two membership functions, where corresponding to each primary MF (which is in $[0, 1]$), a secondary MF is used to the primary one. The Uncertainty in the primary membership function consists of the union of all membership functions. This Uncertainty represents a bounded region that we call the Footprint of Uncertainty (FOU) i.e.,

$$\text{FOU}(\tilde{A}) = \bigcup_{x \in X} u \in J_X$$

The (FOU) represents a complete description of an IT2 FS. It uses an (UMF) Upper Membership Function, and (LMF) Lower Membership Function; The (UMF) and the (LMF) Of \tilde{A} are two T1 MFs that bound the (FOU). (UMF) represents the upper bound of $\text{FOU}(\tilde{A})$ and is denoted $\bar{\mu}_{\tilde{A}}(x)$

$$\bar{\mu}_{\tilde{A}}(x) \equiv \overline{\text{FOU}(\tilde{A})} \quad \forall x \in X$$

$$\underline{\mu}_{\tilde{A}}(x) \equiv \underline{\text{FOU}(\tilde{A})} \quad \forall x \in X$$

For an IT2 FS, $J_X = [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)] \forall x \in X$.

II.5. Uncertainty

Uncertainty is involved with any situation with some lack of information: it may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory or deficient in some other way [40], Different authors define and classify different types of uncertainty. [41]

The classification proposed by Pr. B. Ioos fits to describe uncertainty in electrical engineering [42]. This classification is illustrated in Fig 2-1.

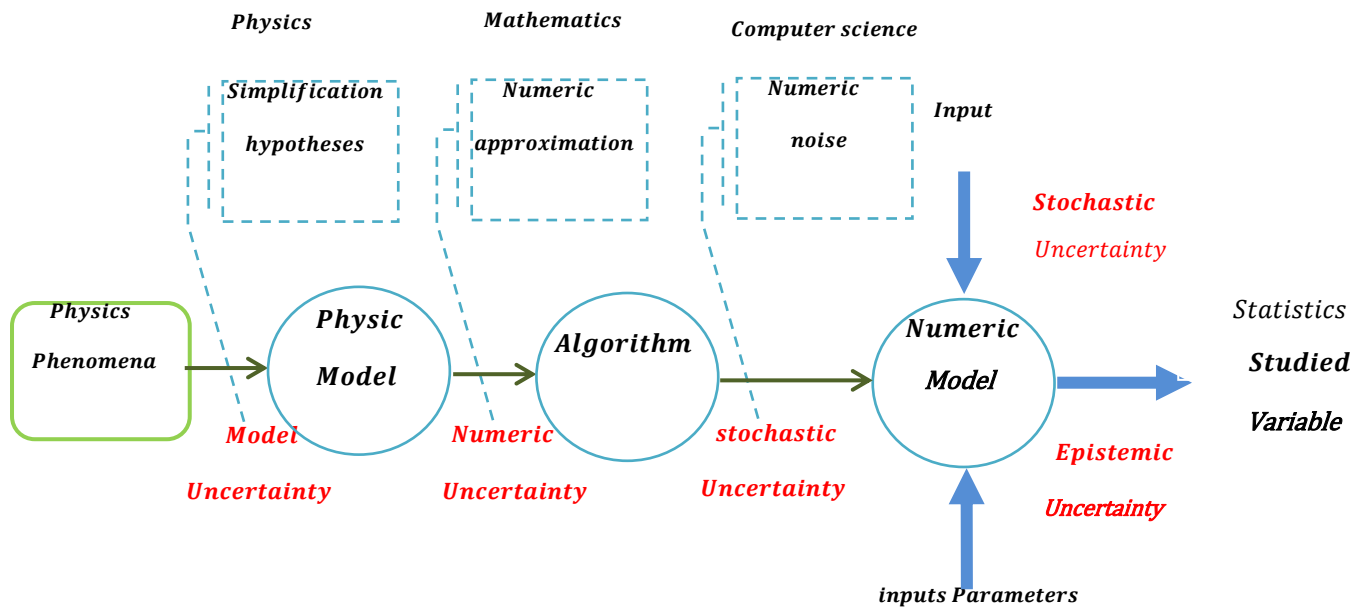


Fig 2- 1 Sources of uncertainty [41][42]

Uncertainty is also present in fuzzy logic systems As shown[39][43]:

- the words used in the antecedents and the consequents of rules can mean different things to different people.
- consequents obtained by polling a group of experts may differ.
- the training data are noisy.
- the measurements that activate the FLS are noisy.

II.6. Membership functions

Membership functions (MFs) enable establishing a relationship between numerical values and linguistic labels.

Type-1 fuzzy MFs (T1-MF) are two dimensional and represent the membership degree μ for a variable x Type-2 fuzzy MFs (T2-MF) are three dimensional:

they consider an uncertainty U of the membership degree. T1-MFs are a particular case of T2-MFs where the uncertainty value is 0. Membership functions are classified as:

- Singleton MFs, if μ value is 0 but in a single point of the domain μ value is 1
- Interval type-1 MFs, if μ value is 0 except in the interval defined by its left and right bounds $[l_b, r_b]$ where $\mu = 1$ if $l_b < x < r_b$

- Type-1 MFs if μ is a crisp value which vary between 0 and 1
- Type-2 MFs if μ is represented by a secondary degree MF $\mu_{\bar{A}}$

Type-2 MFs are classified in:

1. Interval type-2 MFs if $\mu_{\bar{A}}(\mu, x)$ is an interval type-1 MF
2. General type-2 MFs if $\mu_{\bar{A}}(\mu, x)$ is a type-1 MF

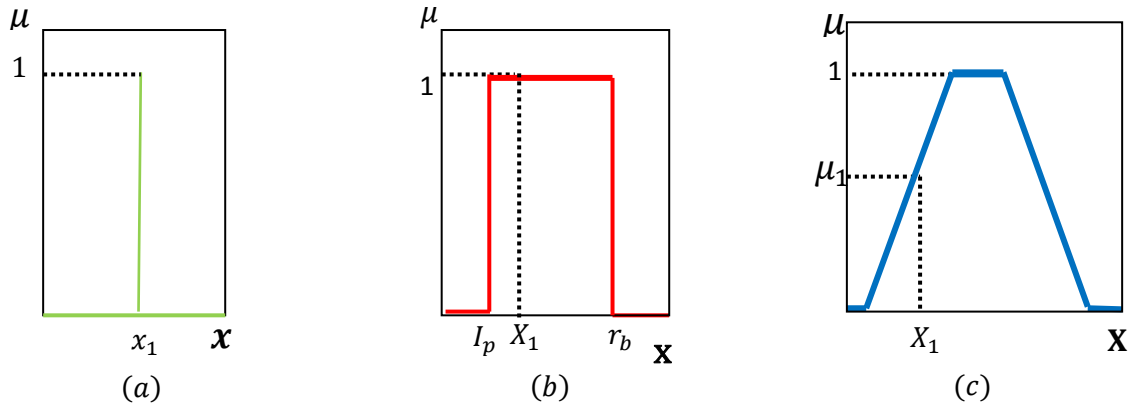


Fig 2- 2 Membership functions (a) singleton, (b) Interval type-1,(c) Type-1.

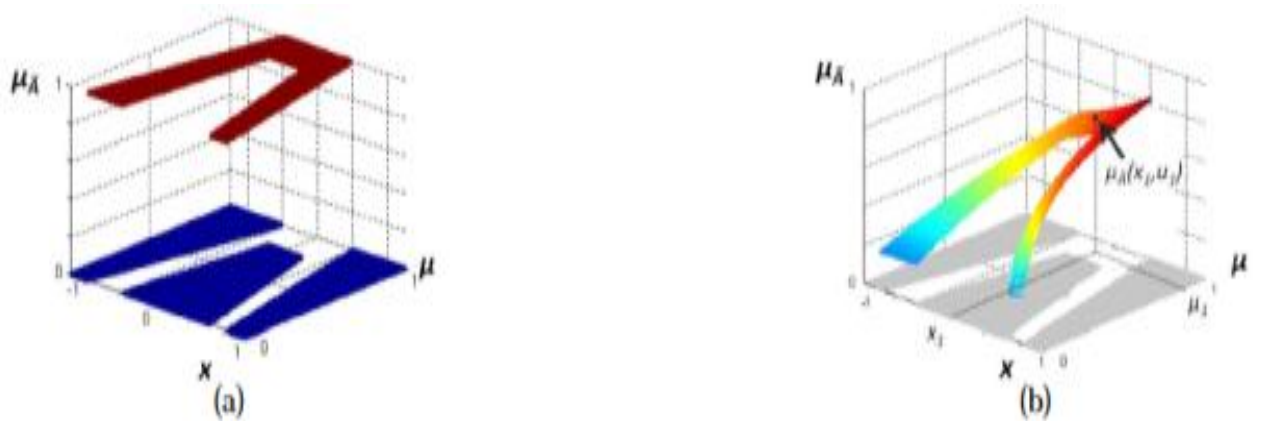


Fig 2- 3 Membership functions (a) Interval type-2, (b), General type-2.

II.6.1. Interval type-2 membership functions

Interval type-2 fuzzy membership functions are easy to implement and then have been used in almost all the works about type-2 fuzzy logic .

For simplicity reasons, interval type-2 membership function has been also selected for this research.

Interval type-2 fuzzy membership functions can be created from two type-1 MFs. An Upper Membership Function (UMF) which represents the maximum value and a Lower Membership Function (LMF) which represents the minimum value of μ for each x . The uncertainty U is represented by the area between the UMF and the LMF. This region is called Footprint of Uncertainty (FOU) and is illustrated in Figure 4.

Type-1 MFs are a particular case of T2-MFs which does not consider the uncertainty, the same MF represents both the UMF and the LMF and the area of the FOU is zero. [41]

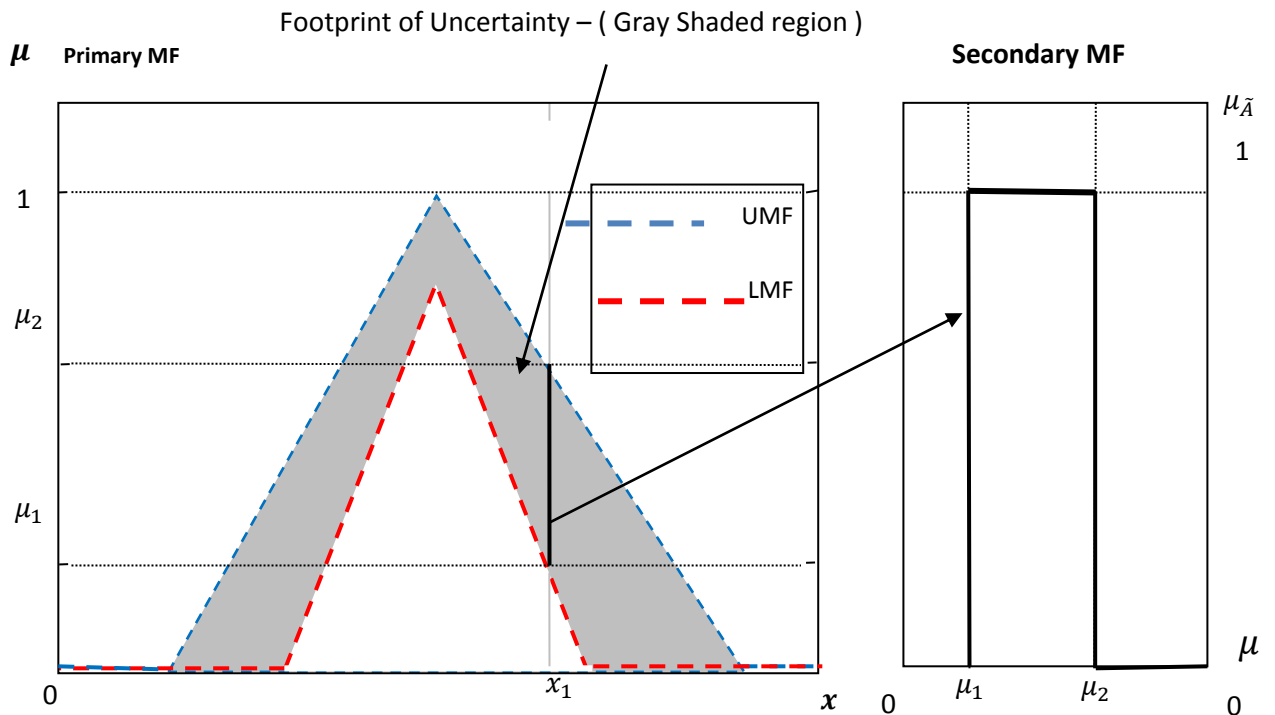


Fig 2- 4 Interval type-2 fuzzy logic primary and secondary MFs

II.7. Interval type-2 fuzzy systems

The difference between the structures of the type-1 fuzzy system and type-2 fuzzy system is only the output processing. When the operators define the type-1 fuzzy sets, the experience and knowledge of human experts are needed to decide both the membership functions and the rules based on the available linguistic or numeric information. As defining the type-2 fuzzy set, the expert knowledge is always represented by linguistic terms and limited uncertainty, leading to the uncertain antecedent part and/or consequent part of the fuzzy rules.

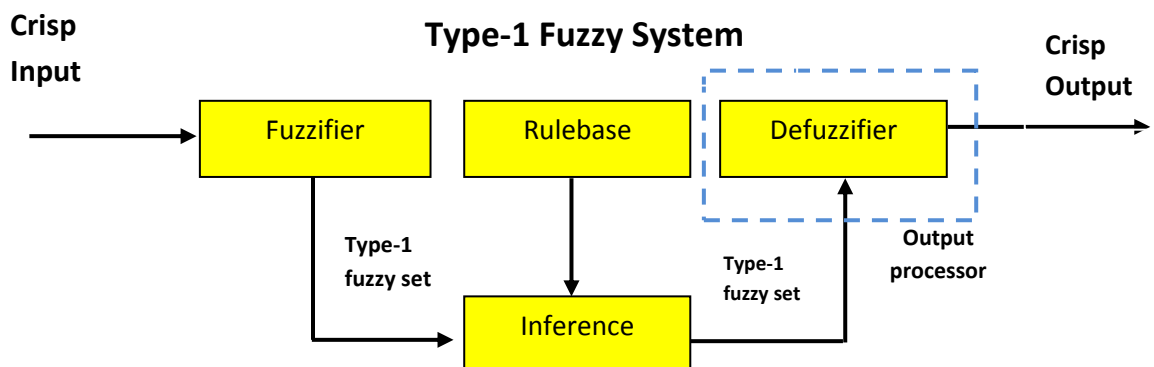


Fig 2- 5 Type-1 fuzzy logic systems [39]

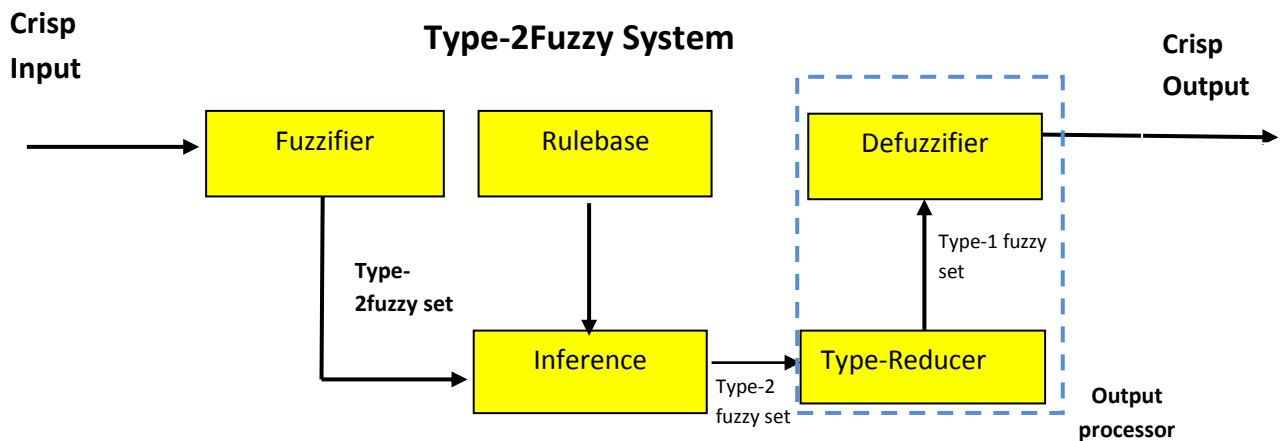


Fig 2- 6 Type-2 fuzzy logic systems [39]

II.7.1. Fuzzification

In this block, we must first define the fuzzy sets of all in-puts' system. Those memberships can contain one or several type-2 fuzzy sets. Second, the fuzzifier maps inputs into the associated fuzzy sets to determine the degree of membership of each input variable. We consider only singleton fuzzification for which the inputs are crisp values.

II.7.2. Inference

This block expresses the relationship that exists between the input variables (expressed as linguistic variables) and the output variables (also expressed as linguistic variables). As in type-1 fuzzy logic, in the design of a type-2 fuzzy logic we generally have IF-THEN rules. The formulation of rules is the same. The only distinction between type-1 and type-2 is associated with the nature of the membership functions. The inference engine combines rules and makes a combination between input type-2 fuzzy sets and output type-2 fuzzy sets.

In practice the computations in an IT2 FLS can be significantly simplified. Consider the rule base of an IT2 FLS consisting of N rules assuming the following form:

$$R^n: \text{IF } x_1 \text{ is } \tilde{X}_1^n \text{ and } \dots \text{ and } x_I \text{ is } \tilde{X}_I^n, \text{ THEN } y \text{ is } Y^n \quad n = 1, 2, \dots, N$$

Where $\tilde{X}_i^n (i = 1, \dots, I)$ are IT2 FSs, and $Y^n = [\underline{y}^n, \bar{y}^n]$ is an interval, which can be understood as the centroid [39][43] of a consequent IT2 FS.

Assume the input vector is $x' = (x'_1, x'_2, \dots, x'_I)$.

Typical computations in an IT2 FLS involve the following steps:

1) Compute the membership of x'_i on each $X'_i, [\mu_{\underline{X}_i^n}(x'_i), \mu_{\bar{X}_i^n}(x'_i)]$
 $i = 1, 2, \dots, I, n = 1, 2, \dots, N.$

2) Compute the firing interval of the n^{th} rule, $F^n(x')$:

$$F^n(x') = [\mu_{\underline{X}_1^n}(x'_1) \times \dots \times \mu_{\underline{X}_I^n}(x'_I), \mu_{\bar{X}_1^n}(x'_1) \times \dots \times \mu_{\bar{X}_I^n}(x'_I)] = [f^l(x'), \bar{f}^l(x')] \equiv [f^n, \bar{f}^n], \quad (2 - 6)$$

$$n = 1, 2, \dots, N.$$

3).The output processor combines the type-2 fuzzy sets (one for each activated rule) to obtain the crisp output of the fuzzy logic system. The output processor is the principal difference between T1-FL and T2-FL systems. However, the output processor of a T2-FL system can be used in T1-FL systems (not vice versa). The type-2 output processor is divided into the type-reducer and the defuzzificator.

II.7.3. Type-Reducer

The type-reducer combines type-2 fuzzy sets into an interval type-1 fuzzy MF called the type-reduced set. The number of input fuzzy sets corresponds to the number of activated rules. There is only one output, the type-reduced fuzzy MF. This MF is computed by using the Karnik-Mendel algorithm.

Perform type-reduction to combine $F^n(x')$ and the corresponding rule consequents. There are many such methods.

The most commonly used one is the center-of-sets type-reducer [39]:

$$Y_{cos}(x') = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^N \in [y_l^N, y_r^N]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^N \in [\underline{f}^N, \bar{f}^N]} 1 / \frac{\sum_{i=1}^N f^i y^i}{\sum_{i=1}^N f^i} \quad (2 - 7)$$

where the multiple integral signs denote the union operation, Y_{cos} This interval set is determined by its two end points, y_l and y_r , $f^i \in F^i = [\underline{f}^i, \bar{f}^i]$ is the interval firing level of the i^{th} rule , y_l^i and y_r^i are the left and right end points of the centroid of the consequent of the i^{th} rules and N is the number of fired rules.

It has been shown that [39][44][45][46]:

$$y_l = \min_{k \in [1, 1-N]} \frac{\sum_{n=1}^k \bar{f}^n \underline{y}^n + \sum_{n=k+1}^N \underline{f}^n \underline{y}^n}{\sum_{n=1}^k \bar{f}^n + \sum_{n=k+1}^N \underline{f}^n} \equiv \frac{\sum_{n=1}^L \bar{f}^n \underline{y}^n + \sum_{n=L+1}^N \underline{f}^n \underline{y}^n}{\sum_{n=1}^L \bar{f}^n + \sum_{n=L+1}^N \underline{f}^n} \quad (2 - 8)$$

$$y_r = \max_{k \in [1, 1-N]} \frac{\sum_{n=1}^k \underline{f}^n \bar{y}^n + \sum_{n=k+1}^N \bar{f}^n \bar{y}^n}{\sum_{n=1}^k \underline{f}^n + \sum_{n=k+1}^N \bar{f}^n} \equiv \frac{\sum_{n=1}^R \underline{f}^n \bar{y}^n + \sum_{n=R+1}^N \bar{f}^n \bar{y}^n}{\sum_{n=1}^R \underline{f}^n + \sum_{n=R+1}^N \bar{f}^n} \quad (2 - 9)$$

where the *switch points* L and R are determined by

$$\underline{y}^L \leq y_l \leq \underline{y}^{L+1} \quad (2 - 10)$$

$$\bar{y}^R \leq y_r \leq \bar{y}^R \quad (2 - 11)$$

And $\{\underline{y}^n\}$ and $\{\bar{y}^n\}$ have been sorted in ascending order, respectively.

y_l and y_r can be computed using the Karnik-Mendel (KM) algorithms [39][46] as follows:

KM Algorithm for Computing y_l :

a) Sort $\underline{y}_n (n = 1, 2, \dots, N)$ in increasing order and call the sorted \underline{y}_n by the same name, but now $\underline{y}^1 \leq \underline{y}^2 \leq \dots \leq \underline{y}^N$. Match the weights $F^n(x')$ with their respective \underline{y}^n and renumber them so that their index corresponds to the renumbered \underline{y}^n

b) Initialize f^n by setting

$$f^n = \frac{f^n + \bar{f}^n}{2} \quad n = 1, 2, \dots, N \quad (2 - 12)$$

and then compute

$$y = \frac{\sum_{n=1}^N \underline{y}^n f^n}{\sum_{n=1}^N f^n} \quad (2 - 13)$$

c) Find switch point $k (1 \leq k \leq N - 1)$ such that

$$\underline{y}^k \leq y \leq \underline{y}^{k+1} \quad (2 - 14)$$

d) Set

$$f^n = \begin{cases} \bar{f}^n, & n \leq k \\ f^n, & n > k \end{cases} \quad (2 - 15)$$

and compute

$$y' = \frac{\sum_{n=1}^N \underline{y}^n f^n}{\sum_{n=1}^N f^n} \quad (2 - 16)$$

e) Check if $y' = y$. If yes, stop and set $y_l = y$ and $L=k$. If no, go to Step 6.

f) Set $y = y'$ and go to Step 3.

KM Algorithm for Computing y_r :

a) Sort \bar{y}_n ($n = 1, 2, \dots, N$) in increasing order and call the sorted \bar{y}_n by the same name, but now $\bar{y}^1 \leq \bar{y}^2 \leq \dots \leq \bar{y}^N$. Match the weights $F^n(x')$ with their respective \bar{y}^n and renumber them so that their index corresponds to the renumbered \bar{y}^n

b) Initialize f^n by setting

$$f^n = \frac{f^n + \bar{f}^n}{2} \quad n = 1, 2, \dots, N \quad (2 - 17)$$

and then compute

$$y = \frac{\sum_{n=1}^N \bar{y}^n f^n}{\sum_{n=1}^N f^n} \quad (2 - 18)$$

c) Find switch point k ($1 \leq k \leq N - 1$) such that

$$\bar{y}^k \leq y \leq \bar{y}^{k+1} \quad (2 - 19)$$

d) Set

$$f^n = \begin{cases} \bar{f}^n, & n \leq k \\ \underline{f}^n, & n > k \end{cases} \quad (2 - 20)$$

and compute

$$y' = \frac{\sum_{n=1}^N \bar{y}^n f^n}{\sum_{n=1}^N f^n} \quad (2 - 21)$$

e) Check if $y' = y$. If yes, stop and set $y_r = y$ and $R=k$. If no, go to Step 6.

f) Set $y = y'$ and go to Step 3.

II.7.4. Defuzzifier

The defuzzificator transforms the type-reduced fuzzy set into a crisp output. It is the simplest subsystem; the crisp output value is computed as the average of the bounds of the type-reduced set.

From the type-reducer, we obtain an interval set Y_{cos} , to defuzzify it we use the average of y_l and y_r , so the defuzzified output of an interval singleton type-2 FLS is [46]

$$y = \frac{y_l + y_r}{2} \quad (2 - 22)$$

II.8. Interval Type-2 TSK Fuzzy Logic systems

Today, the two most popular fuzzy logic systems used by engineers in control are the Mamdani and TSK systems. . Both are characterized by IF-THEN rules, and have the same antecedent structures. They differ in the structures of the consequents. The consequent of a Mamdani rule is a fuzzy set, whereas the consequent of a TSK rule is a function [47].

The differences between a type-1 TSK fuzzy logic controller [12] and a Mamdani T1 fuzzy logic consist essentially of the definition of outputs and then on the consequent part of rules. Consider we have a first order type-1 TSK fuzzy logic with p inputs $x_1 \in X_1, \dots, \dots, \dots, x_p \in X_p$, one output $y \in Y$, and with M rules. The l th rule can be expressed as:

$$R^l : \text{ IF } x_1 \text{ is } F_1^i \text{ and... } x_p \text{ is } F_p^i \text{ THEN } y^l = c_0^l + c_1^l x_1 \dots + c_p^l x_p$$

Type-2 TSK FLS was presented in 1999. Because the universal approximation property and the capability of handling rule uncertainties in a more complete way, interval type-2 FLSs are gaining more and more in popularity. More and more fuzzy experts see the shortcomings of type-1 FLS, and apply type-2 FLS to situations where uncertainties abound.

A type-2 TSK fuzzy logic controller or system (T2 TSK fuzzy logic) was firstly introduced by Liang and Mendel [34]. Although TSK type-1 fuzzy systems have received a lot attention, the literature on TSK type 2 fuzzy systems is few. Liang and Mendel applied type-2 TSK systems in channel equalization of channels [48]. Where, according to them, there are three models of T2 TSK fuzzy logics depending on the kind of the antecedent and consequent part of rules, to have:

- T2 TSK-Model I : Antecedents are IT2 FSs, and consequents are interval T1 FSs

- T2 TSK-Model II: Antecedents are similar to the case of T2 TSK- Model I, however, consequent parameters are crisp (similar to the case of traditional type-1 TSK fuzzy models).
- T2 TSK-Model III: Antecedents and consequents are both type-1 fuzzy sets.

TABLE 2- 1.
RULES OF T2 TSK FLS

TSK FLS	Rules R ^l
Type-1 TSK	IF x_1 is F_1^i and... x_p is F_p^i THEN $y^i = c_0^i + c_1^i x_1 \dots + c_p^i x_p$
T2 TSK-Model I	IF x_1 is \tilde{F}_1^i and... x_p is \tilde{F}_p^i THEN $y^i = C_0^i + C_1^i x_1 \dots + C_p^i x_p$
T2 TSK-Model II	IF x_1 is \tilde{F}_1^i and... x_p is \tilde{F}_p^i THEN $y^i = c_0^i + c_1^i x_1 \dots + c_p^i x_p$
T2 TSK-Model III	IF x_1 is F_1^i and... x_p is F_p^i THEN $y^i = C_0^i + C_1^i x_1 \dots + C_p^i x_p$

Where $i = 1, 2, \dots, M$, C_k^i 's ($k = 1, 2, \dots, p$) are the consequent parameters which are type-1 fuzzy set, c_k^i 's ($k = 1, 2, \dots, p$) are the consequent parameters that are crisp numbers, Y^l are the outputs of the l^{th} rule, \tilde{F}_j^i 's ($j = 1 \dots p$) are type-2 fuzzy sets of input state j in rule M , given an inputs $x_1, x_2 \dots x_p$, F_j^i 's ($j = 1 \dots p$) are type-1 fuzzy sets.

- 1) In an T2 TSK-Model I, The firing strength of the i^{th} rule $F^i(x)$ with meet operation under product or minimum t-norm is an interval type-1 set expressed as:

$$F^i(x) = \left[\underline{f}^i(x), \overline{f}^i(x) \right]$$

Where

$$\underline{f}^i(x) = \underline{\mu}_{\tilde{F}_1^i}(x_1) * \dots * \underline{\mu}_{\tilde{F}_p^i}(x_p)$$

$$\bar{f}^i(x) = \bar{\mu}_{\bar{F}_1^i}(x_1) * \dots * \bar{\mu}_{\bar{F}_p^i}(x_p)$$

The final output is also an interval type-1 set and is calculated as follows [39]:

$$Y(Y^1, \dots, Y^M, F^1, \dots, F^M) = [y_l, y_r] = \int_{y^1} \dots \int_{y^M} \int_{f^1} \dots \int_{f^M} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (2-23)$$

Where M is the number of rules fired, $y_i \in Y^i$, and $Y^i = [y_l^i, y_r^i]$, ($i = 1 \dots M$)

- 2) In an T2 TSK-Model II The final output Is a special case of (2 – 23), because now each Y^i is a crisp value y^i .

so

$$Y(f^1, \dots, f^M) = [y_l, y_r] = \int_{f^1} \dots \int_{f^M} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (2-24)$$

- 3) In an T2 TSK-Model III The final output is special case of (2 – 23), because now each F^i is a crisp value f^i

so

$$Y(Y^1, \dots, Y^M) = [y_l, y_r] = \int_{y^1} \dots \int_{y^M} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (2-25)$$

To compute Y it is only necessary to compute its two end-points y_l and y_r can also be computed more efficient by the KM Algorithm

And the defuzzified output is:

$$y = \frac{y_l + y_r}{2} \quad (2-26)$$

II.9. Chapter Conclusion

Fuzzy control has been used in different applications such as voltage control in power converters or speed control in electric machines.

In the field of Type-2 theory, those both systems are treated. But since the TSK FLSs are widely used in control.

TSK FLS has a powerful capability of explaining complex relations among variables using rule consequents which are functions of the input variables. The universal approximation property of TSK fuzzy systems is well known today.

***CHAPTER III: Type-2 Fuzzy Logic PID Controller
Design for Boost DC-DC Converters***

III.1 Introduction

Traditional PID controllers are featuring with structured simple, reasonable price, and the effectiveness of the systems of linear spectrum. Therefore, several industrial processes are still using the traditional PID controllers [49], but the performance of a PID control system becomes unsatisfactory and fails to guarantee the requirements in most of the practical situations, when the process to be controlled has a high level of complexity, such as, time delay, high order, modeling nonlinearities, vague systems without precise mathematical models, and structural uncertainties.[50]. to achieve better performance over a conventional PID controller, many researchers have attempted to combine the conventional PID with a fuzzy logic controller (FLC) .

Although, the significant improvement of system performance with fuzzy PID controllers over their conventional counterparts, it should be noted that they are usually not effective in cases where the system, to be controlled, has uncertainties as ordinary fuzzy controllers (type-1). In fact, this last (T1-FLC) have limited capabilities to directly handle data uncertainties .Such a fuzzy PID structure (T1F-PID) is simple and can be theoretically analyzed [51]. However, the main drawback of the T1F-PID is its limited capability to directly handle data uncertainties.

DC-DC converters are power electronic systems that convert one level of electrical voltage into another level by the switching action [30]. The DC-DC converters have a lot of uses in personal computers, computer peripherals, and adapters of consumer electronic devices. The DC-DC converters are intriguing subject from the control point of view due to their intrinsic nonlinearity.

The control technique for DC-DC converters have to cope with their wide input voltage and load variations to ensure stability in any operating condition while providing fast transient response. Recently, many researchers have considered the control of DC-DC converters based on output feedback linearization theory [30] sliding-mode control approach and fuzzy control technique [29].

The fuzzy logic controller can ensure the required performance in wide range of operating point variation. Contrarily, the feedback linearization and sliding mode approaches are simple to implement and easy to design, but their performances generally depend on the working point. Thus, the control parameters which ensure proper behavior in any operating conditions are difficult to obtain. In the case of DC-DC converters, the fuzzy logic controller is able to regulate the output voltage to a desired value without steady-state oscillations despite change in load or input voltage. The fuzzy control of DC-DC converters has been implemented successfully at a low cost.[25]

III.2 Type-2 Fuzzy Logic PID Controller Design

III.2.1 The Voltage Controller

The voltage controller requires one single input which is the error signal (e) between the reference and the measured voltage.

The output of the controller is the duty cycle of the converter.

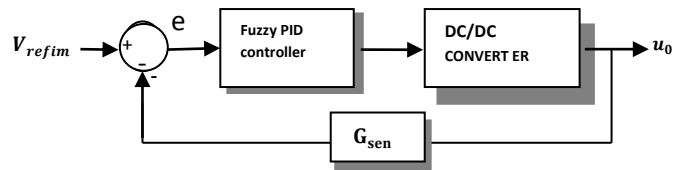


Fig 3-1 DC/DC Boost output voltage controller

Where: sensor of gain G_{sen} and the reference voltage V_{ref}

$$\text{And } V_{refim} = G_{en}V_{ref} \quad , \quad e = V_{refim} - G_{en}u_0 \quad (10)$$

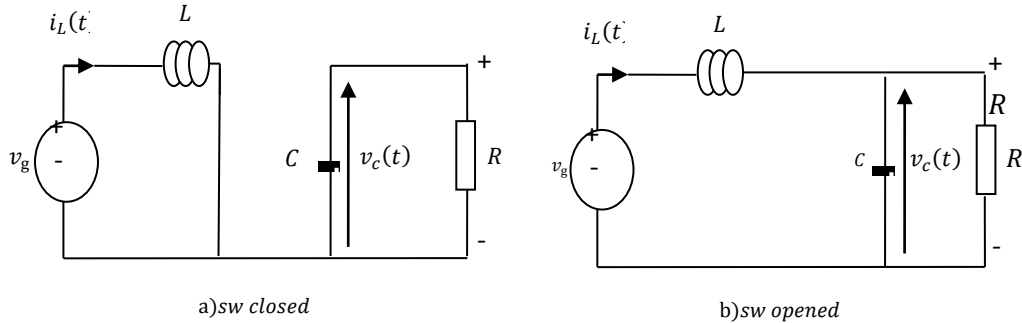


Fig 3-2 Boost converter configurations in CCM.

III.2.2 Fuzzy PID Controller

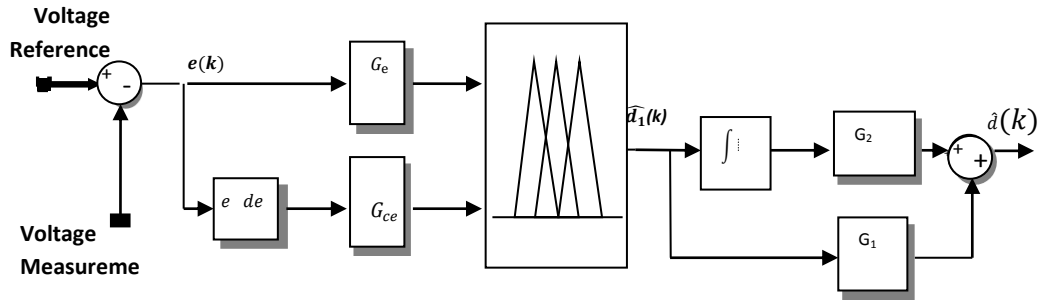


Fig 3-3 Structure of a fuzzy PID controller

Where G_1, G_2 are values of the gains PID controller .

And G_e, G_{ce} values of the gains normalization of the fuzzy system inputs

The control action \hat{d} is obtained by the weighted sum of the fuzzy logic system (FLS) output \hat{d}_1 and its integral action using the gains G_1 and G_2 .

The e and its time derivative de as inputs. This system is constructed from the human experience formulated in a collection of fuzzy rules in the following form:

$$j^{\text{th}} : \text{ IF } e \text{ is } E_0^j \text{ and } de \text{ is } E_1^j \text{ THEN } \hat{d}_1 = C_j(e, de)$$

With E_0^j, E_1^j are respectively the fuzzy sets of the error voltage e and its time derivative de C_j is the j^{th} output singleton.

The strategy of fuzzy control is derived using the following knowledge on the system:

- The change of duty cycle \hat{d}_1 must be large, When u_0 is far from the reference V_{ref} for providing a small response time
- The small change of duty cycle \hat{d}_1 is sufficient to reach the reference providing that u_0 approaches the reference.
- The duty cycle must be unchanged as long as u_0 is in the vicinity of the reference with a sufficient approaching speed, for preventing the output overshoot.
- When u_0 reaches the reference and continue growing up: first, we decrease the

duty cycle change, then if u_0 remains closer to the reference ,the duty cycle changes must be zero otherwise, it must be negative. [4],[52],[24]

Thus the final control action \hat{d} applied to the converter is given by:

$$\hat{d} = G_1 \hat{d}_1 + G_2 \int \hat{d}_1 d\tau \quad (3-1)$$

III.2.2.1 Input Membership Functions Design

The two input fuzzy sets (for e and de) are composed of five membership functions Negative High(NH), Negative Low (NL), Zero (Z), Positive Low (PL) and Positive High (PH) The MFs are symmetric around the zero axes .

The Five type-1 membership functions (Uncertainty =0%) are defined in (Fig3-4).

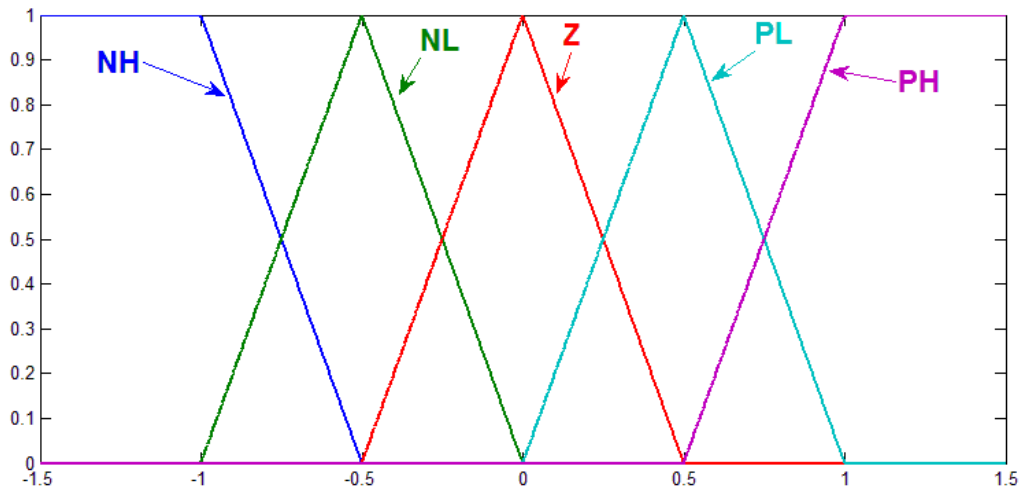


Fig 3-4 Type-1 fuzzy membership functions

To construct the interval type-2 MFs, the type-1 membership functions are modified as illustrated in (Fig 3-5). [41],[53]

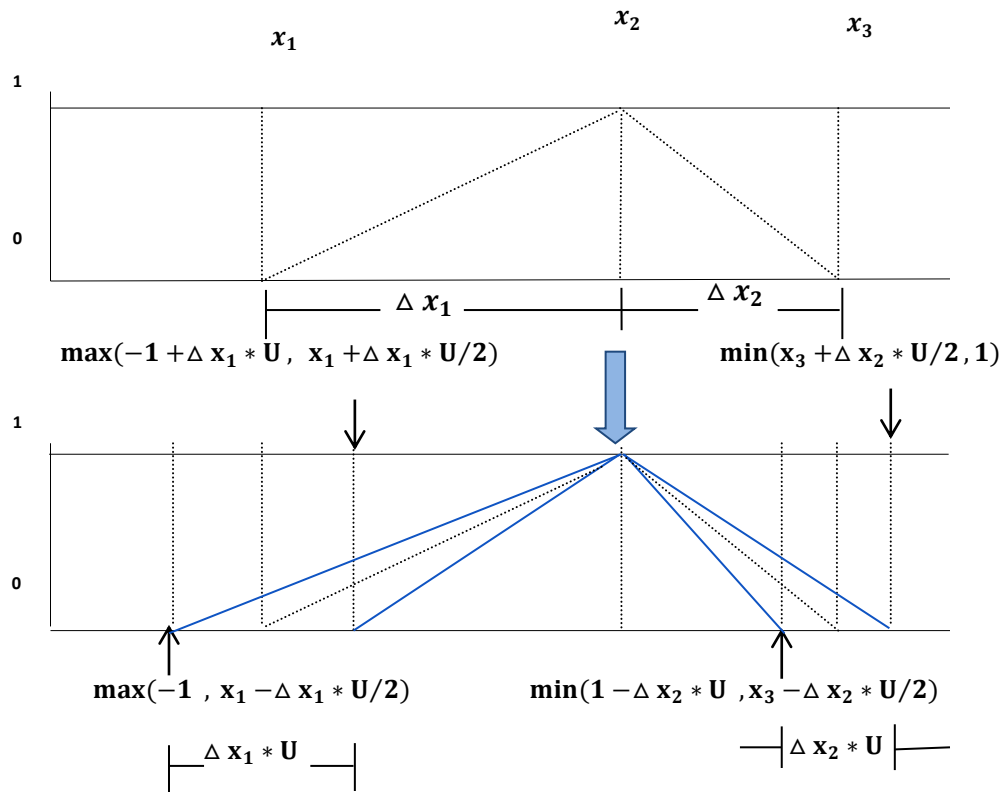


Fig 3-5 From type-1 to interval type-2 membership functions[41],[53]

Here, three different uncertainties were chosen: $U=0\%$, $U=20\%$ and $U=50\%$ in operations of simulation.

The considered fuzzy sets (for uncertainty values of $U=0\%$, $U=20\%$ and $U=50\%$) are presented in the following Membership Functions Design in each case:

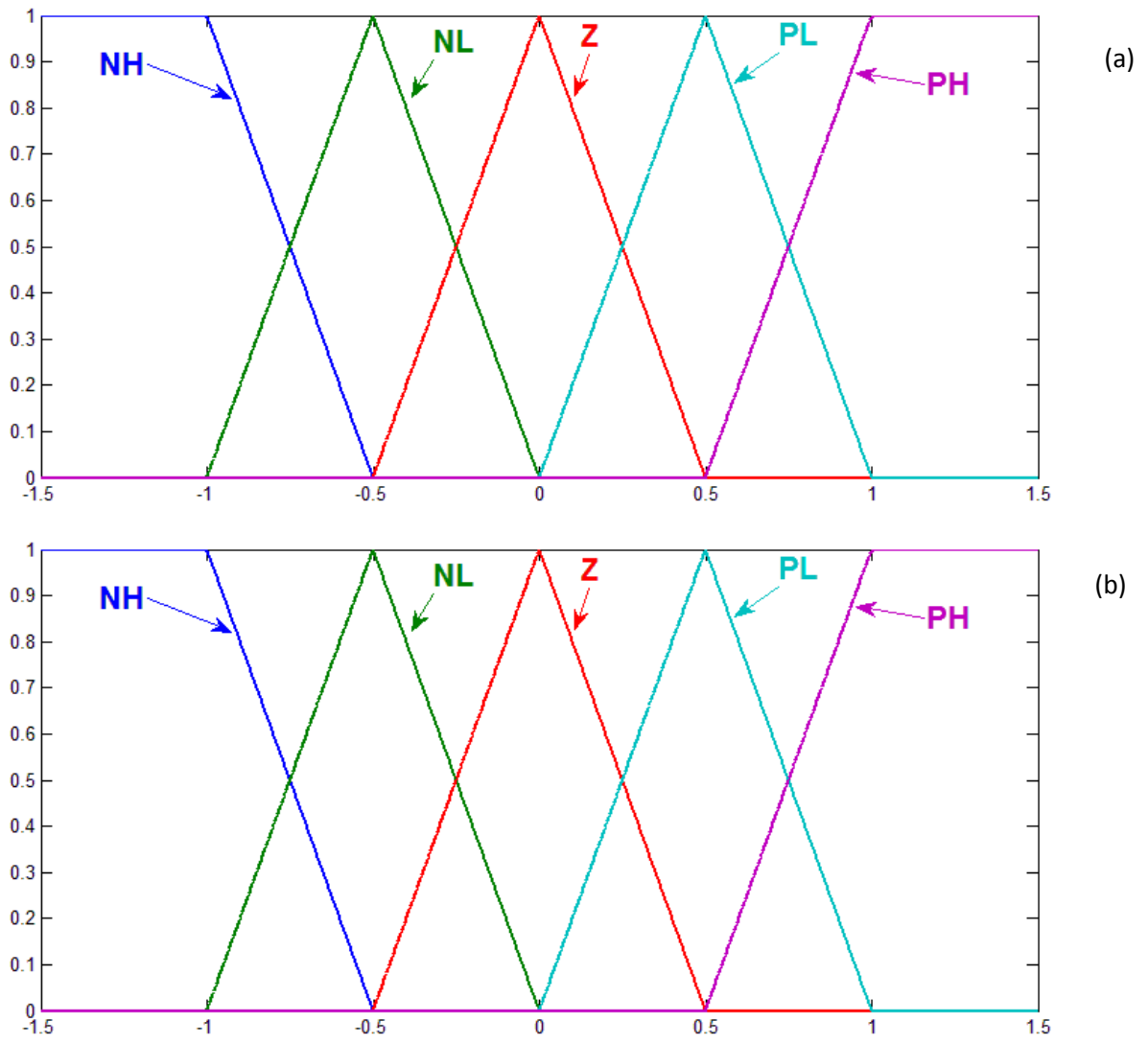


Fig 3-6 Type-1 fuzzy membership functions ($U=0\%$) to represent e (a) and de (b)

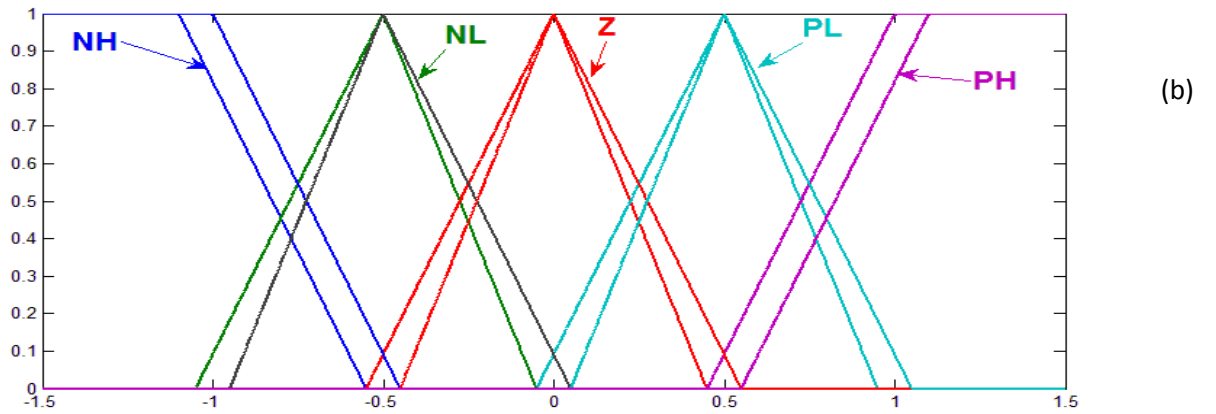
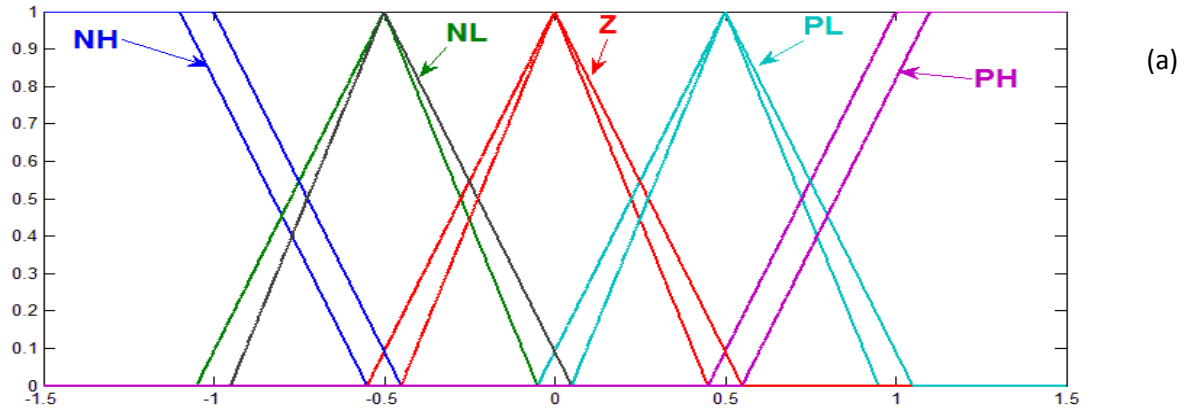


Fig 3-7 Type-2 fuzzy membership functions($U=20\%$) to represent e (a) and de (b)

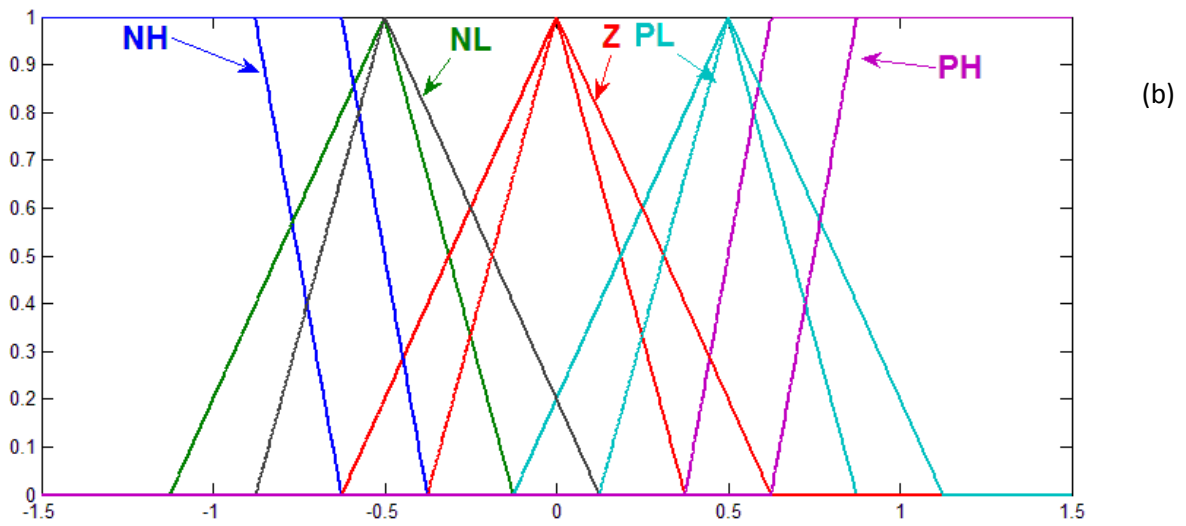
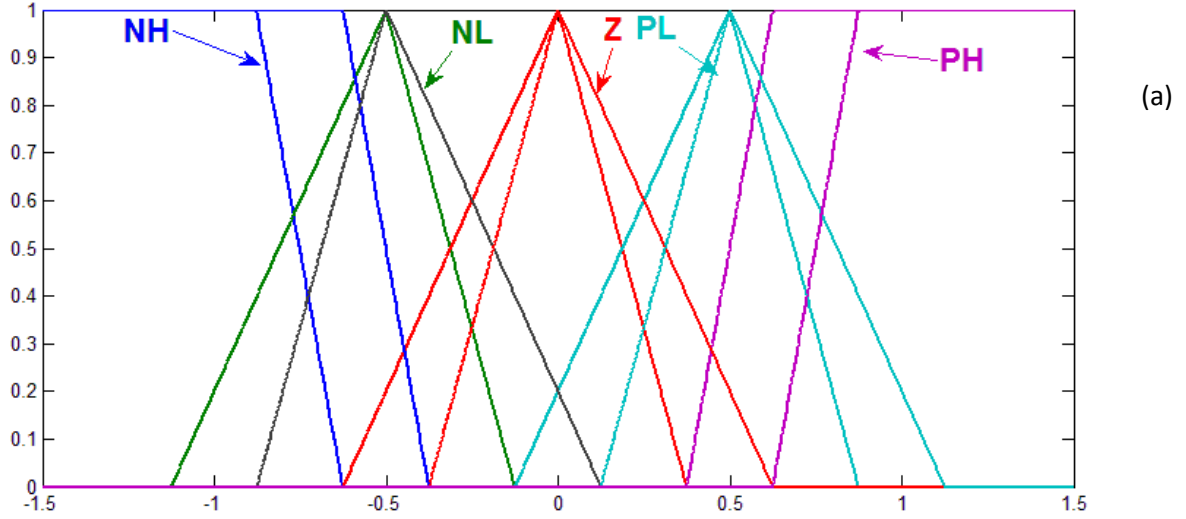


Fig 3-8 Type-2 fuzzy membership functions ($U=50\%$) to represent e (a) and de/dt (b)

III.2.2.2 Rules and Output Membership Functions

We obtain 25 fuzzy rules with 17 output singletons issued from the human expertise [4][52][24].

TABLE 3- 1.
RULES OF T2 TSK FLS

e\de	NH	NL	Z	PL	PH
PH	0,25	0,36	0,49	0,81	1
PL	0	0,04	0,16	0,36	0,64
Z	-0,16	-0,04	0	0,04	0,16
NL	-0,64	-0,36	-0,16	-0,04	0
NH	-1	-0,81	-0,49	-0,36	-0,25

III.3 Results of Simulation

Values of the gains normalization of the fuzzy system: $G_e = 0.77$; $G_{ce} = 6$; $G_{sen} = 0.04$. The reference voltage: $V_{ref}=37.5$ [V].

TABLE 3- 2.
BOOST CONVERTER – VALIDATION PARAMETERS

Description	parameter	value
Series Inductance	L	20[mH]
Parallel Capacitance	C	20[μf]
load resistance	R	30[Ω]
Switching frequency	f_{sw}	5[kHz]
Input Voltage	Vg	15[V]

III.3.1 The Simulation Phases

In the simulation phases we will depend :

- The difference between the FLC is the uncertainty in their membership functions The first FLC has an uncertainty of 0% (Type-1 FLC U=0%) the second and the third consider an uncertainty of 20%(Type-2 FLC U=20%) and 50% respectively (Type-2 FLC U=50%)
- Change in the values of PID.

III.3.1.1 First Simulation:

We are choice the permits: $G1=0.622;G2=255$

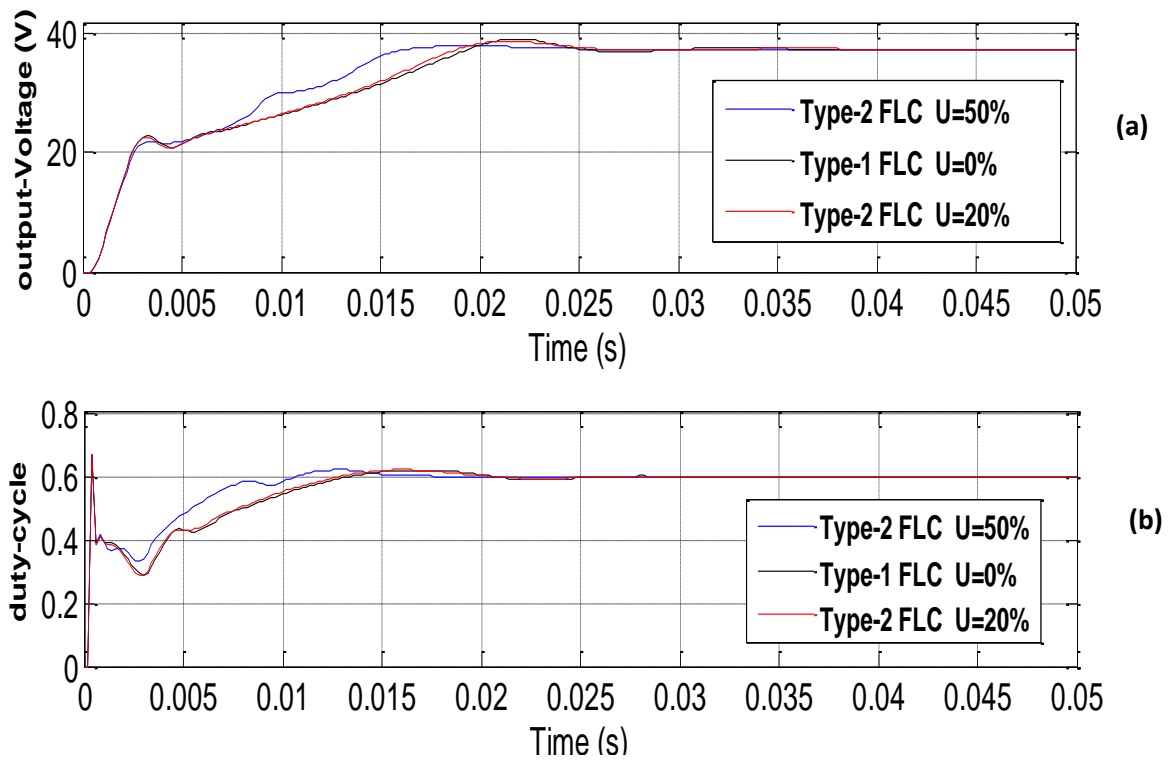


Fig 3-9 .Simulation results of Type-2 fuzzy logic PID controlled Boost converter

Where:

a- Output voltage: for the three fuzzy PID controllers (Type-1 FLC U=0%), (Type-2 FLC U=20%) and (Type-2 FLC U=50%)

b- duty cycle : for the three fuzzy PID controllers (Type-1 FLC U=0%) , (Type-2 FLC U=20%) and (Type-2 FLC U=50%)

To show the visual indications of control performance, we will calculate the integral of square of errors (ISE) and the integral of the absolute errors (IAE) in each case.

- Integral of Square Error (ISE).

$$ISE = \int_0^{+\infty} [e(t)]^2 dt \quad (3 - 2)$$

- Integral of the Absolute value of the Error (IAE).

$$IAE = \int_0^{+\infty} |e(t)| dt \quad (3 - 3)$$

TABLE 3- 3.
NUMERICAL RESULT OF – IAE AND ISE

	IAE	ISE
Type-1 FLC U=0%	0.1983	0.1409
Type-2 FLC U=20%	0.1940	0.1397
Type-2 FLC U=50%	0.1661	0.1283

III.3.1.2 Second Simulation:

We choice the PID values: $G1=0.56$; $G2=255$.

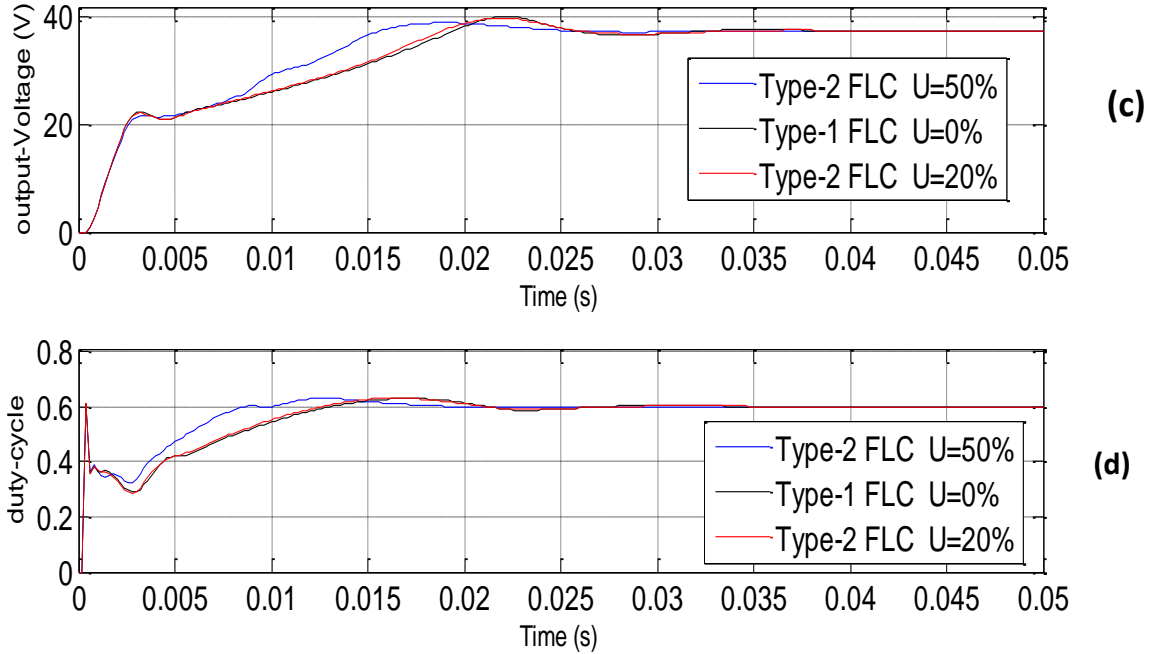


Fig 3-10 Simulation results of Type-2 fuzzy logic PID controlled Boost converter

Where:

c output voltage: for the three fuzzy PID controllers Type-1 FLC ($U=0\%$) , Type-2 FLC ($U=20\%$) and Type-2 FLC ($U=50\%$) .

d Duty cycle: for the three fuzzy PID controllers Type-1 FLC ($U=0\%$) , Type-2 FLC ($U=20\%$) and Type-2 FLC ($U=50\%$) .

TABLE 3- 4.
NUMERICAL RESULT OF – IAE AND ISE

	IAE	ISE
Type-1 FLC U=0%	0.2069	0.1434
Type-2 FLC U=20%	0.2030	0.1422
Type-2 FLC U=50%	0.1735	0.1305

III.3.2 Simulation Analysis

The values of the ISE and the IAE for the proposed Type-2 fuzzy logic PID controller are lower than that the values obtained for the Type-1 fuzzy logic PID controller. So, the proposed Type-2 fuzzy logic PID controller handles with efficiency the system uncertainties rather than the Type-1 fuzzy logic PID controller.

III.4 Chapter Conclusion

In this study, a Type-2 fuzzy logic PID controller was proposed for the control of the DC–DC Boost converter. To show the visual indications of control performance, we had calculated the integral of square of errors (ISE) and the integral of the absolute errors (IAE) in each case.

A Simulation results demonstrated that Type-2 fuzzy logic PID controller reduce the integral of square of errors (ISE),and the integral of the absolute errors (IAE). thus , Type-2 fuzzy logic PID controller ensures better performance to the system ; despite of the changes in the uncertainty in input membership functions and the PID values.

To sum up, Type-2 fuzzy logic PID controller can effectively control a Boost converter DC-DC converter because it was more suitable for application and can handle the different uncertainties of the controlled system.

CONCLUSION

CONCLUSION

In this thesis our objective is to design a type-2 fuzzy logic PID controller for a DC-DC Boost converter and to compare their results to those obtained by a type-1 a fuzzy logic PID controller using the same structure and under the same functioning conditions. This choice is motivated by the fact to take into account the different uncertainties of the controlled system.

In this thesis we introduce three chapters:

Firstly, description of the studied the boost converter and its operating mode and used The State-Space Average method because method is very useful for analyzing the low-frequency, small-signal Performance of switch circuits .It is applicable when the converter switching period is short while comparing to the response time of the output voltage.

Secondly, introduces some motivations for using T2-FL instead of T1-FL control systems and we presented Interval type-2 fuzzy systems in specially Interval Type-2 TSK Fuzzy Logic systems that aide us in our work.

Finally, in this part we presented our method simulation that rely: The difference between the FLC is the uncertainty in their membership functions The first FLC has an uncertainty of 0% (Type-1 FLC $U=0\%$) the second and the third consider an uncertainty of 20%(Type-2 FLC $U=20\%$) and 50% respectively (Type-2 FLC $U=50\%$) Change in the values of PID.

Final result we find Type-2 fuzzy logic PID controller can effectively control a Boost converter DC-DC converter because it was more suitable for application and can handle the different uncertainties of the controlled system.