People's democratic republic of Algeria
Ziane Achour University of Djelfa
Faculty of science and Technology

Course

MODELING AND SIMULATION OF ELECTRIC MACHINES

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Chapter 1

Electromechanical Energy Conversion

1.1 Introduction

This chapter discusses generalities and basic principles that enable a good comprehension and understanding of machine modeling and simulations that will be analyzed in the coming chapters. This chapter is divided into two main parts. The first part deals with electromechanical energy conversion and the equations of energy, force, and torque for an electromagnetic system. The second part explains the structure of electric machines, their principle of operation, their types, and common industrial applications of each type.

1.2 Principal of Electromechanical Energy Conversion

1.2.1 Definition

The electromechanical energy conversion system allows converting electrical energy to a mechanical energy and vice-versa. Using appropriate devices, in general electromagnetic field is used as a coupling medium between mechanical and electrical systems.

When energy is converted from electrical to mechanical form, in this case, the converter is called a motor.

When energy is converted from mechanical to electrical form the converter is called a generator.

Figure 1.1 Simplified form of electromechanical system.

As depicted in figure 1.1 an electromechanical energy conversion converter has three main parts.

- Mechanical system
- Coupling medium
- Electrical system

1.2.2 Energy Distribution

The total energy is given by.

$$W_E = W_e + W_{eL} + W_{eS} \quad 1.1$$

$$W_M = W_m + W_{mL} + W_{mS} \quad 1.2$$

Where

W_E: Is the total electric energy.

W_{eL}: Electrical losses energy.

W_{eS}: Coupling field losses (stored energy)

W_e: Energy transferred to the coupling field.

W_M: Is the total mechanical energy.

W_{mL}: Mechanical losses energy (friction losses).

W_{mS}: Energy stored in the moving member and the compliances of the mechanical system.

W_m: Energy transferred to the coupling field by the mechanical system.

In lossless system

$$W_e = W_f + W_m \quad 1.3$$

W_f: Stored field energy

So:

$$\frac{dW_e}{dt} = \frac{dW_f}{dt} + \frac{dW_m}{dt} \quad 1.4$$

$$\frac{dW_e}{dt} = e.i - P_{mec} \quad 1.5$$

The mechanical power can be calculated using the equations below.

If the movement is linear / f is the force

$$P_{mec} = f\frac{dx}{dt} \quad 1.6$$

If the movement is rotational / T is the torque

$$P_{mec} = T\frac{d\theta}{dt} \quad 1.7$$

a) Single excitation system

Let's consider a single excitation system with linear movement

$$\frac{dW_e}{dt} = e.i - f\frac{dx}{dt} \qquad 1.8$$

By Fraday's law, we get

$$e = \frac{d\lambda}{dt} \qquad 1.9$$

Subtitling 1.9 in 1.10 yield

$$\frac{dW_e}{dt} = i\frac{d\lambda}{dt} - f\frac{dx}{dt} \qquad 1.10$$

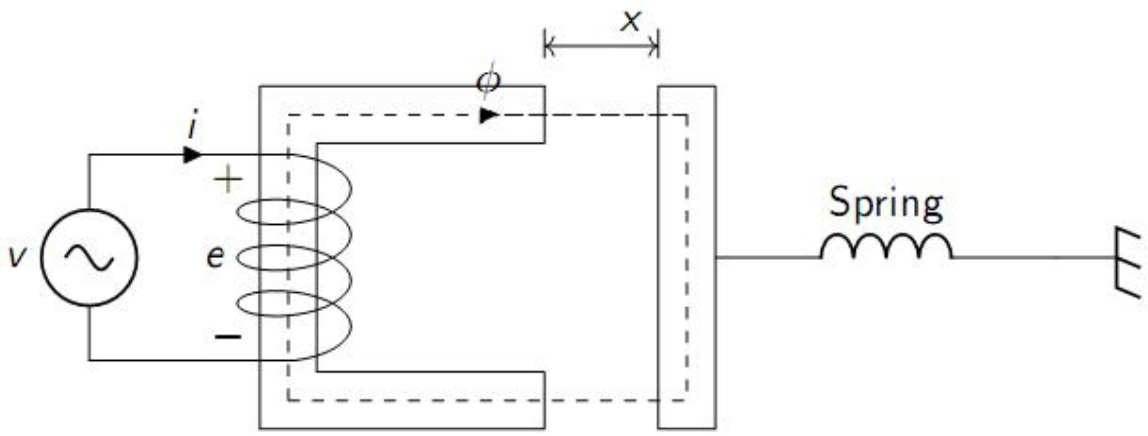


Figure 1.2 Singly excited system

By integrating over this specific range, we can calculate the cumulative area to the left of the magnetization curve. The integral operation focuses on measuring the total space enclosed by the curve on its left side.

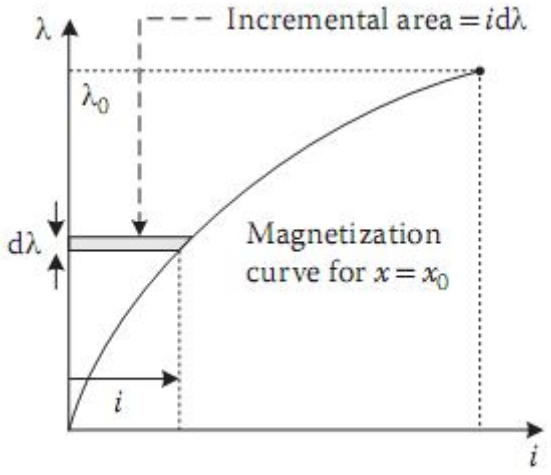


Figure 1.3 Magnetization curve

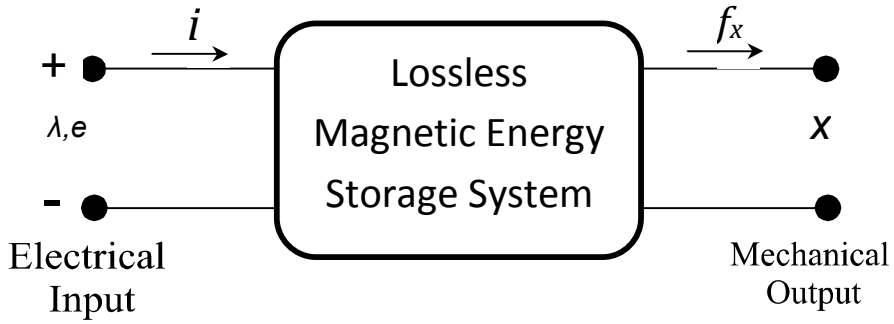


Figure 1.4 Lossless singly excited system

As mentioned above the system is lossless so:

$$v = e = \frac{d\lambda}{dt} \quad 1.11$$

$$W_f = \int i d\lambda \quad 1.12$$

$$W_f = W_f(i, x). \quad 1.13$$

$$dW_f = \frac{\partial W_f}{\partial \lambda} dx + \frac{\partial W_f}{\partial x} d\lambda \quad 1.14$$

In a linear electromagnetic system, the flux λ can be expressed as:

$$\lambda = L(x)i, \quad 1.15$$

L: is the inductance.

Depending on the inductance, the field energy

$$W_f = \int_0^{\lambda} d\lambda \quad 1.16$$

$$W_f = \frac{1}{2}L(x)i^2 = \frac{\lambda^2}{2L(x)} \quad 1.17$$

According to the equation 1.17 The devolped force can be evaluated as .

$$f = -\frac{\partial W_f}{\partial x}d\lambda = \frac{1}{2}\frac{\lambda^2}{L(X)^2}\frac{dL(x)}{dx} \quad 1.18$$

$$f = \frac{1}{2}i^2\frac{dL(x)}{dx}N \quad 1.19$$

The same development for the rotational linear electromagnetic system.

From equation 1.14.

$$T = \frac{1}{2}i^2\frac{dL(\theta)}{d\theta}Nm \quad 1.20$$

b) Multiple Excitation System

In the case of a multi-excited electromagnetic system, the field energy can be described as follows.

$$W_f = \int \sum_{j=1}^{J} i_j d\lambda_j \quad 1.21$$

$$\lambda_j = \sum_{k=1}^{N} L_{jk} i_k \quad 1.22$$

Where $\boldsymbol{L_{jj}}$ is the self-inductance of winding *j* and $\boldsymbol{L_{jk}}$ is the mutual inductance between windings j and k.

$$dW_f(i,x) = \frac{1}{2}\sum_{k=1}^{N} i_k \sum_{j=1}^{N} L_{kj} i_j \quad 1.23$$

For deep comprehension, let's consider a two-electrical input linear electromagnetic system as shown in figure 1.5.

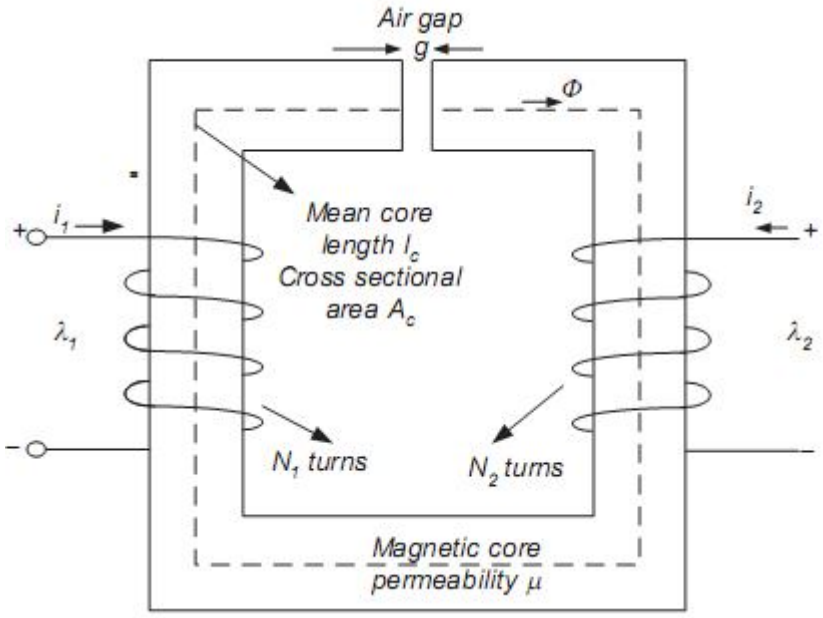


Figure 1.5 Magnetic circuit with two windings

$$e_1 = \frac{\lambda_1}{dt} \quad 1.24$$

$$e_2 = \frac{\lambda_2}{dt} \quad 1.25$$

In this case, the flux linkage is given as :

$$\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2 \quad 1.26$$

$$\lambda_2 = L_{21}\, i_1 + L_{22}\, i_2 \quad 1.27$$

L_{11} ,L_{22} are the self-inductance.

L_{12} ,L_{21} are the mutual inductances between the two windings.

For a linear magnetic system $L_{12} = L_{21} = M$

Equations 1.25 and 1.26 can also be expressed in the matrix form as follows:

$$\begin{bmatrix}\lambda_1 \\ \lambda_2\end{bmatrix} = \begin{bmatrix}L_{11} & M \\ M & L_{22}\end{bmatrix}\begin{bmatrix}i_1 \\ i_2\end{bmatrix}. \quad 1.28$$

Hence we get the incrementalenergy balance equations.

$$dW_e = dW_f + dW_m \quad 1.29$$

Assuming the rotor is statinary, so: $dW_m = 0$.

$$dW_e = dW_f \quad 1.30$$

$$dW_e = e_1 i_1 dt + e_2 i_2 dt \quad 1.31$$

Therefore

$$dW_f = i_1 d(L_1 i_1 + M i_2) + i_2 d(M i_1 + L_2 i_2) \quad 1.32$$

$$dW_f = L_1 i_1 di_1 + L_2 i_2 di_2 + M d(i_1, i_2) \quad 1.33$$

The field energy is

$$W_f = \int_0^{i_1} L_1 i_1 di_1 + \int_0^{i_2} L_2 i_2 di_2 + \int_0^{(i_1,i_2)} M d(i_1, i_2) \quad 1.34$$

$$W_f = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 \quad 1.35$$

1.3 Rotating Electrical Machine Principal

As previously observed, W_f, the magnetic energy stored in the system, is a state variable, and the specific value it takes on is inconsequential. Therefore, if we consider the system to be stationary with no mechanical output, W_f represents the magnetic field energy retained within the system. Therefore,

$$dW_f = e_s i_s dt + e_r i_r dt \quad 1.36$$

$$i_s d\lambda_s + i_r d\lambda_r \quad 1.37$$

For a linear magnetic system,

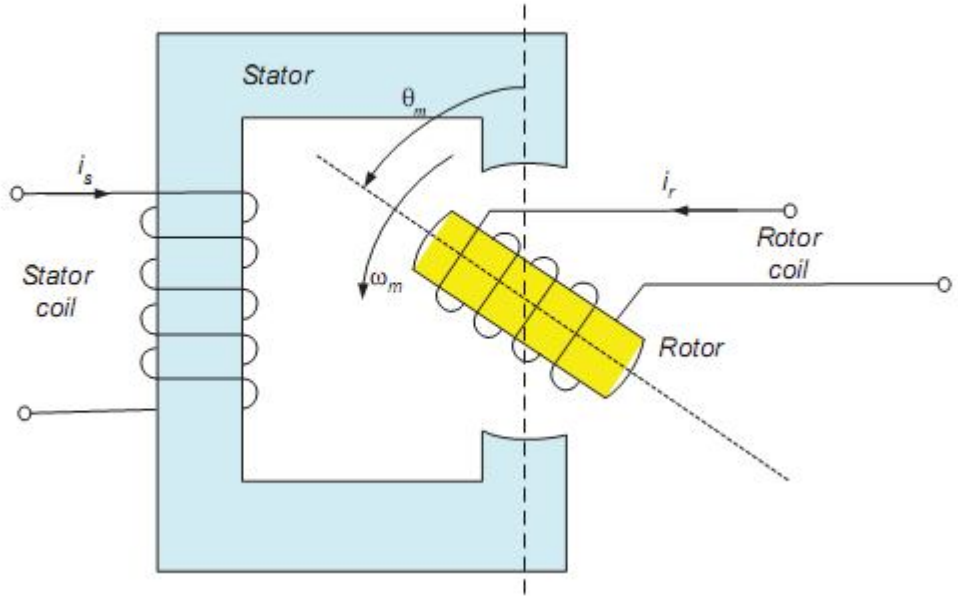


FIGURE 1.6 General structure of a rotating machine.

$$\lambda_s = L_{ss} i_s + L_{sr} i_r \qquad 1.38$$

$$\lambda_r = L_{rs} i_s + L_{rr} i_r \qquad 1.39$$

Here,

L_{ss}, is the self-inductance of the stator winding.

L_{rr}, is the self-inductance of the rotor winding.

L_{rs} and L_{sr} are mutual inductances of the rotor and stator windings.

From equations (1.36), (1.37), (1.38) and (1.39),

$$dW_f = i_s d(L_{ss} i_s + L_{sr} i_s) + i_r d(L_{sr} i_s + L_{rr} i_r) \qquad 1.40$$

$$dW_f = L_{ss} i_s di_s + L_{rr} i_r di_r + L_{sr} d(i_s i_r) \qquad 1.41$$

Total field energy will be

$$W_f = L_{ss}\int_0^{i_s} i_s di_s + L_{rr}\int_0^{i_r} i_r di_r + L_{sr}\int_0^{i_s i_r} d(i_s i_r) \quad 1.42$$

$$W_f = \frac{1}{2}L_{ss}i_s^2 + \frac{1}{2}L_{rr}i_r^2 + L_{sr}i_s i_r \quad 1.43$$

In rotational electro-mechanical energy conversion systems, the developed torque is comparable to the force equation found in earlier chapters.

$$T = \frac{\partial W_c(i,\theta_m)}{\partial \theta_m} \quad 1.44$$

In a magnetic system that follows a straight path, the amount of magnetic energy stored would be the same as the system's co-energy. As a result, based on the given equations, we are able to formulate the following expression.

$$T = \frac{1}{2}i_s^2\frac{dL_{ss}}{d\theta_m} + \frac{1}{2}i_r^2\frac{dL_{rr}}{d\theta_m} + i_s i_r\frac{dL_{sr}}{d\theta_m} \quad 1.45$$

The initial two elements on the left side of the equation indicate "torques generated by changes in self-inductance due to the position of the rotor within the machine." This specific torque factor is known as "reluctance torque." The third element signifies "torque generated by changes in mutual inductance" between the stator and rotor windings. It is essential to remember that the calculation is based on the mechanical (shaft) angle θm.

1.4 Rotating Electrical Machine structure

The electrical machine is an electromechanical device allows to convert electrical energy to mechanical energy and vice versa.

• In Motor operation, electrical energy is transformed into mechanical energy.

• In generator operation, mechanical energy is transformed into electrical energy(it can behave like a brake). In this case it is also called a dynamo.

Electrical machine have two main parts, stator and rotor.

Stator: Fixed part of the machine where the power supply is connected

Rotor: The rotating part which allows the mechanical load to rotate

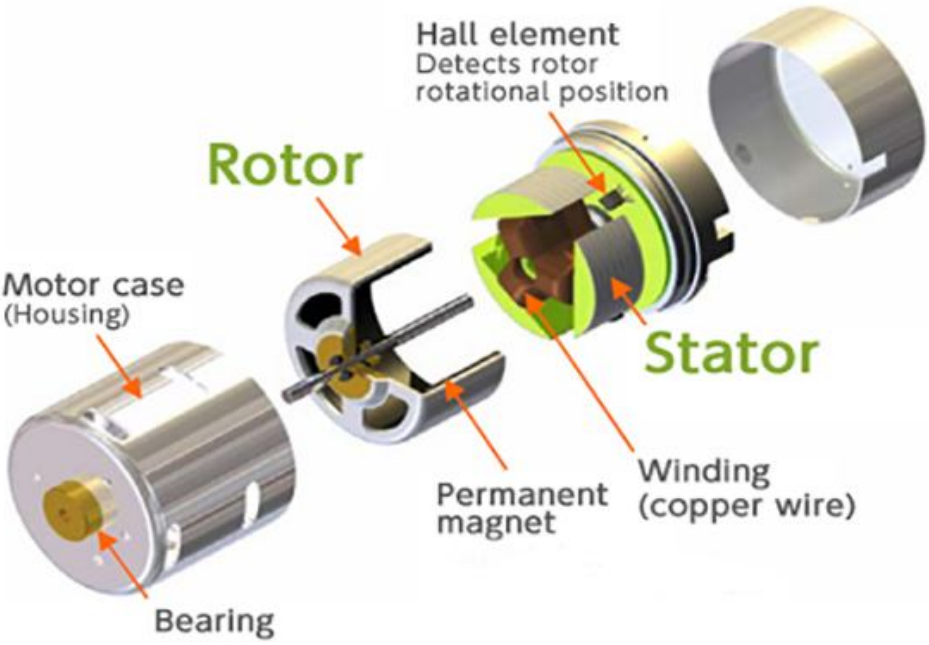
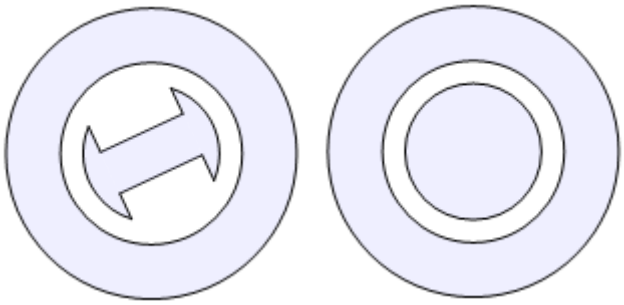


Figure 1.7 Typical electrical machine



Silent pole rotor surface mounted rotor

Figure 1.8 Type of machine Rotors

There are several categories of electrical machines according to:

stator power supply: DC or AC

rotor: excited wound, short circuit wound, permanent magnet,

reluctance, squirrel cage, etc.

There are two main categories of windings: armature windings and field windings. The armature winding is responsible for carrying the main current and generating

electromotive force (emf) or counter-emf through rotation. The flow of electricity in the armature winding is referred to as the armature current. On the other hand, the field winding is responsible for creating.

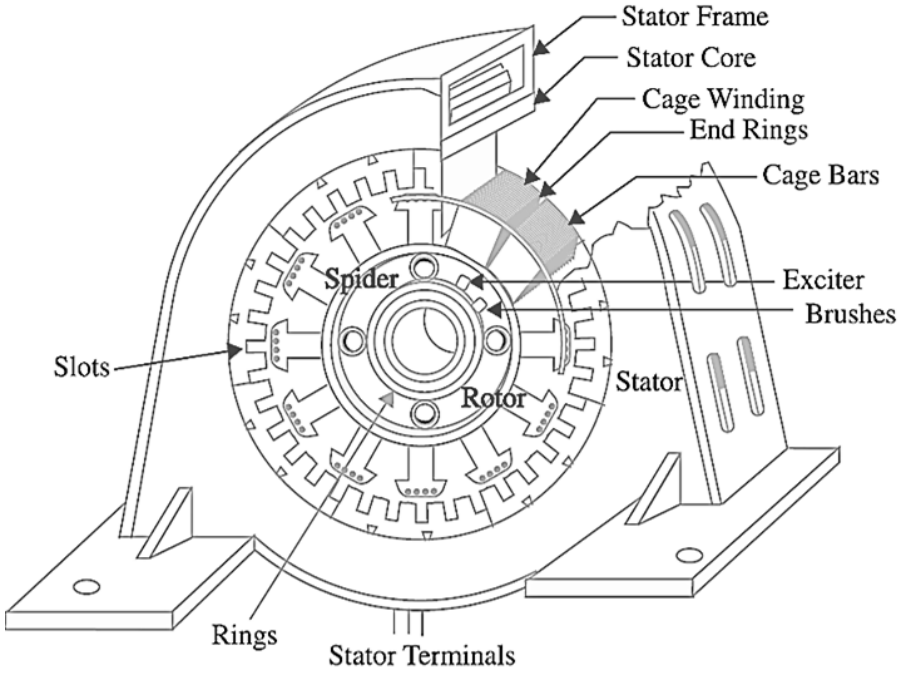


Figure 1.9 Stator, Rotor, and slot of electric machine

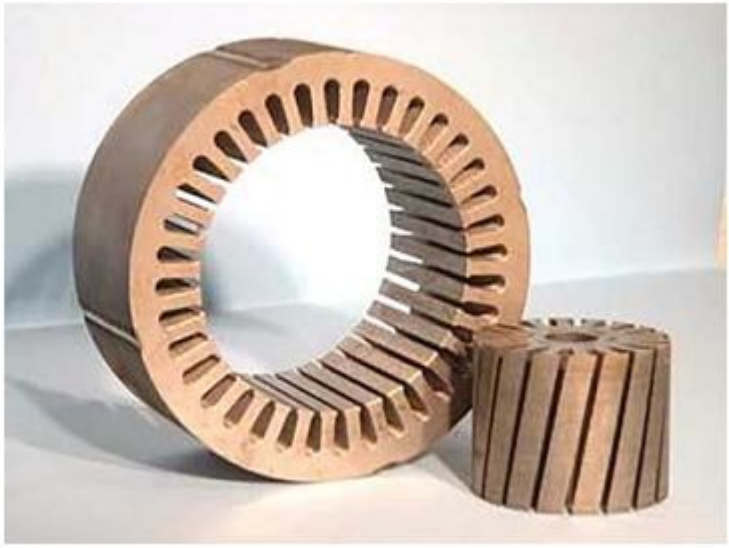


Figure 1.10 Slotted structure of rotor and stator

Motor: Convert electrical energy into mechanical energy.

Generator: Convert mechanical energy into electrical energy.

The operational mechanism of an electric motor is a result of the interplay between two magnetic fields generated by the stator and rotor, respectively. This interaction results in either attractive or repulsive forces, ultimately causing the rotor to spin in relation to the stator.

Chapter 2

General Aspect on Rotation Machine

2.1 Introduction

This chapter is devoted to explaining the operating principle of the three basic types of electric motors that are most commonly used in the industrial field. We mention them consecutively. The movement is a constant current. AC synchronous motor and finally induction (asynchronous) motor.

There are three main types of rotating electrical machine

- ✓ DC Machines
- ✓ AC Machines

2.2 Direct Current DC Machine

A direct current machine is an electrical machine. This is a converter electromechanical device allowing the bidirectional conversion of energy between an electrical installation carried by a direct current and a mechanical device; depending on the energy source.

2.2.1 DC machine structure

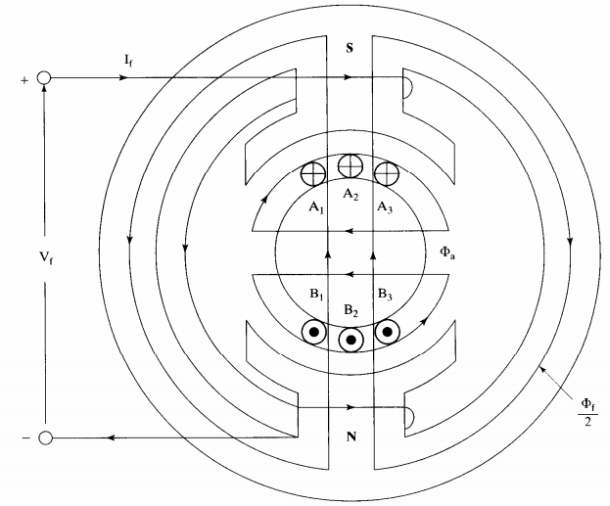


Figure 2.1 Schematic representation of DC machine.

The direct current machine has the following main parts:

a) The inductor

It is formed either from permanent ferrite magnets or from coils placed around the polar cores. When a direct current flows through the coils, they create a magnetic field in the magnetic circuit of the machine, particularly in the air gap, the space separating the fixed part and the part mobile, where the drivers are located. This flow and this field are oriented from the North Pole to the South Pole

b) The armature

The armature core is made of iron to channel the field lines, the conductors are housed in notches on the rotor, and two conductors form a turn.

c) Collector and brushes

The collector is a set of insulated copper blades, arranged on the end of the rotor, the brushes carried by the stator rub on the collector.

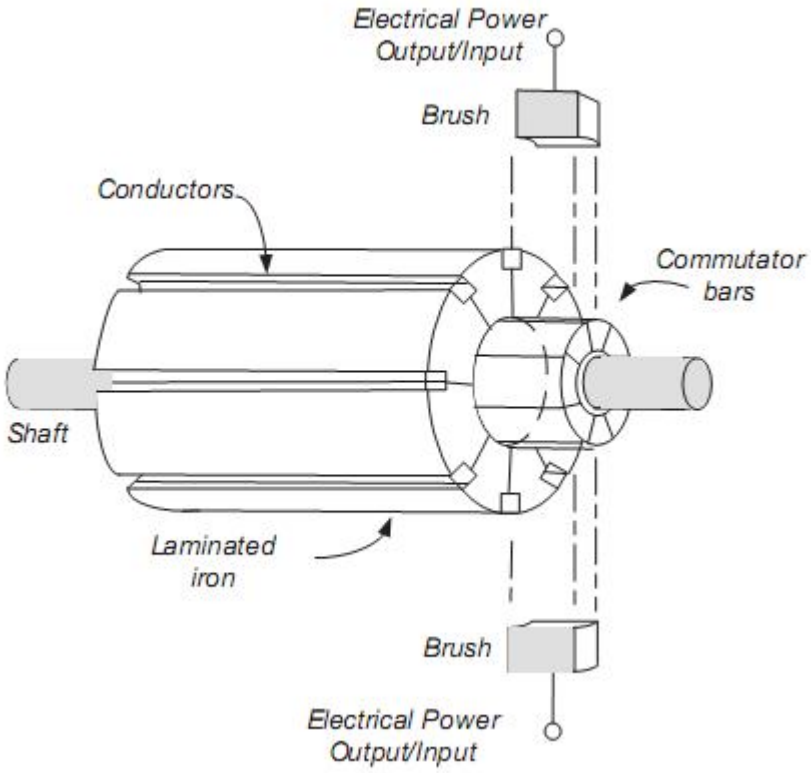


Figure 2.2 Rotor structure of DC machine

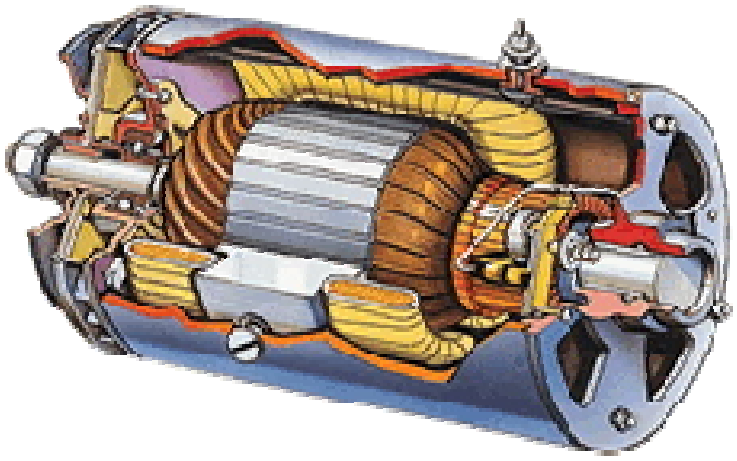


Figure 2.3 Panoramic view of industrial DC machine

2.2.2 Induced EMF

The interaction between the current-carrying rotor conductors and the magnetic field causes them to experience a force in a specific direction. This force is a crucial component in the operation of electric motors.

$$\overrightarrow{dF} = I\overrightarrow{dl}\,\hat{}\,\overrightarrow{dB} \qquad 2.1$$

This force will make the rotor spin, generating a motor torque as shown in the diagram below. :

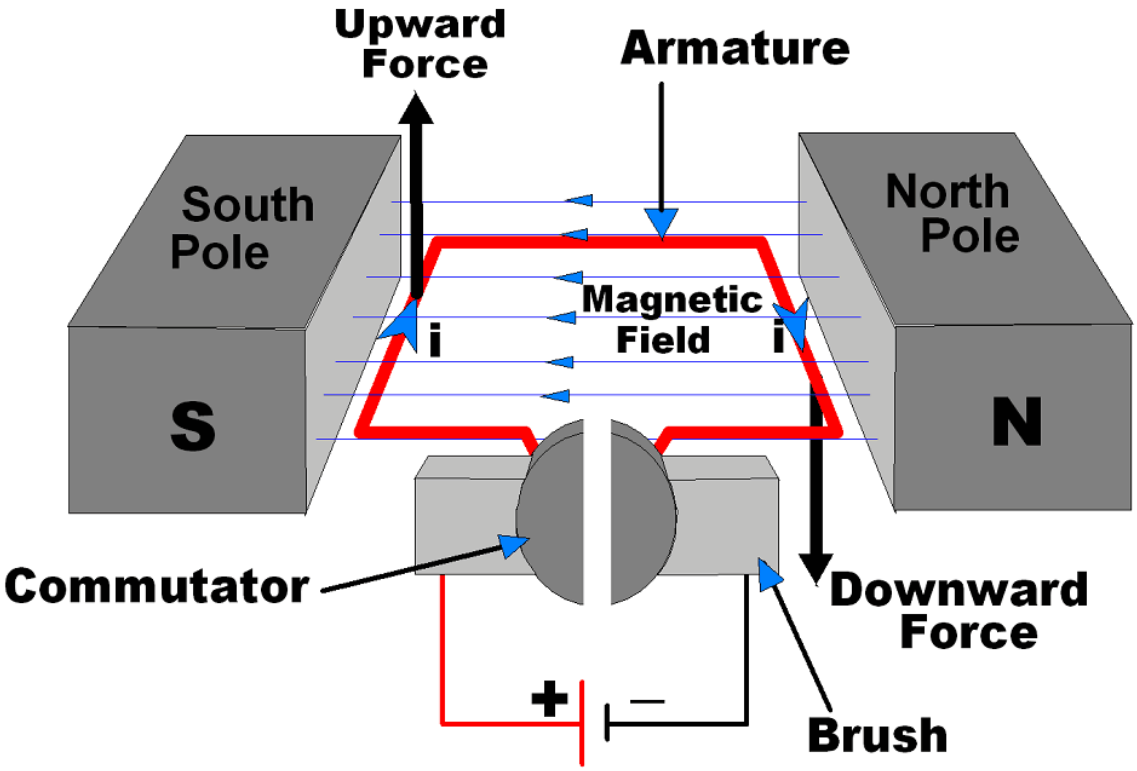


Figure 2.4 Working principal of DC machine (https://ampmotorworks.com)

The magnitude of the back EMF induced in the armature is given by:

$$e = Z\frac{d\phi}{dt} = Z\frac{\phi}{t} \qquad 2.2$$

"t" is the time taken by the conductors to cut the flux lines. Therefore,

$$t = \frac{1}{2xfrequancy} = \frac{1}{2\left(\frac{P}{2}\right)\left(\frac{N}{60a}\right)} \qquad 2.3$$

$$e = \frac{NP\phi Z}{60a} \qquad 2.4$$

E: Induced emf

N: Rotor speed

P: Number of poles

Z: Total number of conductors in the armature

a: is the number of parallel paths through the armature between the brushes of opposite polarity

Ø: the useful flux per pole.

For control purposes the equation 1.20 can be simplified as

$$\boldsymbol{e = K\emptyset\omega_m} \quad 2.5$$

$$\boldsymbol{\omega_m = \frac{2\pi N}{60} rad/sec} \quad 2.6$$

$$\boldsymbol{K = Z\left(\frac{P}{a}\right)\left(\frac{1}{2\pi}\right)} \quad 2.7$$

If the field flux is constant then

$$\boldsymbol{e = K_b\omega_m} \quad 2.8$$

$$\boldsymbol{K_b = K\emptyset} \quad 2.9$$

2.2.3 Electrical equivalent circuit

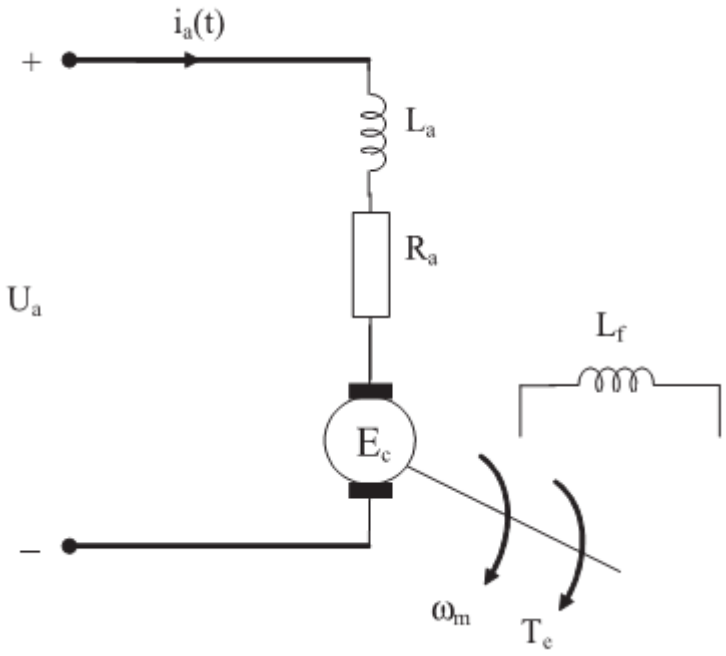


Figure 2.5 scheme of electrical equivalent circuit of dc machine

a) In the inductor

$$v_e = R_e I_e \quad (2.10)$$

$$\emptyset_e = R_e I_e \quad (2.11)$$

b) In the armature

$$V_a = E + R_a I_a \quad (2.12)$$

The mechanical power is expressed as:

$$P_m = e.i_a = T_e.\omega_m \quad (2.13)$$

The electromagnetic torque

$$P_m = T_e.\omega_m \quad (2.14)$$

Combining the equation 2.13 and 2.14 yield

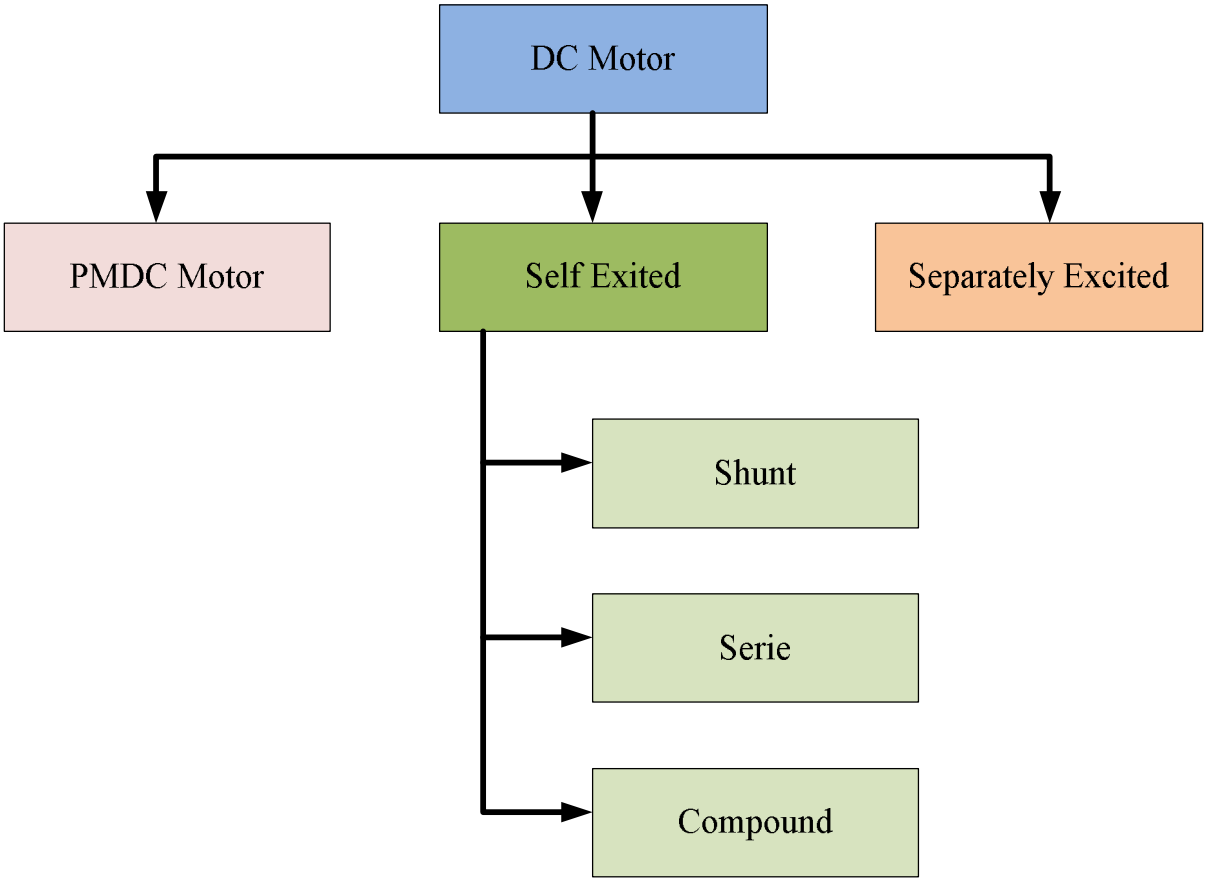
$$T_e = \frac{e.i_a}{\omega_m} \quad (2.15)$$

Subtitling 2.9 in 2.15

$$T_e = \frac{K_b \omega_m i_a}{\omega_m} = K_b i_a \quad (2.16)$$

2.2.4 The different types of direct current machines

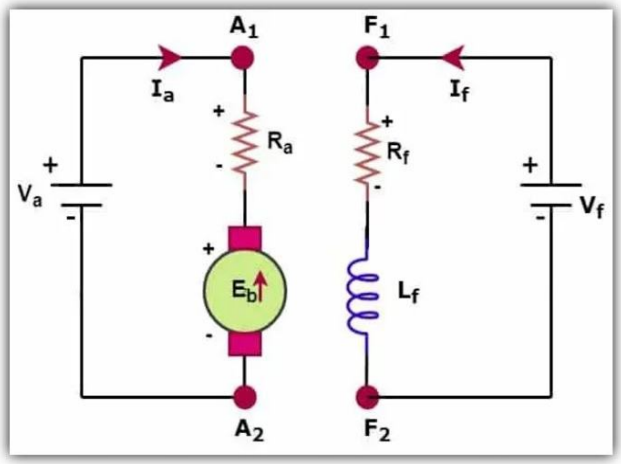
Depending on the application, the inductor and armature windings may be connected in different ways. We generally find five types as illustrated in the diagram bellow.



2.6 Dc machine types diagram

a) Separately excited dc machine

In the separately excited motor the inductor and the armature are independent and can be supplied by separate sources: for example variable power supply by electronic device for the armature and fixed power supply for the inductor.



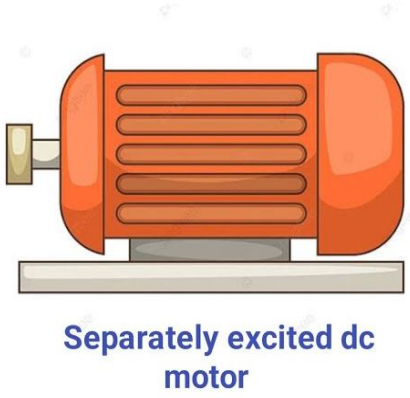


Figure 2.7 Scheme of separately excited DC machine

b) Dc shunt motors

In the shunt motor the stator is connected in parallel with the rotor. So the voltage across the rotor is the same as that across the stator.

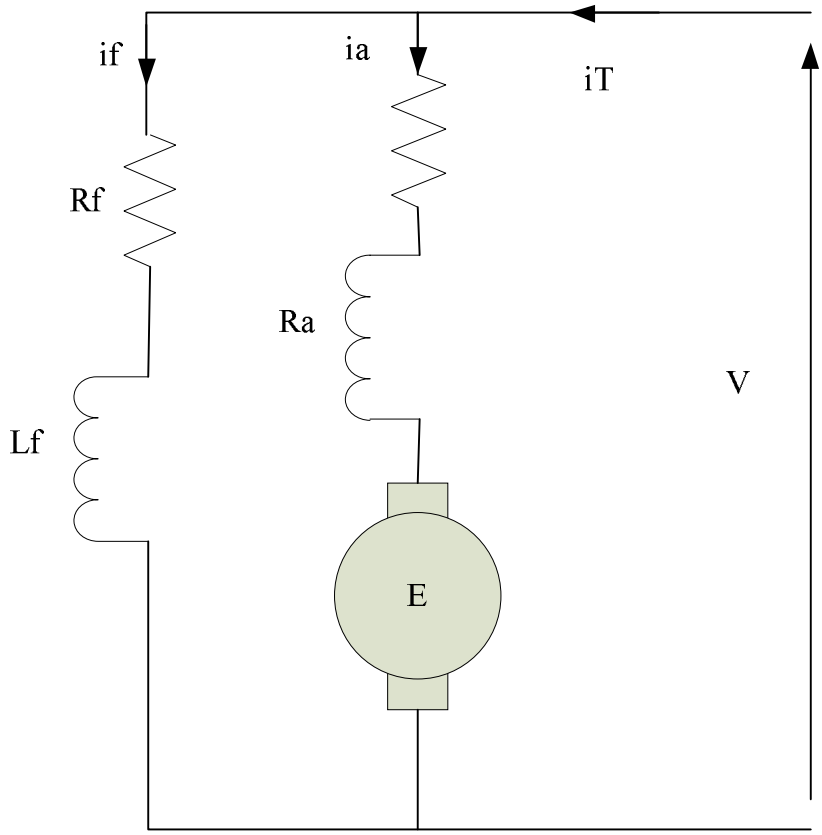


Figure 2.8 Scheme of dc shunt machine

$$\boldsymbol{V_a = V_e = V} \qquad 2.17$$

$$\boldsymbol{V_a = E + R_a I_a} \qquad 2.18$$

c) Series machine.

Serial excitation is today reserved for motors application, This type of motor is characterized by the fact that the stator is connected in series with the rotor.

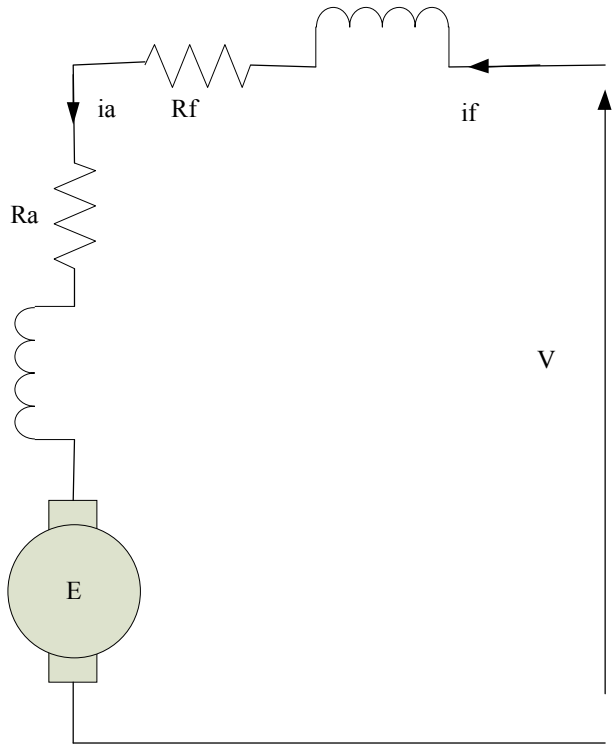


Figure 2.9 scheme of dc series machine

So the same current flows through the rotor and the stator:

$$\boldsymbol{I_a = I_e = I} \qquad 2.19$$

The supply voltage

$$\boldsymbol{V = V_e + V_a} \qquad 2.20$$

$$\boldsymbol{V_a = E + R_a I_a + R_e I_e} \qquad 2.21$$

$$\boldsymbol{T = K_a K_e . I = K I^2} \qquad 2.22$$

The above equations show that series excitation motors can develop very high torque, particularly at low speed, this being proportional to the square of the current.

d) DC compound machine

In the compound motor, part of the stator is connected in series with the rotor and another is of the parallel or shunt type.

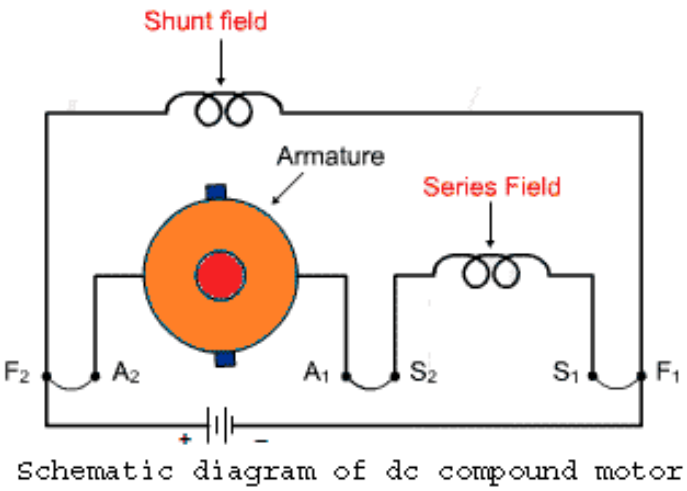


Figure 2.10 scheme of dc compound machine

e) Permanent magnet motor.

a constant inductor flux it is created by permanent magnets.

$$e = K_b \omega_m \quad (2.23)$$

$$T_e = K_b i_a \quad (2.24)$$

$$V_a = E + R_a I_a \quad (2.25)$$

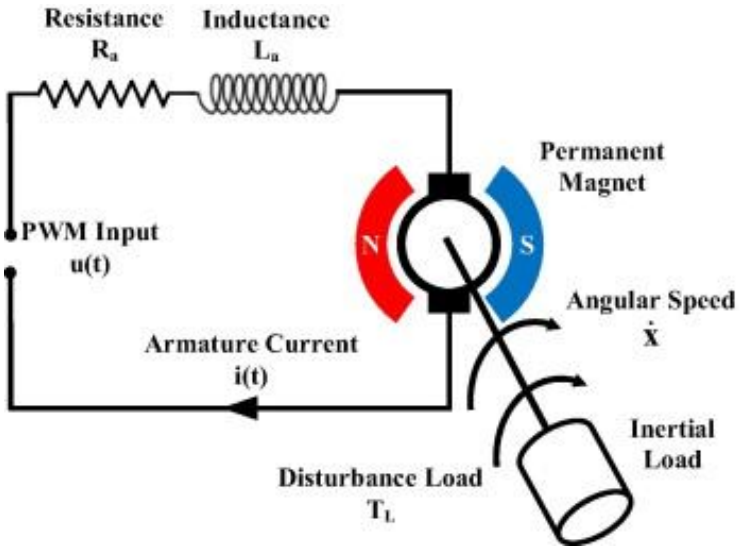


Figure 2.11 Scheme of permanent magnet dc machine

2.2.5 Four quadrant operation of DC machine

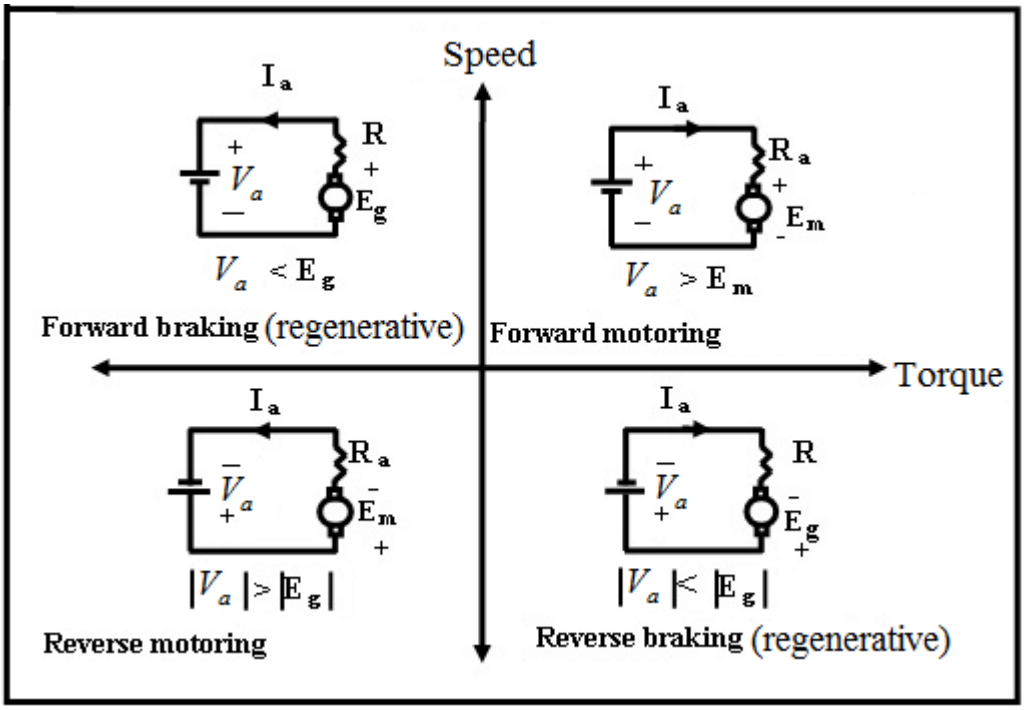


Figure 2.12 Four quadrant operation

Quadrant 1.

The machine is working as a motor supplying mechanical energy

Quadrant 2.

The machine is working as generator with negative torque.

Quadrant 3.

Reverse motoring mode. The motor speed and torque are negative while the the power is positive.

Quadrant 4.

The torque is positive, and the speed is negative. This quadrant corresponds to the braking in the reverse motoring mode.

Table 2.1 four quadrant operation

Function	Quadrant	Speed	Torque	Power Output
Forward Motoring	1	+	+	+
Forward Braking	2	+	-	-
Reverse Motoring	3	-	-	+
Reverse Braking	4	-	+	-

2.2.6 Power flow diagram of DC Machine

a) Losses In A DC Machine

- Copper losses
  - Armature Cu loss
  - Field Cu loss
  - Loss due to brush contact resistance
- IronLosses
  - Hysteresisloss
  - Eddy currentloss
- Mechanical losses
  - Friction loss
  - Windageloss

b) Generator mode

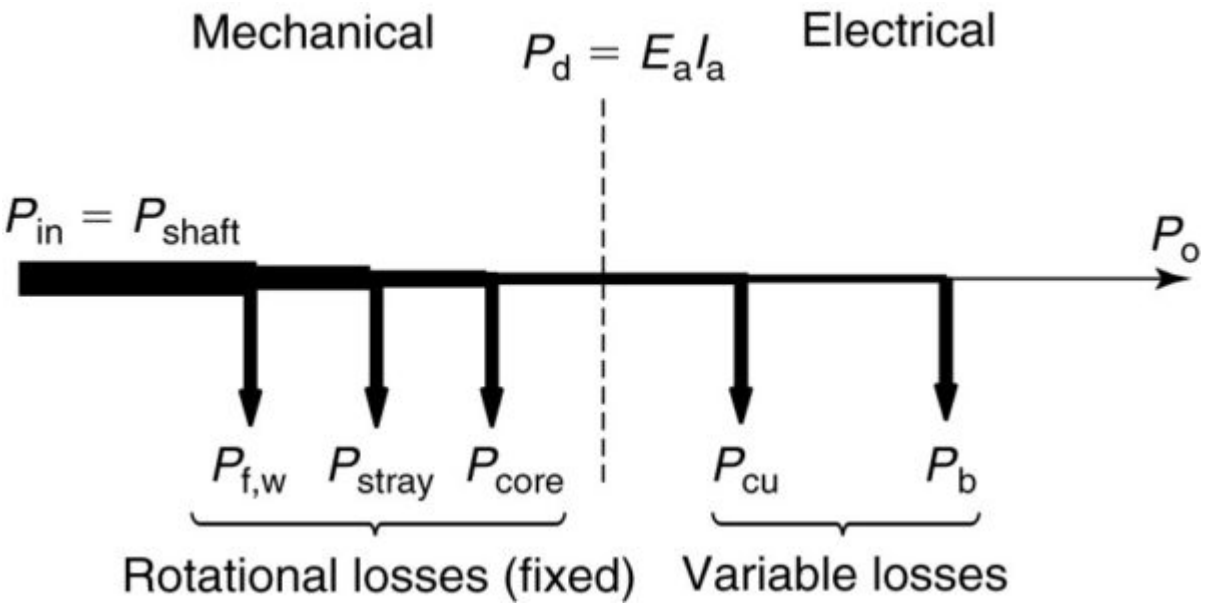


Figure 2.13 power flow in generator mode (https://electricalacademia.com)

c) Motor mode

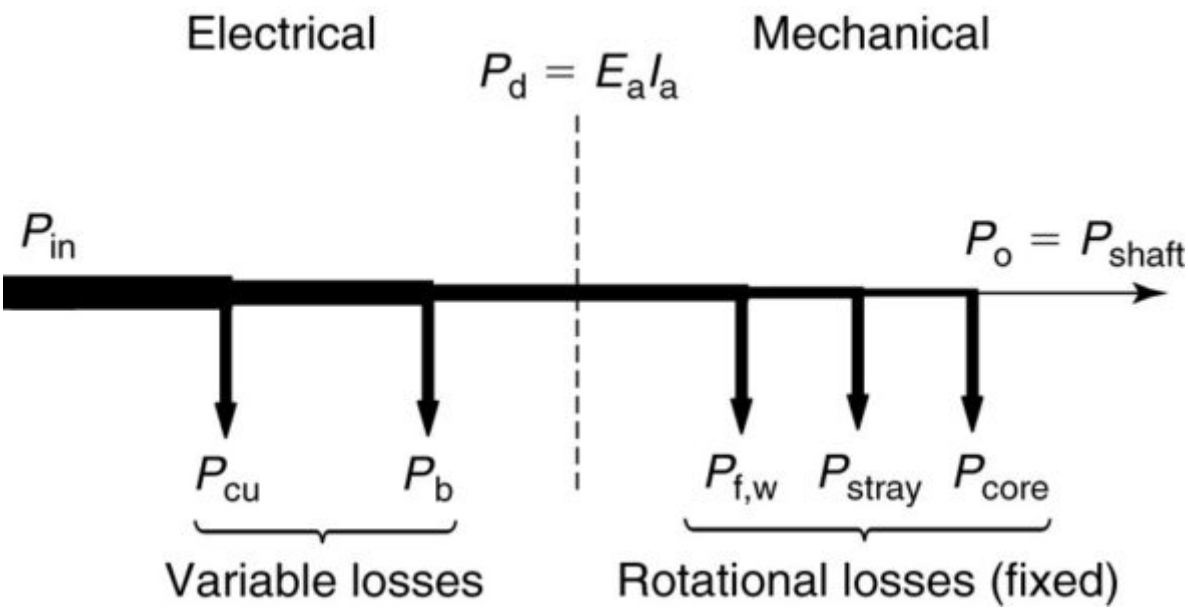


Figure 2.14 power flow in motor mode (https://electricalacademia.com)

2.2.7 Common Industrial Application

Fabrication and Production Unit

Machinery that demands constant power such as vacuums, elevators, and sewing machines Warehouse sorting equipment.

2.3Alternating current (AC) Machine

2.3.1 Principles of Alternating Current Machines

a) MMF of single-phase single pole single phase machine

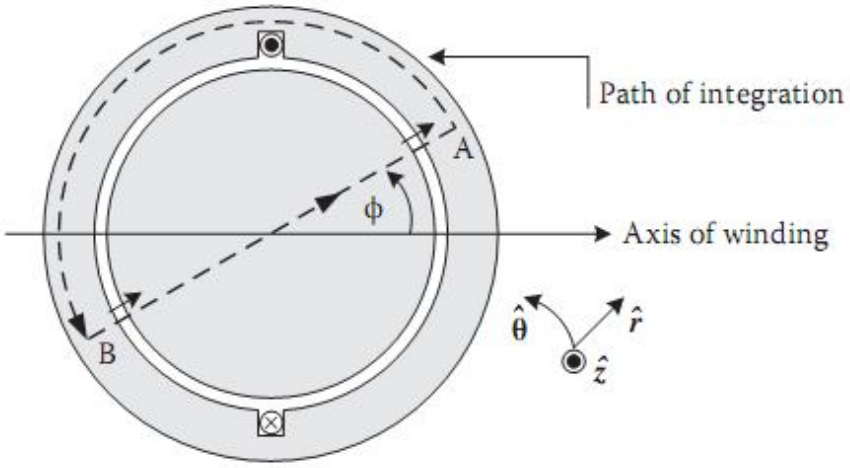


Figure 2.17 single pole single phase machine

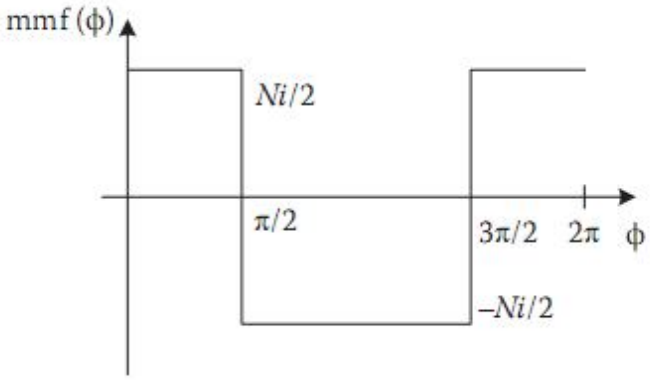


Figure 2.15 MMF wave form

The square wave shown in the image is rich in harmonics, so in order to improve the waveform of the magnetic field modulation mechanism, the coils must be spread out across several slots. When the number of slots is enough to generate a sine wave magnetic field modulation, the modulation can be expressed as:

$$\boldsymbol{mmf(\theta) = \frac{Ni}{2} cos(\theta)} \qquad 2.26$$

N : The total number of turns in the winding

b) Three-phase monopolar machine

Assuming that a three-phase machine is powered by a balanced three-phase source with a sinusoidal waveform, we will determine the total magnetic flux produced in the air gap.

.

$$\boldsymbol{mmf_a(\theta) = \frac{Ni_a}{2} cos(\theta)} \quad 2.27$$

$$\boldsymbol{mmf_b(\theta) = \frac{Ni_b}{2} cos\left(\theta - \frac{2\pi}{3}\right)} \quad 2.28$$

$$\boldsymbol{mmf_c(\theta) = \frac{Ni_c}{2} cos\left(\theta + \frac{2\pi}{3}\right)}$$

2.29

$$\boldsymbol{mmf_T(\theta) = mmf_a(\theta) + mmf_b(\theta) + mmf_c(\theta)}$$

2.30

$$\boldsymbol{mmf_T(\theta) = \frac{NI_m}{2} cos(\omega t) cos(\theta) + \frac{NI_m}{2} cos\left(\omega t - \frac{2\pi}{3}\right) cos\left(\theta - \frac{2\pi}{3}\right) + \frac{NI_m}{2} cos\left(\omega t + \frac{2\pi}{3}\right) cos\left(\theta + \frac{2\pi}{3}\right)} \quad 2.31$$

After simplification, we get:

$$\boldsymbol{mmf_T(\theta) = \frac{3}{2}\frac{NI_m}{2} cos(\omega t + \theta)} \quad 2.32$$

$\boldsymbol{I_m}$ and $\boldsymbol{\omega}$ are the magnitude and the frequency of phase current.

The calculation demonstrates that the magnetic field produced by a three-phase coil powered by a balanced three-phase electricity flows in a sinusoidal pattern through the air gap and rotates at a speed matching the frequency of the power source, which is also referred to as the synchronous speed..

In case of multipole machine (P ploles) , the electrical angle $\boldsymbol{\theta_e}$ is related to the mechanical angle $\boldsymbol{\theta_m}$ by :

$$\boldsymbol{\theta_e = \frac{P}{2}\theta_m} \quad 2.33$$

2.3.2 Types of AC machine

AC machines are divided into two Types:

i. **Synchronous machines:** the rotor can contain permanent magnets, an electromagnet, or simply a ferromagnetic material in case of reluctant machine. In all cases, it rotates at the same speed as the rotating field (synchronous speed).

ii. **Asynchronous or induction machines:** the speed of the rotor is slightlydeferent from that of the rotating field; the relative speed betweenthe rotor and the rotating field is called slip. The rotor of aasynchronous machine is generally short-circuite.

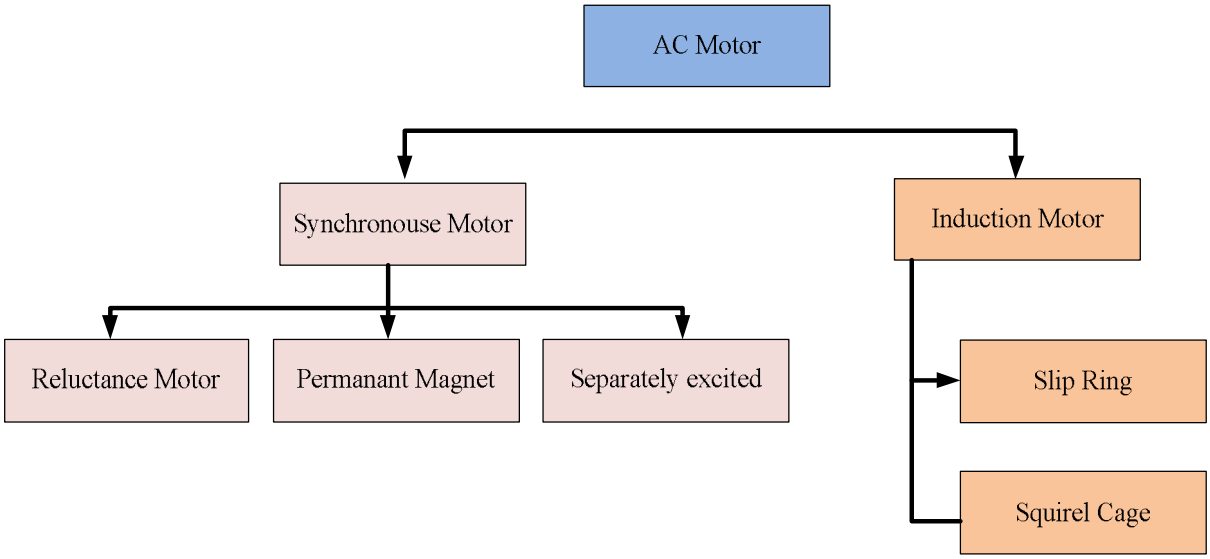


Figure 2.17 type of ac machines

2.4 Synchronous machine

Synchronous machines are electrical machines whose rotation speed of the shaft is equal to the rotation speed of the rotating field. Depending on their mode of operation, they either generate an electric current (“generator” mode) or produce a rotational movement of the rotor (“motor” mode).

Speed at which RMF rotates or Synchronous speed can easily be derived as follows.

$N_s = 120f/P$

The connection between the speed of a synchronous motor, measured in Ns (revolutions per minute), and the frequency of the electricity, in hertz, is evident. The number of poles on the rotor is denoted by P. Essentially, by manipulating the frequency of the electricity, one can effectively regulate the speed of the synchronous motor. This precision control ability is what makes them ideal for applications that require high accuracy..

a) Rotor

At the rotor, we have the inductor (or excitation). Is it a magnet or electromagnet powered by direct current by through brooms. The inductor creates a rotating field.

b) Stator

At the stator, we have the armature (circuit of power). These are three windings offset from.

2.4.1 Type of synchronous machine

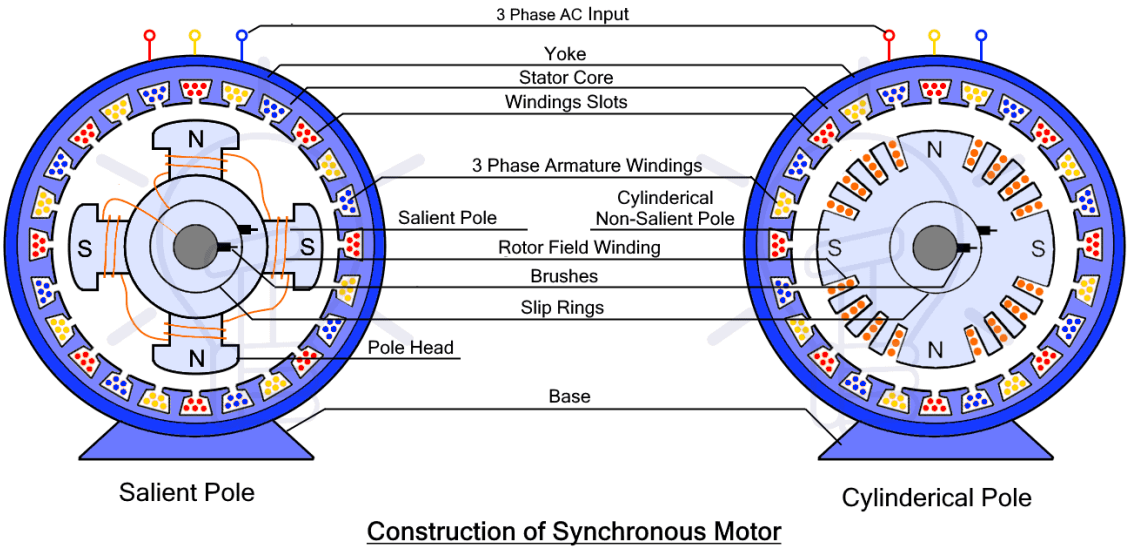


Figure 2.18 Type of Synchronous machine

Direct Current (DC) Excited Synchronous Motor

The DC Excited Synchronous Motor needs a DC power source in order for the rotor to produce a magnetic field. It consists of both stator and rotor windings. The rotor can be powered by either a distinct DC power supply or a DC generator that is linked to the motor's shaft.

Permanent Magnet Motor

A constant magnetic flux is produced by permanent magnets in the steel rotor of a Permanent Magnet Motor. Once the speed is close to synchronous speed, the rotor becomes stationary. An AC supply is used to power the winding in the stator, generating a magnetic field that rotates. Permanent magnet motors and brushless DC motors share

similarities.

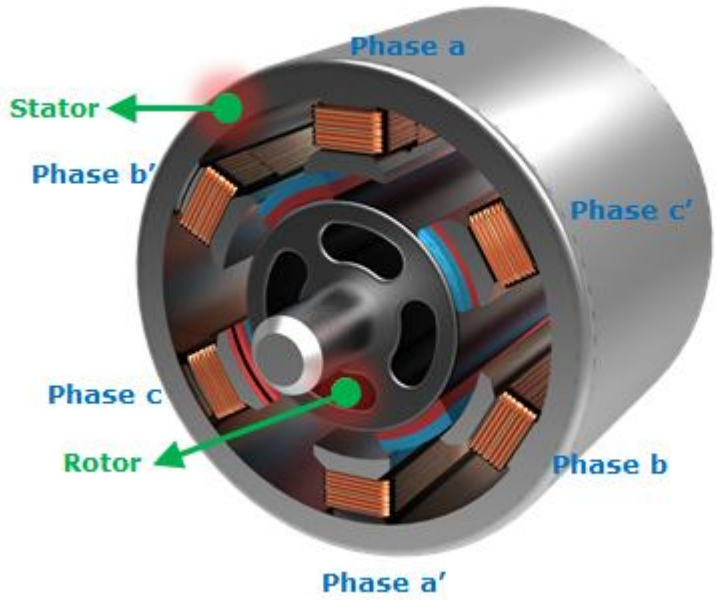


Figure 2.19 Permanent magnet motor structure (https://enmotor.com/)

Reluctance motor

The variable reluctance synchronous machine is structurally a salient pole synchronous machine without excitation. Its stator is the same as that of ordinary AC machines. The electromagnetic torque consists exclusively of the saliency torque.

Figure 2.20 Syn. RM Rotor (https://electengmaterials.com/)

2.4.2 Electrique equivalent circuit of synchronous machine

a) Motor mode

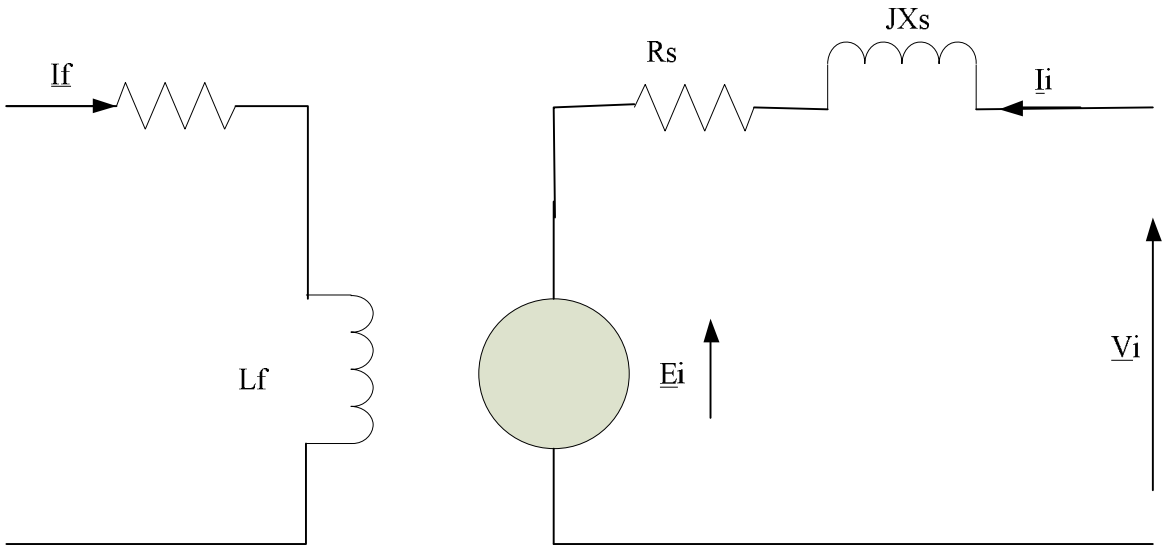


Figure 2.22 Electrique equivalent circuit of synchronous machine

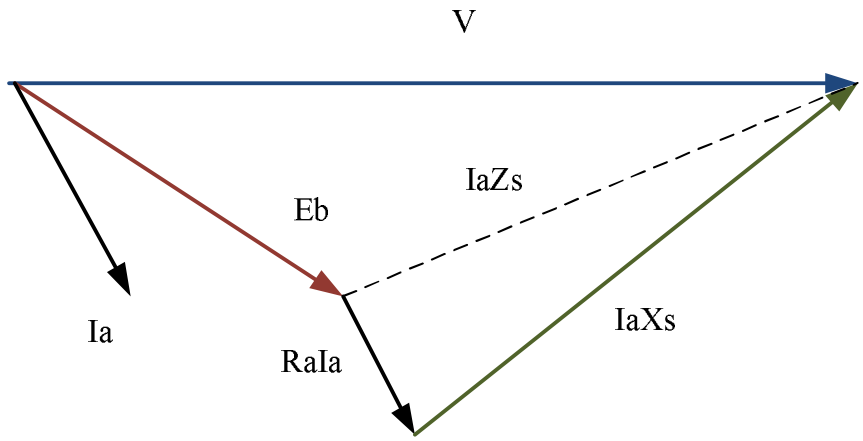


Figure 2.23 space vector syschrounous machinne

$$V = E + I_a Z_s \qquad 2.24$$

$$Z_s = R_a + jX_s \qquad 2.25$$

The per phase input power of a star connected motor is:

$$P = VI_a cos\phi \quad (2.26)$$

Ø: is the phase shift between current and voltage.

From the diagram we have

$$I_a cos\phi = \frac{VE}{Z_s} cos\delta \quad (2.27)$$

δ:internal angle of the machine.

The mechanical power is equal to

$$P_m = P - I_a^2 R_a \quad (2.28)$$

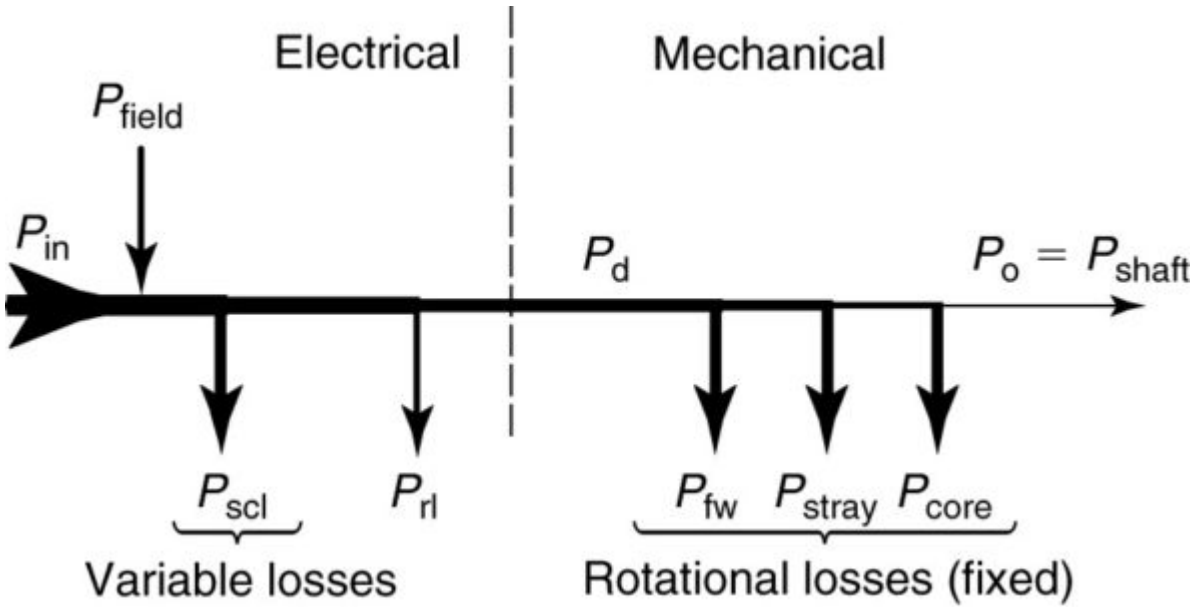
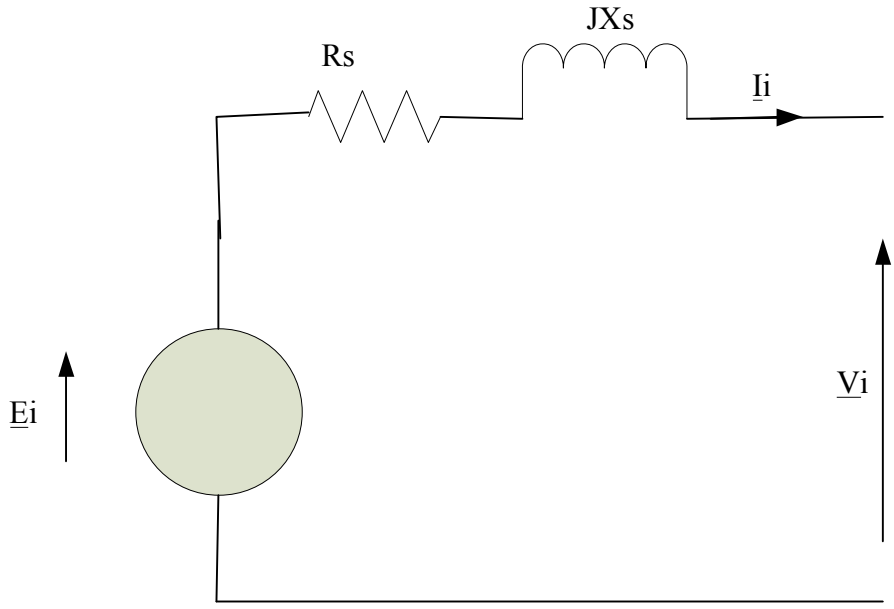


Figure 2.24 Power flow diagram of synchronous motor (https://electricalacademia.com)

b) Generator mode



2.25 Electrical equivalent circuit of synchronous generator

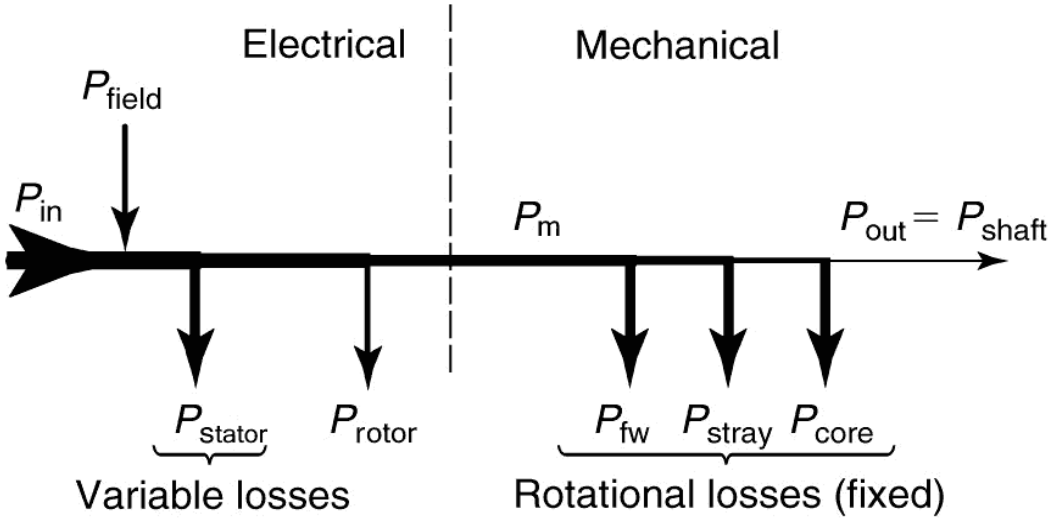


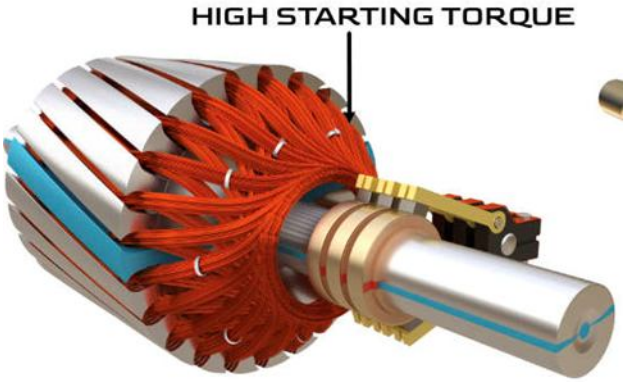
Figure 2.26 Power flow diagram of synchronous generator (https://electricalacademia.com)

2.5 Induction machine

2.5.1 Types of induction machine

a) Slip Ring Motor

The slip ring induction motor, a wound rotor motor type, is a variety of the 3-phase induction motor. It is commonly employed in situations where high torque, such as in cranes and elevators, is needed due to its benefits such as low initial current, high starting torque, and enhanced power factor. The rotor windings of this motor contain a greater number of windings and are linked to external resistance through slip rings to regulate the motor's torque/speed.



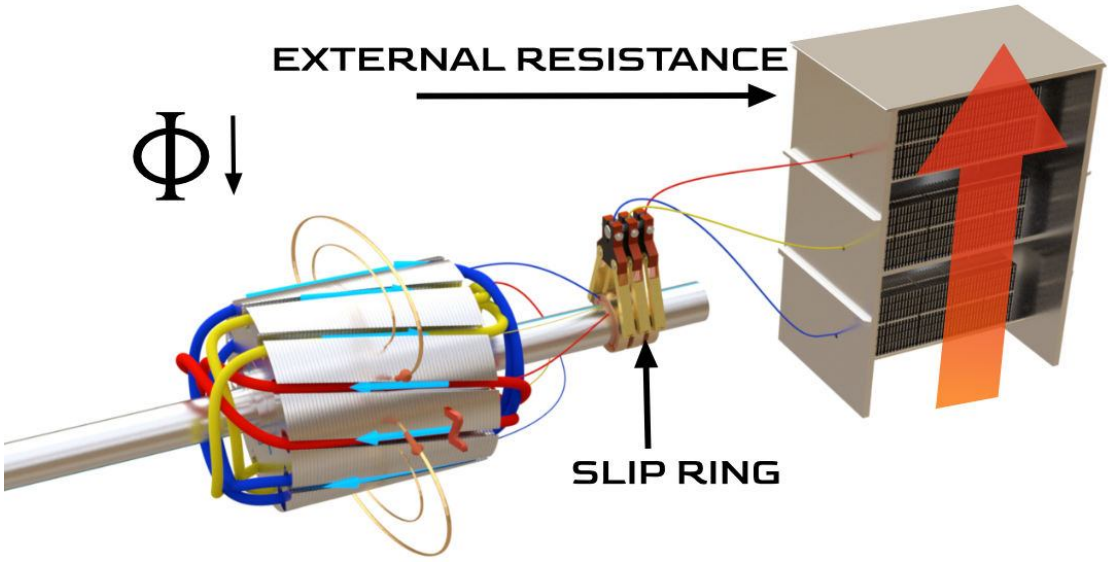


Figure 2.27 Slip Ring Motor (https://www.lesics.com/)

b) Squirrel cage

The squirrel cage is a cage in the shape of a cylinder that surrounds the shaft and has bars connecting its two ends. End rings are added to each end of the squirrel cage to create a short-circuit for the induced current to pass through. Squirrel cages are commonly constructed from copper or aluminum. .

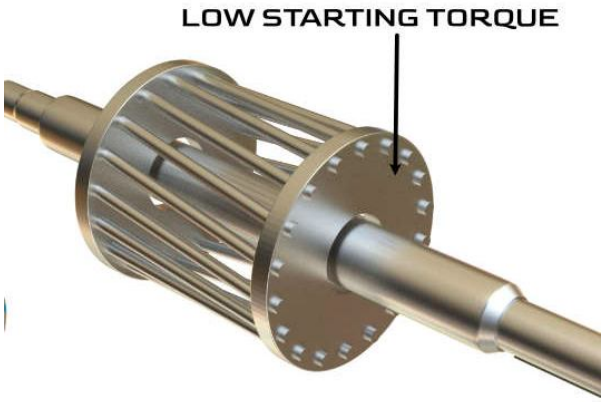


Figure 2.28 Squirrel Cage Rotor (https://www.lesics.com/)

2.5.2 Slip of asynchronous machine

Slip is a quantity which accounts for the difference in the rotational speed of an asynchronous machine in relation to the rotational speed of its stator field. There must be a difference in speed for this type of machine to work because it is the offset between the rotor and the stator field which causes the induced currents to appear at the rotor

$$n_s = \frac{f}{p} tr/s$$

2.29

$$n_s = \frac{60f}{p} rpm \qquad 2.30$$

$$n_s - n = g.n_s \qquad 2.31$$

$$s = \frac{n_s - n}{n_s} \quad 2.32$$

$$s = \frac{\omega_s - \omega}{\omega_s} \quad 2.33$$

$\omega_s$the angular synchronous speed of the stator field in the machine.

$\omega$the angular speed of rotation of the machine

2.5.3 power flow of induction machine

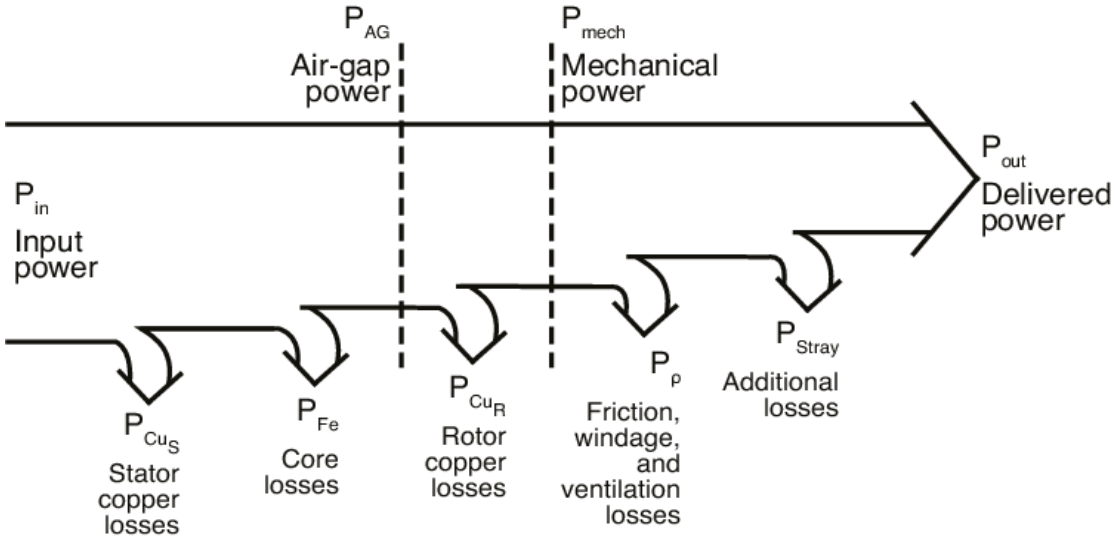


Figure 2.29 power flow of induction machine

P_a : electrical power supplied to the machine.

P_u: mechanical power transmited to the load .

Losses generally are

P_{js}: joule effect losses in the stator winding

P_{fs}: iron losses in the stator

P_{jr}: joule effect losses in the rotor winding

P_{fr}: iron losses in the stator

The diagram below represents the transmission of power through the machine:

2.6 Common industrial application of AC machines

Since AC motors deliver continuous energy changes, they are ideal for demanding applications like those seen in industrial environments. While DC motors are preferred for

hand tools or large high-torque applications like steel mill rolling equipment and paper machines, AC motors are commonly used in equipment such as:

Compressor power drives

Air conditioning compressors

Hydraulic pumps

Irrigation pumps

Conveyor systems

Shredders

Die casting machines

Sheet metal presses

Machine tools

Fans and blowers

Compressor drives and systems

Computers

Conveyer systems

Fans and air conditioners

Hydraulic and irrigation pumps

2.7. Conclusion

Approximately half of the world's electricity is consumed by electric motors, and the demand for accurately controlled motor drives is on the rise. Electric drives offer optimal control capabilities for various processes. Electric motor torque can be precisely managed, and the conversion processes between power electronics and electromechanical systems are highly efficient. Ultimately, a controlled electric motor drive can result in substantial energy savings. In the future, electric drives are expected to play a significant role in powering cars and industrial machinery. Due to the large energy usage, electric drives have a notable impact on the environment. Inefficient or poorly designed drives can unnecessarily harm the environment. The environmental challenges faced today motivate electrical engineers to develop innovative and efficient electric drive systems.

PRACTICAL EXERCISES N° 1

DC MACHINES

Exercise 01:

The rotational frequency of a constant independent excitation motor is 1400 min^{-1} when it absorbs a current of 30 A at a voltage of 115 V. The armature sound resistance is 0.3 Ω. The Joule losses in the inductor are 150 W and the collective losses, assumed to be independent of intensity I, are equal to 220 W. Calculate:

1. The counter-electromotive force E';
2. The power consumptionPa;
3. The useful power Pu;
4. The engine's η performance;
5. The moment of the useful couple Tu.

Exercise 02:

For an independently excited motor, the following information is available:

1. Inductor: resistance r = 150 Ω , nominal supply voltage u = 120 V;
2. Armature: resistance R = 0.5 Ω, nominal supply voltage U = 220 V.

The tests are carried out under the respective nominal voltages of the armature and the inductor.

A load test gave for the armature rotating at the rotational frequency of 1450 rpm^{-1} a current 18 A.

In a no-load test, Iv = 1.2 A and Pv = 320 W (armature alone) were recorded.

A. Calculate for the load test:

1. the electromagnetic power Pem (or useful electrical power Little);
2. losses due to the stator Joule effect p_{js} and rotor effect p_{jr} ;
3. collective losses p_c ;
4. the useful power P_u ;
5. the moment of the useful torque T_u ;
6. The η efficiency of the engine.

B. Calculate for the no-load test:

1. the F.C.E.M. E'_v;

2. The rotation frequency n_v.

Exercise 03:

The measurement of the internal characteristic of an independently excited motor gave the following values for a rotational frequency n = 1500 rpm^{-1}.

i (A)	0	0,3	0,4	0,45	0,5	0,55	0,6	0,7
E (V)	15	150	200	230	245	250	255	265

A no-load test yielded the following measurements: U_v = 22 V, I_v = 1.5 A for i_v= . The resistance of the armature is R = 1 Ω. The sum of iron losses and mechanical losses will be 0,45

Considered constant. The excitation losses are 100 W.

The armature is supplied with a constant voltage U = 220 V and is passed through by a current of I = 10 A. The inductor current i is set to .Calculateundertheseconditions:0,45 A

1. the e.c.m.f' of the engine;
2. the speed of the engine in revolutions per minute;
3. the moment of the electromagnetic torque Tem;
4. 4- Useful power;
5. 5- Engine efficiency.

Exercise 04:

A mine shaft extraction machine is driven by an independently excited DC motor. The motor ratings are: U_n = 600 V; I_n = 1,500 A and n_n = 30 rpm^{-1}.

The resistance armature R = 12 m□ is powered by a converter group providing a variable voltage from zero to the nominal voltage.

The inductor is powered by a voltage of 600 V and a current of 30 A.

Motor losses other than Joule losses are considered constant for the same rotational frequency and equal to pc = 27 kW.

The extraction machine is designed in such a way that, for a given load, the tensile force remains constant regardless of the level reached by the cage containing that load.

1. At start-up, what voltage must be applied to the armature so that the current in the circuit is equal to 1.2 I_n ?
2. Nominal operation is considered during a ramp-up. Calculate

1. the total power absorbed;
2. the total power lost by the Joule effect
3. useful powerperformance.
4. the moment of the useful torque as well as the moment of the electromagnetic torque.

SOLUTION #1

EXERCISE N°01

1- $E' = U - RI$ $\quad E' = 115 - 0{,}3 \times 30$ $\quad$ **$E' = 106$ V**

2- $P_a = UI + p_{Jinducteur}$ $\quad P_a = 115 \times 30 + 150$ $\quad$ **$P_a = 3600$ W**

3- $P_u = E'I - p_c$ $\quad P_u = 106 \times 30 - 220$ $\quad$ **$P_u = 2960$ W**

ou $\quad P_u = UI - p_{Jinduit} - p_c$ $\quad P_u = 115 \times 30 - 0{,}3 \times 30^2 - 220$

4- $\eta = \frac{P_u}{P_a}$ $\quad \eta = \frac{2960}{3600}$ $\quad$ **$\eta \approx 82{,}22$ %**

5- $T_u = \frac{P_u}{2\,\pi\,n}$ $\quad T_u = \frac{2960}{2\,\pi\frac{1400}{60}}$ $\quad$ **$T_u \approx 20{,}19$ Nm**

EXERCISE N°02

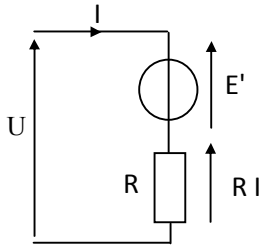
A. Load Testing

1- Electromagnetic power $\quad Pem = E'.\ Ohm$

's law at the terminals of a receiver, the armature: $U = E' + RI$ or $\quad E' = U - RI$ hence:

$Pem = (U - RI).\ I = [(220 - 0.5 \times 18) \times 18)]$ $\quad$ **$Pem \approx 3798$ W**

2- Losses due to the joule effect in the inductor. This circuit is traversed by a current of intensity i which can be calculated. Since we know the supply voltage of this circuit, we prefer to use:



$pJinductor = \frac{u^2}{r}$ $\quad pJinductor = \frac{120^2}{150}$

pJinductor≈ 96 W

Joule losses in armature

$pJinduit = RI2$ $\quad pJinduit = 0.5 \times 182$ $\quad$ **pJinduit≈ 162 W**

3- Collective losses

These losses can be determined during a no-load test because they are practically independent of the current I absorbed by the armature $PV = pc + RIV2$

$pc = PV - RIV2$ $\quad pc = 320 - 0{,}5 \times 1{,}22$ $\quad$ **pc ≈319,3 W**

4-Usefulmotor power

$Pu = Pem - pc$ $\quad Pu = 3798 - 319.3$ $\quad$ **Pu ≈ 3478.7 W**

5-Useful torque moment

$Tu = \frac{P_u}{2\,\pi\,n}$ $Tu = \frac{3478,7}{2\,\pi\,1450/60}$ **Tu ≈ 22.9 Nm**

6- Engine efficiency

$\frac{P_u}{P_a} = \frac{P_u}{UI + p_{Jinducteur}} = \frac{3478,7}{220 \times 18 + 96} \approx$ **85,8 %**

B- No-load test

1. No-load emf

$E' = U - RIV$ $E' = 220 - 0.5 \times 1.2$ **E' = 219.4 V2.** The excitation conditions are the same for both operations. The flow is constant.

$E' = n\,N$ hence $\frac{E'_V}{E'} = \frac{n_V}{n} nV = n\frac{E'_V}{E'}$ $nV = 1450 \times \frac{219,4}{211}$ **nV≈ 1508 rpm**

EXERCISE N°03

1- $E' = U - RI$ $E' = 220 - 1 \times 10$ **E' = 210 V**

2- $E' = n\,N$□□□: $\frac{E'}{E} = \frac{n'}{n}$ $n' = n\frac{E'}{E}$ In the test for an inductor current of 0.45 A, we have E = 230 V for a rotational frequency n = 1500 rpm.

Let : $n' = 1500\frac{210}{230}$ **n' ≈ 1370 rpm**

3-$Tem = \frac{P_{em}}{\Omega}$ $Tem = \frac{E'I}{2\,\pi\,n'}$ $Tem = Tem\frac{210 \times 10}{2 \times \pi \times 1370/60}$ **≈ 14.6 Nm**

4- $Pu = Pem - pc$ avec $pc = PV - pJinduit$

$PV = UV.IV$ $PV = 22 \times 1.5$ $PV = 33$ W

$pJinduit = R.IV2$ $pJinduit = \times 1\ 1,52$ $pJinduit = 2,25$ W

$pc = 33 - 2,25$ $pc = 30.75$ W

d'où $Pu = Pem - pc$ $Pu = 210 \times 10 - 30,75$ **P_u = 2069 W**

5- Do not=UI+ui No=220 × 10+100 Pa=2300 W

$\frac{P_u}{P_a} = \frac{2069}{2300} \approx$ **90 %**

EXERCISE N°04

1- At start-up, the emf E' is zero. $Ud = E'_d + RId$ with $E'_d = 0$ we have: $Ud = RId$

On impose : $Id = 1,2\ In$ $Ud = 0,012 \times 1,2 \times 1500$ **Ud = 21,6 V**

2- Nominal operation One = 600 V In = 1500 A nn = 30 rpm R = 0.012 Ω

u = 600 V i = 30 A pc = 27 kW

1. By =UnIn+ui Pa =600 × 1500+600 × 30=600 × 1530 **Pa=918 kW**

2. pJ = RIn2 + ui pJ = 0,012 × 15002 + 600 × 30 **pJ = 45 kW**

3. Pu = Pa – pertesPu = Pa – (pJ + pc)=Pu = 918.103 – (45.103 + 27.103)**Pu = 846 kW.**

4. $\frac{P_u}{P_a} = \frac{846.10^3}{918.10^3}$ **= 92,16 %**

5. Lo = $\frac{P_u}{2\,\pi\,n}$ Tu = $\frac{846.10^3}{2\pi \times 30/60}$ ~~En ≈ 4488 Nm~~ **Tu = 269 kNm.**

6. Tem = $\frac{P_{em}\ P_u + p_c}{2\,\pi\,n\ 2\,\pi\,n} = = \frac{P_a - p_J}{2\,\pi\,n} = \frac{846.10^3 + 27.10^3}{2\,\pi\ 30/60}$ ~~≈ 4631 nm~~ **Tem = 277.88 kNm**

PRACTICAL EXERCISES N°2

ASYNCHRONOUS MACHINES

Exercise 01:

We study a machine whose equivalent diagram for a phase is shown opposite.

Indications on the nameplate: 400 V ; 45 A ; 24 kW ; 1450 tr/min

A no-load test at the synchrony speed measured the power P = 1300 W, the RMS value of the compound voltages U = 400 V and the RMS intensity of the line currents I = 15 A.

A blocked rotor short-circuit test measured the power P = 1450 W, the RMS value of compound voltages U = 63 V and the RMS of the line currents I = 45 A.

1. During a no-load test at the synchronous speed (short-circuited rotor), the equivalent scheme becomes:

□ 31.2 □ 10.4 □ 18.0 □ 54.0

2. During the no-load test, the reactive power shall be equal to (in kvar):

□ 31.1 □ 10.3 □ 17.9 □ 54.0

3. The equivalent resistance Rf is valued:

□ 110 □ □ 40.7 □ □ 123 □

4. The magnetizing inductance Lm has the following value (in mH):

□ 16.3 □ 49.4 □ 28.4 □ 9.4

5. In the short-circuit test, the reactive power is equal (in kvar):

□4.69 □ 8.38 □,759

6. In the short circuit test, the power for the resistor Rf is equal to:

□ 36 □ □ 97.5 □ □ 32.3 □

7. In the short-circuit test, the current in the resistance R is equal to:

□ 38.0 A □ 45.0 A □ 42.6 A

8. The equivalent resistance R is valued:

□ 0.781 □ □ 0.260 □ □ 0.233 □

9. In the short-circuit test, the reactive power for the inductance Lm is equal to (in var):

□,775 □,256 □,445 □ 1344

10. The equivalent inductance L is set to:

□ 2.32 mH □ 7.77 mH □ 2.59 mH

Exercise 02: Continuous and Load Testing

Data: 3~ 400V / 50Hz network. MAS: iron stator losses: 200W.

Continuous test: voltage between 2 phases = 4.50V; Line current = 2.30A

Load test: RMS line current = 6.30A; Cosφ 0.820 inductive; 1405 rpm.

1° Calculate the values of:

- The active power consumed,
- Joule losses at the stator,
- The electromagnetic power transmitted,
- The moment of the electromagnetic torque.

2° Justify the number of pairs of poles. From this, we can derive the value of the slippage.

3° Calculate the value of the rotor joule losses.

4° Is it possible to know the value of the mechanical useful power? What for?

Exercise 03: This is a three-phase asynchronous motor with a cage rotor whose stator windings are star-coupled.

Engine Ratings

- Nominal voltage between phases: UN = 585 V
- Nominal stator frequency: fN = 88 Hz
- Stator current rating: IN = 35.4 A
- Nominal Power Factor: $\varphi_{cos\,N}$ = 0.732
- Nominal synchronization frequency: Ns = 2640 rpm-1
- Nominal rotor rotation frequency: NN = 2610 rpm-1

In the following, the following is neglected:

- stator leakage resistors and inductors;

- losses in iron;

- Mechanical losses.

A.1 - Study of the nominal operation of the engine

A.1.1- Determine the number p of pairs of poles of the motor given that the nominal stator frequency is fN = 88 Hz.

For the nominal point of operation:

A.1.2 - Calculate the slippage gN.

A.1.3 - Calculate the electrical power PN absorbed by the motor and specify the value of the electromagnetic power PTrN transmitted to the rotor.

A.1.4 - Calculate the NC electromagnetic torque

A.1.5 - Express the Joule losses at the pJr rotor as a function of PTr. Calculate pJrN.

A.1.6 - Calculate the useful power PUN developed by the engine.

A.2 - Simplified expression of the electromagnetic torque moment

For each phase of the engine, the simplified single-phase equivalent model shown in Figure 2 is adopted.

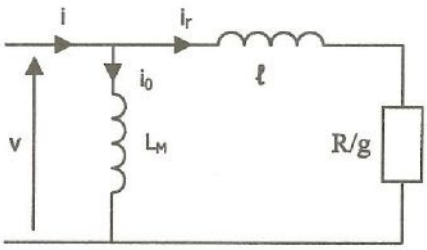


figure 2

-R/g is the resistor modeling the active power transfer to the rotor

LM is the magnetizing inductor

L is the total leakage inductance as seen from the stator

g is the slippage

v is a simple voltage of the RMS supply network

i is the magnitude of the line current

On donne LM=26,6mH ℓ=2,38mH R=0,147 Ω

A.2.1 - Calculate, under nominal power supply (V = 338 V and f = 88 Hz), the rms value Io of the current i0.

A.2.2 - From the equivalent single-phase model in Figure 2, express the RMS value Ir of the current ir as a function of V, lω, R and g.

A.2.3 - Show that for the low values of the slip (g « 1), the previous relation becomes: $I_r = \frac{V}{R} \cdot g$

A.2.4 - Express the power transmitted to the PTr rotor as a function of Ir and $\frac{R}{g}$

A.2.5 - Deduct then that, for the low values of the slip, the moment of the electromagnetic torque is written: C = K.g

Express K as a function of the rms voltage V, the resistance R, and the synchronous velocity Ω_S expressed in rad.s-1. Calculate the numerical value of K under nominal power supply (V = 338 V and f = 88 Hz).

A.3 - Tensile operation

We consider the case where the motor develops an electromagnetic torque C = 170 N.m when operating under nominal power supply (V = 338 V and f = 88 Hz).

The slippage will be assumed to be low and the following results will be assumed:

I0=23A $I_r = \frac{V}{R} \cdot g$ C=8433.g

A.3.1 - Determine the slip value corresponding to the operation under study.

A.3.2 - Deduct the value of the rotation frequency of the rotor N in rpm-1.

A.3.3 - Determine the value of Ir.

A.3.4 - Let φ_r be the phase shift of Ir with respect to the simple voltage v taken as a phase reference. Referring to the equivalent single-phase model in Figure 2, express tanφ as a function of lω and $\frac{R}{g}$. From this we can deduce that $\varphi_r \approx 10.2°$.

A.3.5 - Since the simple voltage v is always chosen as the reference for the phases, draw the Fresnel vectors $\vec{I}_0$ associated $\vec{I}_r$ with the currents io and ir (the scale 5 A / cm will be adopted).

A.3.6 - Derive the construction of the Fresnel vector $\vec{I}$ associated with i.

A.3.7 - Determine graphically the rms value I and the phase shift φ of i with respect to v.

A.3.8 - Calculate the electrical power P received by the motor in this operating case

SOLUTION #2

Exercice N°01

Question	1	2	3	4	5	6	7	8	9	10
Answer	10,4	10.3	123 Ω	49,4	4,69	32,3 W	42,6	0,260 Ω	256	2.59 mH

Exercice N°02

1.1. $P_a = \sqrt{3}UI\cos\varphi = \sqrt{3}\times 400\times 6,3\times 0,82 = 3579W$

1.2. The coupling is not known, but the measurement given the resistance between two phases.

And whatever the coupling with $P_{JS} = \frac{3}{2}R_B I^2 \; R_B = \frac{4,5}{2,3} = 1,96\ \Omega$

Therefore $P_{JS} = \frac{3}{2}\left(\frac{4,5}{2,3}\right)6,3^2 = 116\ W$

1.3. $P_{em} = P_a - P_{JS} - P_{fs} = 3579 - 116 - 200 = 3263\ W$

1.4. $T_{em} = \frac{P_{em}}{\Omega_S} = \frac{3263}{2\pi\times\frac{1500}{60}} = 20,8\ Nm$

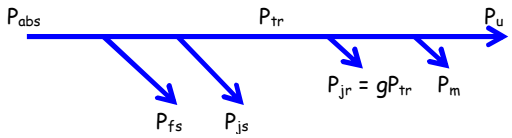
2°) If we consider that nS is equal to 1500 rpm (in 50 Hz nS takes the values 3000, 1500, 1000, 750...) then $p = \frac{f}{\underbrace{n_S}_{tr/s}} = \frac{50}{1500/60} = 2$ 2 pairs of poles

$g = \frac{n_S - n}{n_S} = \frac{1500 - 1405}{1500} = 6,3\%$

3°) $P_{jr} = gP_{tr} = 0,063\times 3263 = 205,6W$

4°) To know the mechanical power, we must neglect the mechanical losses.

Then $P_u = P_a - P_{JS} - P_{fs} - P_{jr} = 3579 - 116 - 200 - 205,6 = 3057,4\ W$



Exercise N°03

A.1. Study of the nominal operation of the engine

A.1.1. from $f = p \cdot \underbrace{n}_{tr/s}$ we pull $f_N = p \cdot \frac{n_S}{60}$ with the notations of the problem

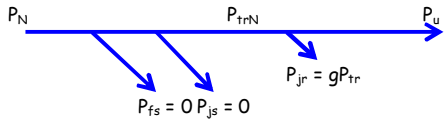
$\Rightarrow p = 60 \cdot \frac{f_n}{n_S} = 60 \cdot \frac{88}{2640} = 2$ The number of pole pairs is 2.

A.1.2. By definition $g = \frac{N_S - N}{N_S} = \frac{2640 - 2610}{2640} = 0,01136$ The slippage is therefore

g=1.14%

A.1.3. $P_n = \sqrt{3} \cdot U_N I_N \cos\varphi_N = \sqrt{3} \cdot 585 \times 35,4 \times 0,732 = 26,25\ kW$

Since the losses in iron and the joule stator losses are negligible, we will have



$P_N = P_{trn} = 26,3\ kW$

A.1.4. From $P_{trn} = C_n \cdot \Omega_S = C_n \cdot \frac{2\pi}{60} \cdot n_S$ then on $C_n = \frac{P_{trN}}{\frac{2\pi}{60} \cdot N_S} = \frac{26,3 \cdot 10^3}{\frac{2\pi}{60} \cdot 2640} = 95,13\ Nm$

The nominal torque is therefore $C_n = 95,13\ Nm$

A.1.5. It is known that the joule stator losses are

$p_{jrN} = g_n P_{trN} = 0,01136 \times 26,3 \cdot 10^3 = 298\ W$

The rotor joule losses are therefore $p_{jrN} = 298\ W$

A.1.6. Since mechanical losses are neglected, the useful power developed by the engine is given by $P_{uN} = P_{trN} - p_{jrN} = 26300 - 298 = 26,0\ kW$.

The useful power developed by the engine is therefore $P_{uN} = 26,0\ kW$

A.2. Simplified Expression of the Electromagnetic Torque Moment

A.2.1. According to the diagram, we have $I_0 = \frac{V}{L_M \omega} = \frac{338}{26,6 \cdot 10^{-3} \times 2\pi \times 88} = 23,0\ A$.

The magnetizing current is therefore $I_0 = 23,0\ A$

A.2.2. According to the model, we have $\underline{V} = \frac{R}{g}\underline{I_r} + j\ell\omega\underline{I_r}$ $I_r = \frac{V}{\sqrt{\left(\frac{R}{g}\right)^2 + (\ell\omega)^2}}$

A.2.3. As the slippage decreases, it can be expected that $\frac{R}{g}$ it will become larger and larger in front $\ell\omega$ of . Indeed $\left(\frac{R}{g} = \frac{0,147}{0,01} = 14,7 >> \ell\omega = 2,38 \cdot 10^{-3} \times 2\pi \times 88 = 1,31\right)$

So $I_r = \frac{V}{\sqrt{\left(\frac{R}{g}\right)^2}} = V\frac{g}{R}$ so $I_r = V\frac{g}{R}$

A.2.4. We have $\boxed{P_{tr} = 3\frac{R}{g} I_r^2}$ therefore $P_{tr} = 3\frac{R}{g} g^2 \frac{V^2}{R^2}$

A.2.5. Substituting $I_r = V\frac{g}{R}$ in $P_{tr} = 3\frac{R}{g} I_r^2$, we get $P_{tr} = 3\frac{\cancel{R}}{\cancel{g}} g^{\cancel{2}} \frac{V^2}{R^{\cancel{2}}}$

Let $\boxed{P_{tr} = 3g\frac{V^2}{R}}$ and as $P_{tr} = C \cdot \Omega_S$ therefore $C = \frac{P_{tr}}{\Omega_S} = 3g\frac{V^2}{R \cdot \Omega_S}$ of the form

$C = k \cdot g$ with $\boxed{k = 3\frac{V^2}{R \cdot \Omega_S} = 8433\ Nm}$

A.3. Tensile operation

A.3.1. With $C = 8433 \cdot g$ and C=170 Nm then $g = \frac{C}{k} = \frac{170}{8433} = 2,02 \cdot 10^{-2}$ let

$\boxed{g = 2,02 \cdot 10^{-2} = 2\%}$

A.3.2. So $N = N_S(1-g) = 2640 \times (1 - 0,0202)$ so $\boxed{N = 2587 \text{ tr/min}}$

A.3.3. So $I_r = V\frac{g}{R} = 338 \times \frac{0,0202}{0,147}$ $\boxed{I_r = 46,4\ A}$

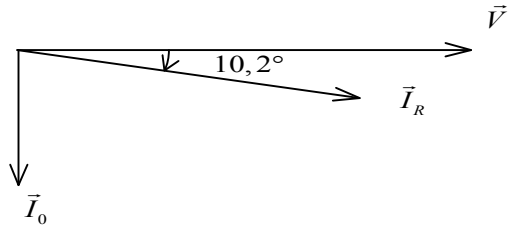
A.3.4. We can see that $\boxed{\tan\varphi_r = \frac{\ell\omega}{\frac{R}{g}} = \frac{g\ell\omega}{R}} = \frac{0,0202 \times 2,38 \cdot 10^{-3} \times 2\pi \times 88}{0,147} = 0,180$ either

$\boxed{\varphi_r = \arctan 0,180 = 10,2°}$

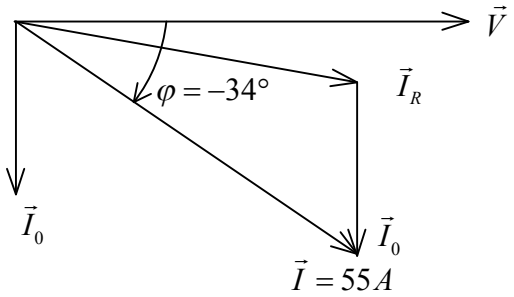
A.3.5. Fresnel Construction

$\vec{I}_0$ is 23 A and 90° behind on $\vec{V}$ / $\vec{I}_r$ is 46.4 and 10.2° behind with $\vec{V}$

the 5A/cm scale we have



A.3.6. Hence $i = i_0 + i_r$ $\vec{I} = \vec{I}_0 + \vec{I}_r$ the construction below



A.3.7 On mesure sur la construction $\boxed{I = 55\ A \text{ et } \varphi = 34,5°}$

A.3.8. Avec $P = 3VI\cos\varphi = 3\times 338\times 55\times\cos 34,5 = 45961$ on trouve donc P=46000 W (bien loin de P_{un})

PRACTICAL EXERCISES N°3

SYNCHRONOUS MACHINES

Exercise 01: ***Reactive Power Compensation Using a Synchronous Machine***

In order to operate, many loads require the current flowing through them to lag behind the voltage across them, which is why they are said to consume reactive power. The following study focuses on a synchronous motor, used to compensate for the reactive energy consumed by the lighting of an office building.

II.1. DISTURBANCE STUDY

In a company, the offices are located in a building, which is 850 m away from the technical room containing the general power transformer. The connection is made in 230 V/400 V three-phase, via 3 cables with a cross-section of 35 mm² for the phases and a cross-section of 10 mm² for the neutral. The lighting is provided by fluorescent lamps which correspond to a balanced three-phase load of 50 kW with an inductive power factor k = 0.76. Remember that currents will be considered sinusoidal.

Online Loss Study

II.1.1. Intensities

It is assumed that the voltage drop in the cables has been taken into account and that the compound voltage for the office building is indeed 400 V.

1. Determine the RMS current I in each phase conductor.
2. What is the RMS current in the neutral conductor?

II.1.2. Cable Strength

The cables are cylindrical conductors made of aluminum, with a resistivity under normal operating conditions: ρ = 2.7 x 10-8Ω.m. Calculate the total strength of each cable.

Subsequently, we will take the following values, which take into account the resistances of the connections: phase: 1.0 Ω, neutral: 2.5 Ω.

II.1.3. Joule losses

Determine the total losses for all cables that supply power to the building for this power factor value.

II.2. CORRECTION OF PERTURBATIONS

To improve the power factor of the building, it was decided to use a synchronous motor placed in the building. This motor will operate as an automatic synchronous compensator, so that at any given moment the power factor of the vessel will be equal to 1.

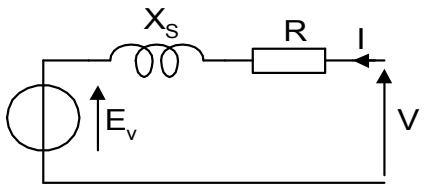


figure 4

The electrical characteristics of the motor are as follows: 4-pole, star coupling for operation on the 230 V/400 V three-phase grid, Pnominal = 50 kW. For the study, it will be modelled according to the synchronous reactance method (linear model known as the "Behn-Eschenburg" model), in accordance with figure 4 corresponding to a phase of the machine whose windings are assumed to be star-coupled; in this hypothesis we can write Ev = βIe with β = 0.83 x 103 V.A-1.

II.2.1. Paramètres du modèle

To determine the parameters of the synchronous machine model, the following tests were performed in alternator operation.

II.2.1.1. No-load characteristic

The r.m.s. value of the no-load voltage Ev between phase and neutral as a function of the current in the excitation circuit Ie was recorded.

Ie(A)	0	0,05	0,10	0,15	0,20	0,25
Ev(V)	0	41,5	83	124	166	207

From this table of measurements, justify the value of the coefficient β = 0.83 x 103 V.A-1.

II.2.1.2 Short-circuit tests

A symmetrical short circuit is performed on the three phases, the following two tests are obtained:

Ie = 0 A, Icc = 0 A, Ie = 0,38 A, Icc = 70 A.

A measurement of the resistance between phase and neutral gave R = 0.1 Ω.

From these results, justify the value of XS = 4.5 Ω.

For the rest, we will neglect the resistance R in front of the synchronous reactance XS, as well as the mechanical losses and the losses in the iron.

II.2.2. Synchronous compensator

The machine works as a synchronous compensator: it does not provide any mechanical power but it operates at no time by absorbing a current in advance of $\pi/2$ rad on the corresponding single voltage. It therefore provides a reactive power that compensates for that consumed by the lamps.

II.2.2.1. Using the assumptions in **II.1**, determine the total reactive power consumed by the lamps. From this we can derive the RMS value of the current that must flow through the machine to provide the same reactive power.

II.2.2.2. Give the relationship between $\underline{V}$, $\underline{Ev}$ and $\underline{I}$, and then plot these quantities on a Fresnel diagram.

II.2.2.3. Determine the value of the excitation current corresponding to this operation.

II.2.3. Improving the power factor using the synchronous machine

II.2.3.1. The compound supply voltage at 400 V is maintained. As the machine operates as a synchronous compensator, in parallel with the load (the whole building), it provides, as before, a reactive power equal to that consumed by the lamps. Determine the new current value in the phase cables.

II.2.3.1. Determine the new value of online losses.

II.2.3.1. Propose one or more solutions to further reduce these online losses.

SOLUTION #3

Exercice N°01

II.1.1. Current: A balanced load consuming P=50 kW with a power factor k powered in three-phase absorbs a current $I = \frac{P}{\sqrt{3} \times U \times k} = \frac{50 \cdot 10^3}{\sqrt{3} \times 400 \times 0,76} = 95A$

As the system is three-phase balanced and the sinusoidal currents the current IN = 0

II.1.2. Cable Strength

The strength of the cables is $R = \rho \frac{\ell}{S}$

So the cables are the line conductors $R = 2,7 \cdot 10^{-8} \frac{850}{35 \cdot 10^{-6}} = 0,656\ \Omega$

For Neutral $R = 2,7 \cdot 10^{-8} \frac{850}{10 \cdot 10^{-6}} = 2,3\ \Omega$

II.1.3. Joule Losses

$P_J = 3R_{ph}I^2 = 27,1kW$

In neutral: no current so no losses

Note: very high losses compared to the 50kW payload, which we will do by lowering the power factor

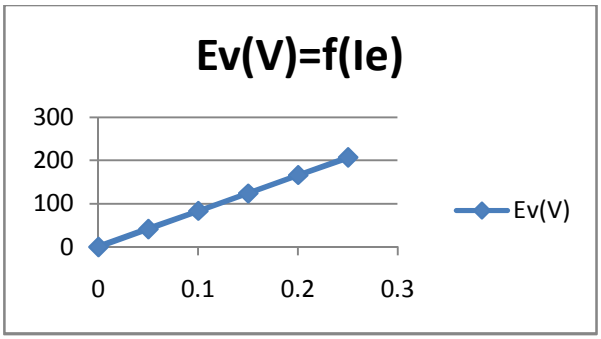
II.2. Correction des perturbations

II.2.1. Model parameters

II.2.1.a. Coefficient β

Since the model chosen is the Behn-Eschenburg model by definition of this model, its no-load characteristic coincides with the tangent at the origin of the no-load characteristic of the real machine. When this characteristic is described by an array of values, it is agreed that the tangent at the origin is the line passing through the origin of the axes and the first point of the array of values (after 0) So here the tangent passes through the points of coordinates [0;0] and [0.05A; 41.5V]. Then

$\beta = \frac{41,5}{0,05} = 0,83 \cdot 10^3\ VA^{-1}$



II.2.1.b. Valeur de X_S

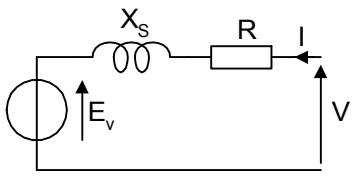


figure 4

If we make a symmetrical short circuit between the 3 phases, we can use the diagram reduced to one phase to study the short circuit. By calling EVCC the fem corresponding to Ie = 0.38 A, we then know that the current I is ICC = 70A .

Since then V=0 we get, according to the diagram in Figure 4: $Z_S = \frac{E_{VCC}}{I_{CC}} = \sqrt{X_S^2 + R_S^2}$

With $E_{V(i_e=0,38)} = E_{VCC} = \beta \cdot 0,38$

$$\Rightarrow Z_S = \frac{E_{VCC}}{I_{CC}} = \frac{\beta \cdot 0,38}{I_{CC}} = \frac{830 \cdot 0,38}{70} = \sqrt{X_S^2 + R^2} = 4,51$$

And as $Z_S = \sqrt{X_S^2 + R^2}$

So with $R = 0,1\ \Omega \Rightarrow X_S = \sqrt{Z_S^2 - R^2} = \sqrt{4,51^2 - 0,1^2} = 4,5\Omega$

This justifies the value of the statement: $X_S = 4,5\Omega$

II.2.2. If the machine seen in receiver convention absorbs a current in advance of π/2 over the simple voltage, it is therefore seen as a bank of star capacitors by the network.

The machine does not consume any active power, so it does not consume any active power $P = \sqrt{3}UI \cos \underbrace{\varphi}_{\pi/2} = 3VI \cos\varphi = 0$

It provides reactive power $Q = \sqrt{3}UI \sin\varphi = 3VI \sin\varphi = 3VI$

II.2.2.1. If the lamps absorb 50 kW with fP = 0.76 inductive, then they consume the reactive power:

$$Q_{lampes} = P \cdot \tan(\arccos 0,76) = 42,8\,kVAr$$

The synchronous machine must provide this power so $3VI = 42,8 \cdot 10^3 \Rightarrow I = 61,7\ A$

II.2.2.2. According to Figure 4, neglecting R, he comes by writing the law of meshes

$\underline{V} = \underline{E_V} + jX_S \cdot \underline{I}$ either $\underline{V} = \underline{E_V} + j4,5 \cdot \underline{I}$

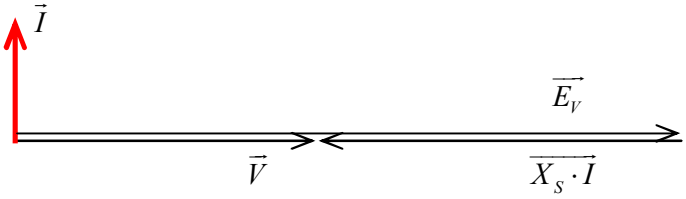
Fresnel diagram:

$\vec{I}$ ahead $\vec{V}$ of $.\frac{\pi}{2}$

$\overrightarrow{X_S \cdot I}$ will be ahead of $\frac{\pi}{2}$ on $\vec{I}$ so we will have $X_S \cdot I = 4,5 \times 61,7 = 278V$

And V=230 V and lagging behind $\frac{\pi}{2}$ the current

Therefore $\underline{E_V} = \underline{V} - jX_S \cdot \underline{I}$



II.2.2.3. Value of Ie

According to the Fresnel diagram, we have , in modulus, $V = E_V - X_S \cdot I$ so with

$V = 230V$ and $X_S \cdot I = 4,5 \times 61,7 = 278V$ then

$I = 61,7\ A \Rightarrow E_V = 508\ V \Rightarrow I_e = 0,612\ A$

II.2.3. Power Factor Improvement with the Synchronous Machine

II.2.3.a. Current in phase cables.

The whole as seen by the grid (lamps + MS) will consume 50kW, without using any reactant since the reactive power supplied by the synchronous machine is exactly equal to the reactive power absorbed by the lamps and the MS running at no load and having no losses does not involve any active power.

The network therefore "sees" a receiver (lamps consumes P and Q and MS consumes no P and compensates Q) with a power factor = 1

The new line current I' is therefore given by:

$$P = \sqrt{3} \cdot U \cdot I' \cdot \underbrace{\cos \underbrace{\varphi'}_{0}}_{1} = 50000 \Rightarrow I' = \frac{50000}{\sqrt{3} \cdot 400} = 72,2 \ A$$

II.2.3.b. New Online Losses

The set is always balanced, so the current in the neutral is always zero. Then

$p'_J = 3 \cdot R_{phase} \cdot I'^2 \Rightarrow p'_J = 15,6 \ kW$

It is noticeable that these losses are lower than those found previously.

II.2.3.c. Reduction of line losses

Leaving aside the economic aspect, since $p_J = 3 \cdot \frac{\rho \cdot \ell}{S} I^2$ we can decrease pJ by decreasing ρ (Cu instead of Al), by decreasing l or, above all, by increasing S.

Chapter 3

DC Machine Modeling

3.1 Introduction

DC machines are the important machine in the most control systems such as electrical systems in homes, vehicles, trains, and process control. It is well known that the mathematical model is very crucial for a control system design. For a DC motor, there are many models to represent the machine behavior with a good accuracy. However, the parameters of the model are also important because the mathematical model cannot provide a correct behavior without correct parameters in the model.

3.2 Dynamic modeling of separately excited dc machine.

The equivalent electrical circuit separately excited dc machine is demonstrated in figure 3.1.

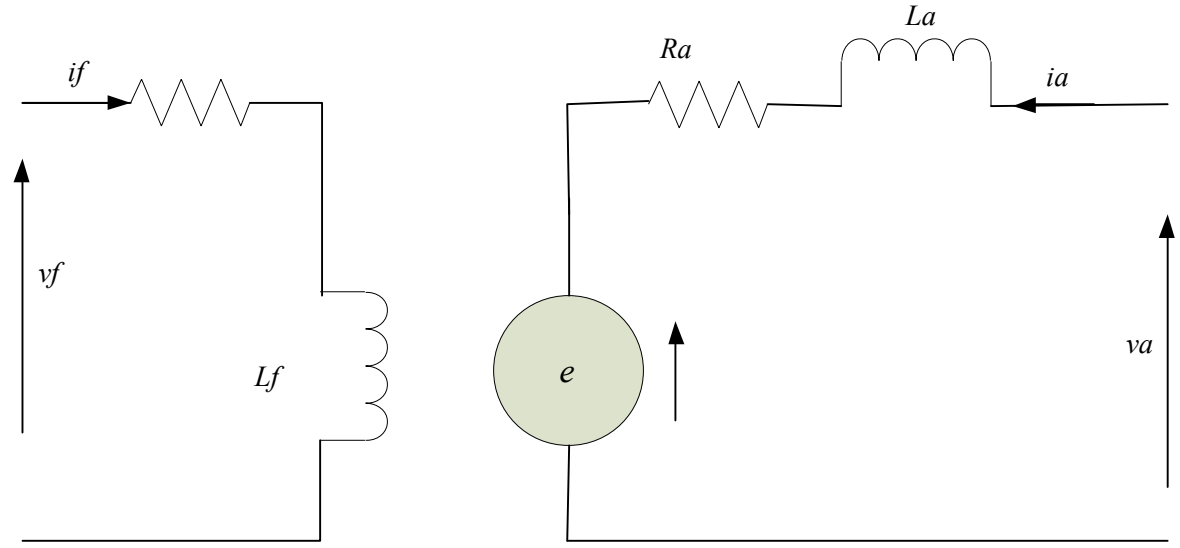


Figure 3.1 equivalent circuit of separately excited dc machine.

3.2.1 Field circuit equation.

$$v_f = R_f i_f + L_f \frac{di_f}{dt} \quad 3.1$$

R_f: Resistance of the field windings.

L_f: Self-inductances of the field windings.

i_f : Currents flowing through the field windings.

L_{af} : Mutual inductance between the field and armature coils.

3.2.2 Armature circuit equation.

$$v_a = e_a + R_a i_a + L_a \frac{di_a}{dt} \quad (3.2)$$

R_a: Resistance of armature windings.

L_a: Self-inductances of armature windings.

i_a : Currents flowing through the field and armature windings.

3.2.3 Mechanical equation

The electromechanical behavior of the machine can be described by the following differential equation

$$T_e - T_L = J \frac{d\omega_m}{dt} + B\omega_m \quad (3.3)$$

ω_m : Mechanical speed of the shaft.

B and J are the damping and the moment of inertia, respectively.

3.2.4 Time domain model

As shown in the previous chapter the back emf and electrical torque can be expressed as:

$$e_a = K_f i_f \omega_m \quad (3.4)$$

$$T_e = K_f i_f i_a \quad (3.5)$$

Combining the above equation and after simplification, we get the set of the following three differential equations that describe the dynamic model of a separately excited machine:

$$\begin{cases} \frac{di_a}{dt} = \frac{1}{L_a}\left(v_a - R_a i_a - K_f i_f \omega_m\right) \\ \frac{di_f}{dt} = \frac{1}{L_f}\left(v_f - R_f i_f\right) \\ \frac{d\omega_m}{dt} = \frac{1}{J}\left(K_f i_f i_a - T_L - B\omega_m\right) \end{cases} \quad (3.6)$$

3.2.5Frequency domain model

By applying Laplace transformation on system 3.6 we find the dynamic model in frequency domain of DC machine.

$$I_a(s) = \frac{1}{R_a + sL_a}\left(V_a(s) - L_{af}\omega_m I_f(s)\right) \quad (3.7)$$

$$I_f(s) = \frac{1}{R_f + sL_f} V_f(s) \quad 3.8$$

$$\omega_m(s) = \frac{1}{B + sJ} (L_{af} I_a(s) I_f(s) - T_L) \quad 3.9$$

From the above equations, the block diagram of the dynamic model of dc machine can be represented as figure 3.2.

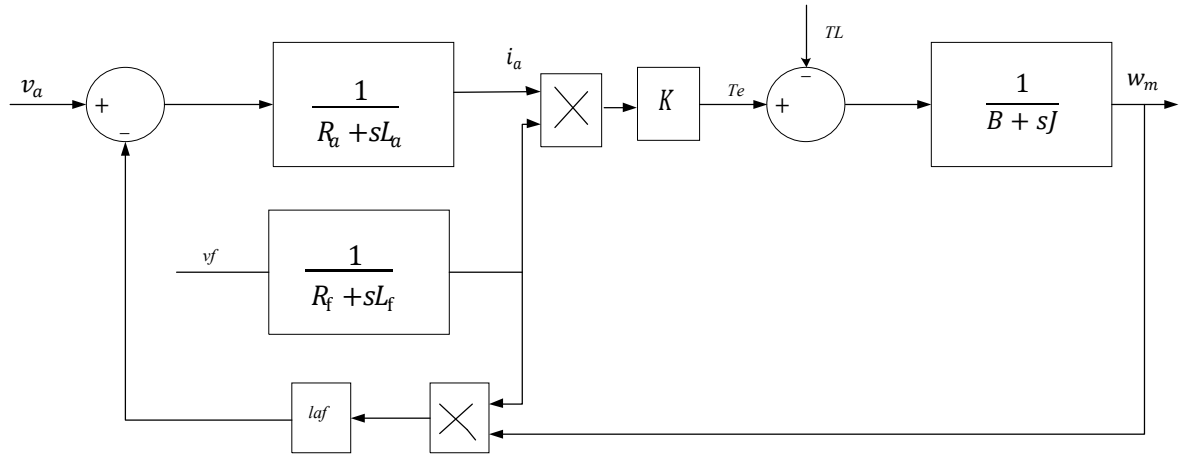


Figure 3.2. Block diagram of dynamic model of separately excited DC machine.

3.2.6 State space model

Selecting the state variables as:

$$x_1(t) = i_a(t) \quad 3.10$$

$$x_2(t) = i_f(t) \quad 3.11$$

The following equations can be derived from the system 3.6

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_f \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b \omega_m}{L_a} \\ 0 & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_a \\ i_f \end{bmatrix} + \begin{bmatrix} \frac{v_a}{L_a} \\ \frac{v_f}{L_f} \end{bmatrix} \quad 3.13$$

$$\frac{d\omega_m}{dt} = \frac{1}{J}(K_f i_f i_a - T_L - B\omega_m) \quad 3.14$$

3.3 Dynamic modeling of permanent magnet motor.

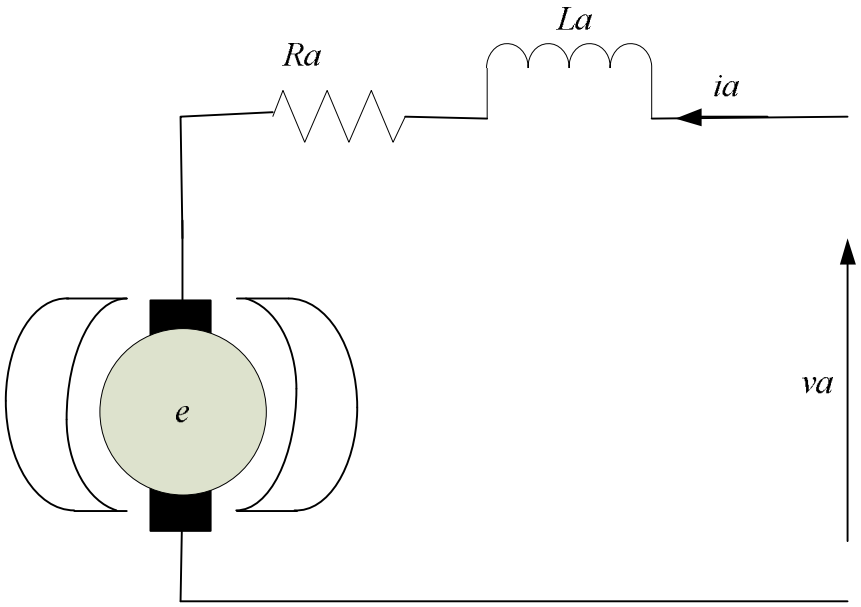


Figure 3.3Equivalent circuit of permanent magnet motor

3.3.1 Time domain modeling

In this type of machinea constant inductor flux it is created by permanent magnets. So the above equations of separatly excited machine can be reproduced as:

$$\frac{di_a}{dt} = \frac{1}{L}(v_a - R_a - K\omega_m) \quad 3.15$$

$$\frac{dw_m}{dt} = \frac{1}{J}(Ki_a - C_r - B\omega_m) \quad 3.16$$

$$K = K_f i_f \quad 3.17$$

3.3.2Frequency domain modeling

Equations 3.15, 3.16 and 3.17 can be represented in frequency domain as:

$$I_a(s) = \frac{1}{R_a + sL_a}(V_a(s) - K\omega_m) \quad 3.18$$

$$\omega_m(s) = \frac{1}{B + sJ}(T_e - T_L) \quad 3.19$$

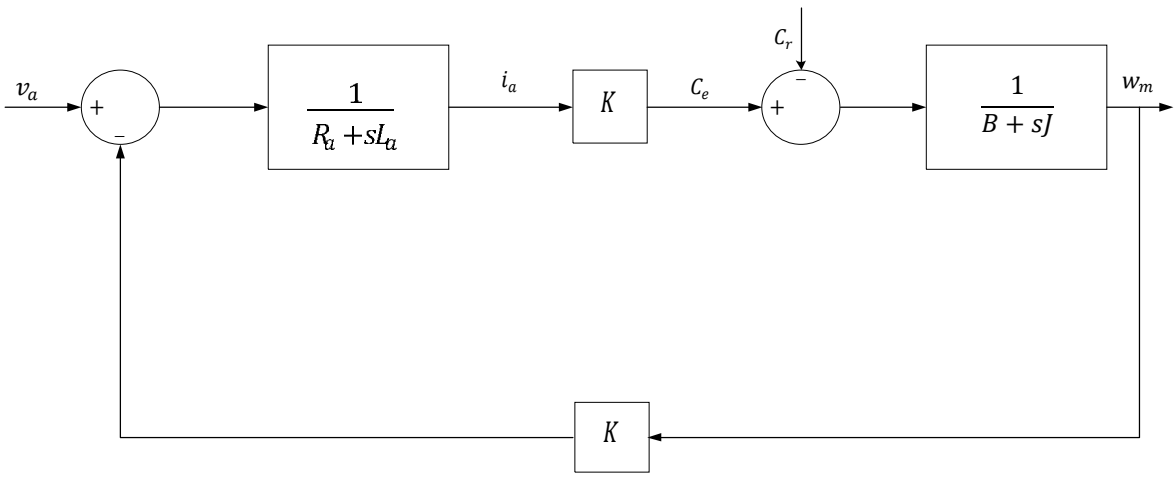


Figure 3.4 Block diagram of dynamic model of PMDC machine

3.3.3 State Space Representation

Selecting the state variables as:

$$x_1(t) = i_a(t) \tag{3.20}$$

$$x_2(t) = \omega_m(t) \tag{3.21}$$

$$\frac{dx_1(t)}{dt} = -\frac{R_a}{L_a}x_1(t) - \frac{K_b}{L_a}x_2(t) + \frac{1}{L_a}v_a \tag{3.22}$$

$$\frac{dx_2(t)}{dt} = -\frac{K_a}{J}x_1(t) - \frac{B}{J}x_2(t) - \frac{1}{J}T_L \tag{3.23}$$

The state space representation of PMDC model in matrix form is:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_a}{J} & -\frac{B}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_L \end{bmatrix} \tag{3.24}$$

3.4 Modeling of shunt connected dc motors

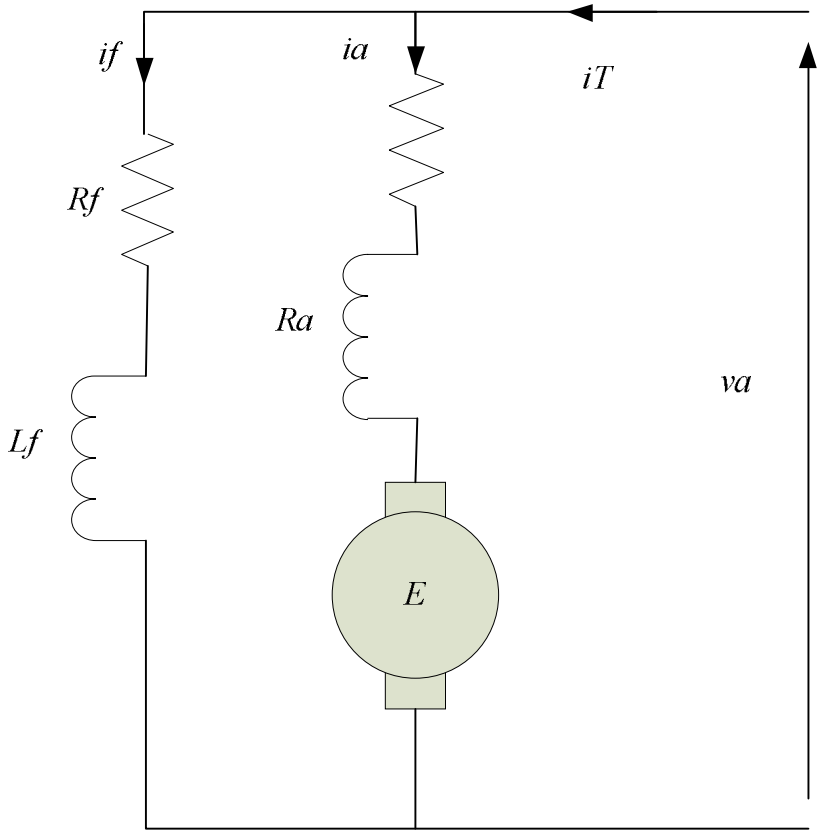


Figure 3.5 Equivalent circuit of shunt connected DC machine

The voltage across the inductor and the armature are the same.

$$v_a = v_f = v$$

The total currant equal to the sum of inductor and armature currant.

$$i_t = i_f + i_a$$

3.4.1 Time Domain Model

$$\begin{bmatrix} V_f \\ V_a \end{bmatrix} = \begin{bmatrix} R_f + \frac{d}{dt}L_f & 0 \\ \omega_r L_{af} & R_a + \frac{d}{dt}L_a \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix} \quad 3.25$$

3.4.2 Frequency Domain Model

$$V_f(S) = V_a(S) = R_a I_a(S) + SL_a I_a + \omega_r L_{af} I_f \quad 3.26$$

$$I_a(s) = \frac{1}{R_a + sL_a}(V_a(s) - L_{af}\omega_m I_f(s)) \quad 3.27$$

$$I_f(s) = \frac{1}{R_f + sL_f} V_f(s) \quad 3.28$$

$$\omega_m(s) = \frac{1}{B + sJ}(L_{af} I_a(s) I_f(s) - T_L) \quad 3.29$$

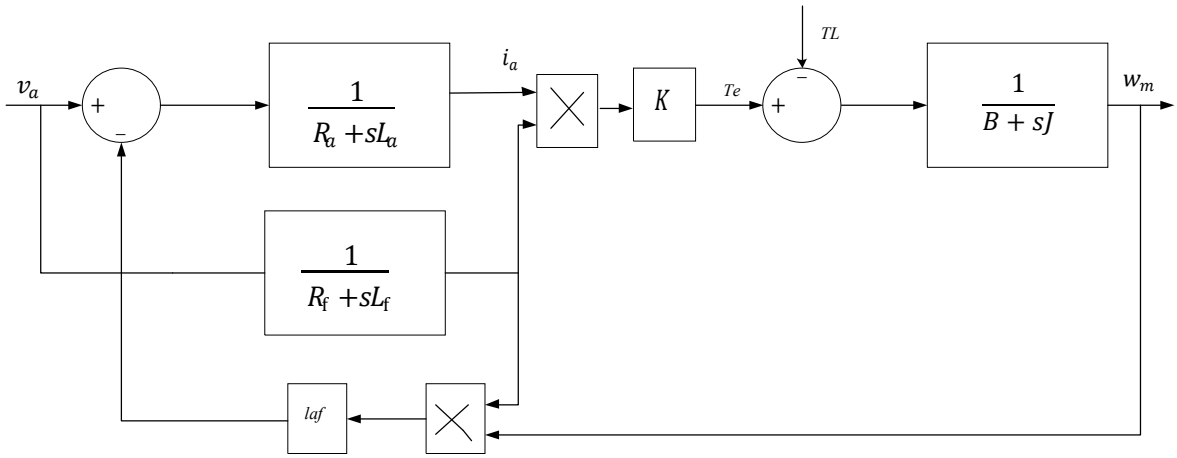


Figure 3.6 Block diagram of dynamic model of shunt connected machine

3.4.3 State space representation

$$\frac{di_f}{dt} = -\frac{R_f}{L_f} i_f + \frac{1}{L_f} V_f \quad 3.30$$

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{L_{af}}{L_a} i_f \omega_r + \frac{1}{L_a} V_a \quad 3.31$$

$$\frac{d\omega_r}{dt} = -\frac{B}{J} \omega_r + \frac{L_{af}}{J} i_f i_a - \frac{1}{J} T_L \quad 3.32$$

In matrix form

$$\frac{d}{dt}\begin{bmatrix} i_f \\ i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_f} & 0 & 0 \\ 0 & -\frac{R_a}{L_a} & 0 \\ 0 & 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i_f \\ i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{L_f}{L_a} i_f \omega_r \\ \frac{L_f}{J} i_f i_a \end{bmatrix} + \begin{bmatrix} \frac{1}{L_f} & 0 & 0 \\ 0 & \frac{1}{L_a} & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} V_f \\ V_a \\ T_L \end{bmatrix} \quad 3.33$$

3.5 Modeling of series connected dc machine

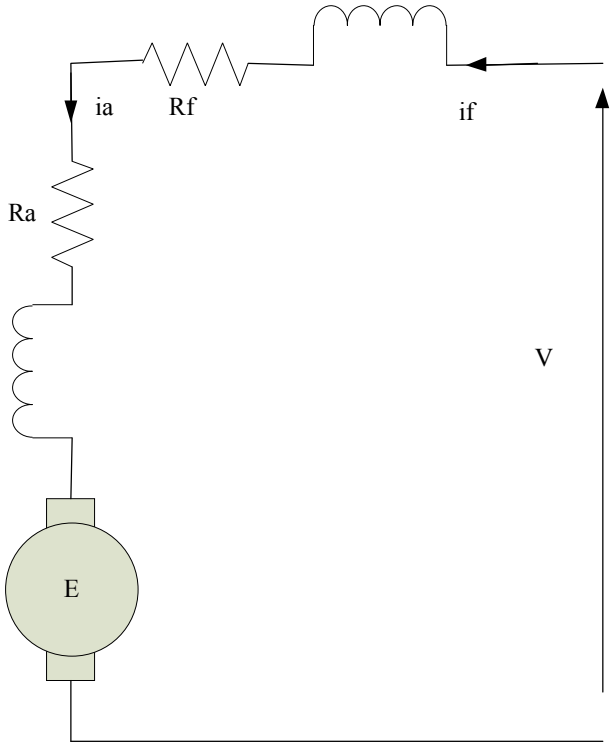


Figure 3.7 equivalent electrical circuit of dc series machine

So the same current flows through the rotor and the stator:

$$\boldsymbol{I_a = I_f = I}$$

The supply voltage

$$\boldsymbol{V = V_e + V_a}$$

3.5.1 Time domain model

$$\begin{bmatrix} V_f \\ V_a \end{bmatrix} = \begin{bmatrix} R_f + \frac{d}{dt}L_f & 0 \\ \omega_r L_{af} & R_a + \frac{d}{dt}L_a \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix} \quad 3.34$$

$$V = V_a + V_f = \left(R_a + R_f + SL_a + SL_f\right)I_a + \omega_r L_{af} I_a \quad 3.35$$

3.5.2 Frequency Domain model

$$V_a(S) = R_a I_a(S) + SL_a I_a + \omega_r L_{af} I_f \quad 3.36$$

$$V_f(S) = R_f I_f(S) + SL_f I_f \quad 3.37$$

$$V = V_a + V_f = \left(R_a + R_f + SL_a + SL_f\right) I_a + \omega_r L_{af} I_a \quad 3.38$$

$$I_a(s) = \frac{1}{R_a + R_f + sL_a + sL_f}\left(V_t(s) - L_{af}\omega_m I_a(s)\right) \quad 3.39$$

$$T_e = L_{af} i_f i_a = L_{af} I_a^2 \quad 3.40$$

$$\omega_m(s) = \frac{1}{B + sJ}\left(L_{af} I_a^2 - T_L\right) \quad 3.41$$

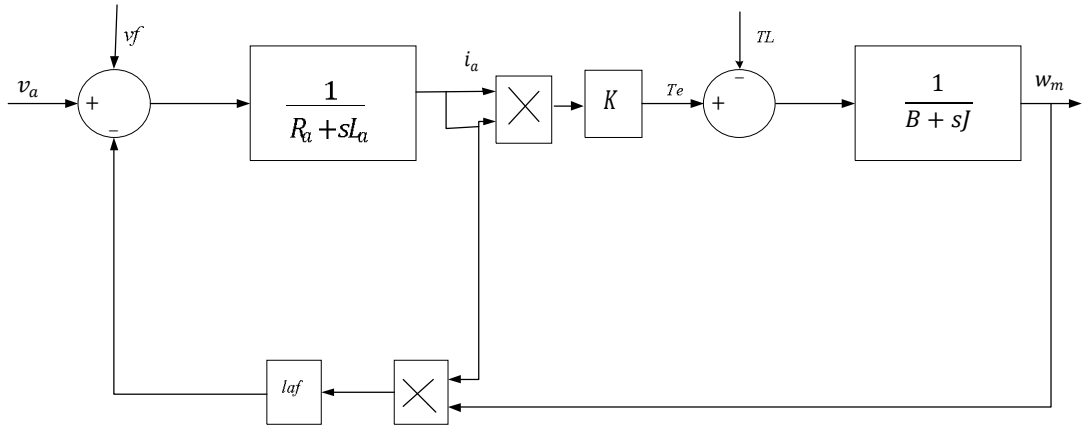


Figure 3.8 Block diagram of dynamic model of dc series machine

3.6 Modeling of dc compound dc machine

3.6.1 Short shunt

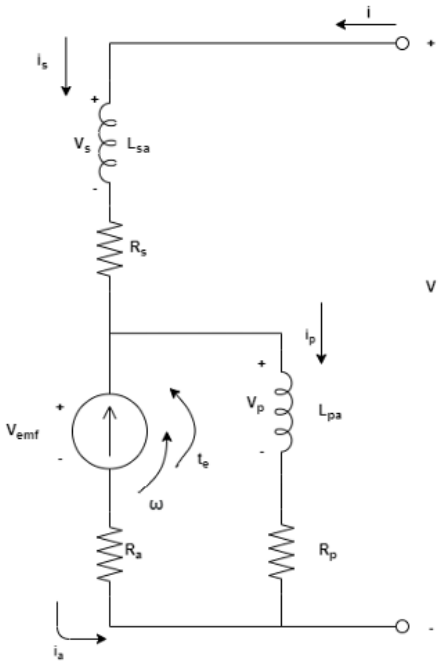


Figure 3.9 electrical equivalent circuit of short shunt dc compound machine

i is the total current.

i_s is the series field winding current.

i_p is the parallel field winding current.

i_a is the armature current.

v is the total voltage.

v_s is the series field winding voltage.

v_p is the parallel field winding voltage.

v_a is the armature voltage.

.

If you set the **Steady-state parameterization** parameter to By equivalent circuit parameters, you can specify the equivalent circuit parameters for this model:

R_a Armature resistance.

R_s Series field winding resistance.

R_p Shunt field winding resistance.

L_{sa} Series field winding to armature back EMF constant.

L_{pa} Shunt field winding to armature back EMF constant.

$$\boldsymbol{i_T = i_s = i_a + i_p}$$

$$\boldsymbol{v = v_a + v_s}$$

$$\boldsymbol{v_a = v_p}$$

The bak emf

$$e_a = \omega\left(L_{sa} i_s + L_{sb} i_p\right) \tag{3.42}$$

$$v_s = R_s i_s + L_s \frac{di_s}{dt} + L_{sp} \frac{di_p}{dt} \tag{3.43}$$

$$v_p = R_p i_p + L_p \frac{di_p}{dt} + L_{sp} \frac{di_s}{dt} \tag{3.44}$$

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \tag{3.45}$$

3.6.2 Long shunt

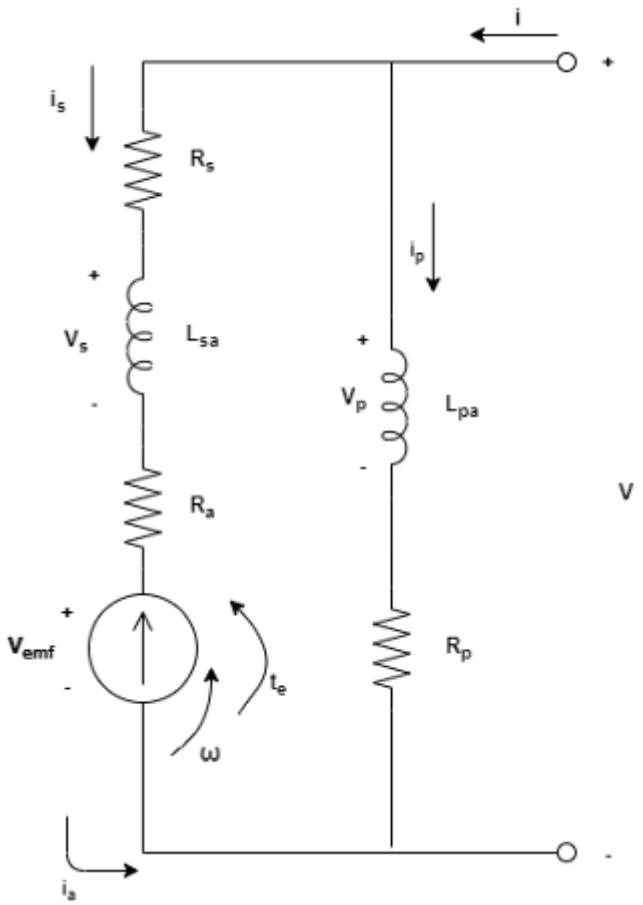


Figure 3.10 electrical equivalent circuit of long dc compound machine

"https://www.mathworks.com/help"

$$\boldsymbol{i_T = i_p + i_a}$$

$$\boldsymbol{i_s = i_a}$$

So the total current of long shunt coumpound machine is given by

$$\boldsymbol{i_T = i_p + i_s}$$

The voltage equation is,

$$\boldsymbol{v = v_a + v_s}$$

The dynamic equations are derived as:

$$e_a = \omega\left(L_{sa} i_s + L_{sb} i_p\right) \quad 3.46$$

$$v_s = R_s i_s + L_s \frac{di_s}{dt} + L_{sp} \frac{di_p}{dt} \quad 3.47$$

$$v_p = R_p i_p + L_p \frac{di_p}{dt} + L_{sp} \frac{di_s}{dt} \quad 3.48$$

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + e_a \quad 3.49$$

3.7 Conclusion

DC motor drives have become increasingly popular, following recent developments in rare-earth permanent-magnet materials and the semiconductor devices used to control the stator input power and to sense the rotor position. They are now frequently used in applications such as flight control systems and robot actuators, and for drives which require high reliability, long life, little maintenance and a high torque-to-weight ratio.

Chapter 4

Transformation Matrices

4.1 Introduction

The study of the behavior of the alternative machine for different operating points and in static and variable (dynamic) regimes is essential for the sizing or choice of the machine to be used with these accessories, as well as the design of the control adopted.

The Transformation Matrix simplify the study of electrical machines by converting three phase quantities in two axis stationary and rotating reference frames , making modeling, simulation and control more strain forward.

4.2Spacevector

A balanced three-phase system can be written in the following form:

$$\begin{aligned} f_a(t) &= F\cos(\omega t + \theta_0) \\ f_b(t) &= F\cos\left(\omega t + \theta_0 - \frac{2\pi}{3}\right) \\ f_b(t) &= F\cos\left(\omega t + \theta_0 - \frac{4\pi}{3}\right) \end{aligned} \qquad 4.1$$

Where

F: Peak value

ω: Angular frequency

θ_0: Initial phase angle

The system described in equation (4-1) can be represented by the complex form:

$$f_{abc}(t) = \frac{2}{3}[f_a(t) + a f_b(t) + a^2 f_c(t)] \qquad 4.2$$

$a = e^{j\frac{2\pi}{3}}$: Fortescue operator / $a^2 = e^{j\frac{4\pi}{3}}$.

The variable $f\ (t)$ may represents any physical entity (Voltage, Current, Flux……).

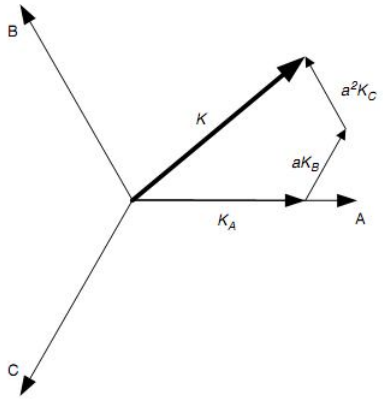


Figure 2.8 Space vector representation for three-phase variables

Figure 4.1 space vector of three phase system

4.3 ClarkTransform

Clark transform converts a three-phase system (abc) to two phase system (α,β).

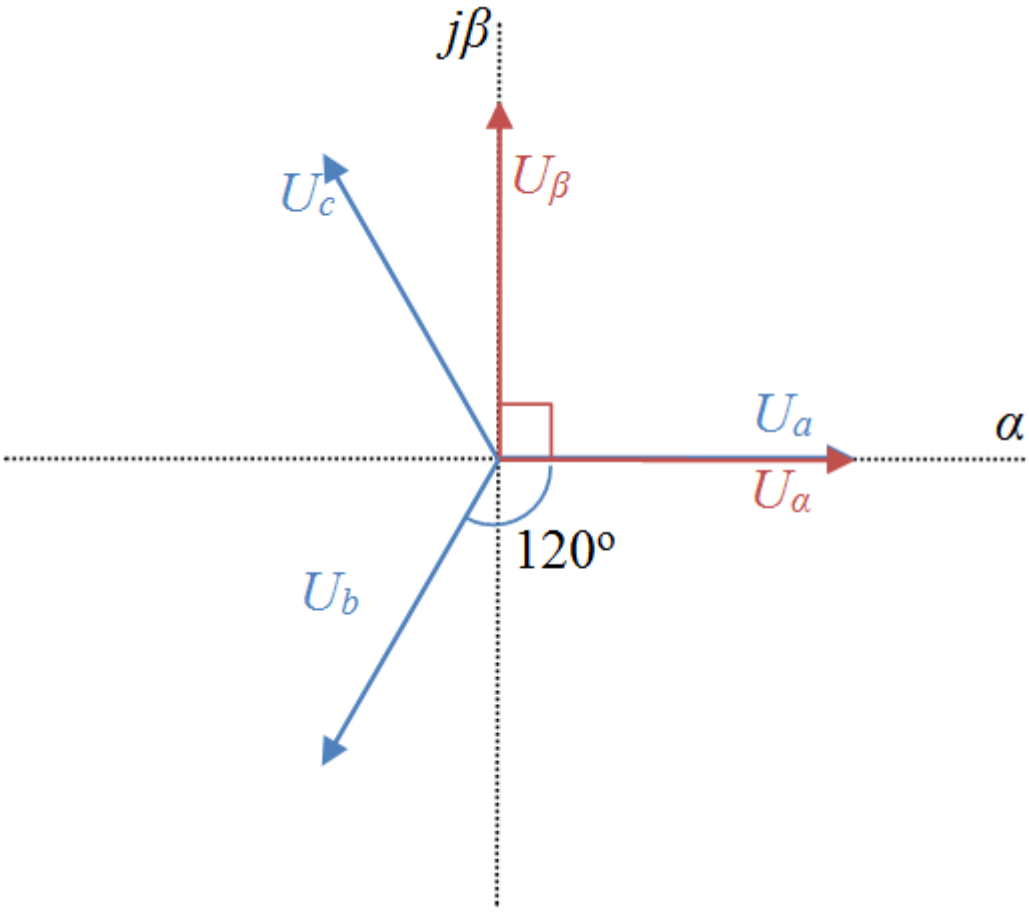


Figure 4.2 αβ Component representation

Transformation Matrices

The transformation between the natural frame and the fixed frame is done by the transformation matrix abc/αβ.

From Figure 4-2 the complex vector can be written in the form

$$f_{\alpha\beta}(t) = \left[f_\alpha(t) + jf_\beta(t)\right] \quad 4.3$$

The phase angle of the complex vector can be deduced as follows

$$\theta(t) = tan^{-1}\left[\frac{f_\beta(t)}{f_\alpha(t)}\right]. \quad 4.4$$

$$f_\alpha = Re\left(\frac{2}{3}[f_a + af_b + a^2 f_c]\right) \quad 4.5$$

$$f_\beta = Im\left(\frac{2}{3}[f_a + af_b + a^2 f_c]\right) \quad 4.6$$

$$\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \\ f_o(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix} \quad 4.7$$

$$\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \\ f_o(t) \end{bmatrix} = T_{abc/\alpha\beta}\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix} \;/\; T_{abc/\alpha\beta} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad 4.8$$

The coefficient 2/3 is added to mountain the amplitude across the transform.

$$f_\alpha = Fcos(\omega t) - \frac{1}{2}Fcos\left(\omega t - \frac{2\pi}{3}\right) - \frac{1}{2}Fcos\left(\omega t - \frac{4\pi}{3}\right) \quad 4.9$$

Adopting the trigonometric identity

$$\mathrm{Cos}(A).\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B) \quad 4.10$$

$$cos\left(\omega t - \frac{2\pi}{3}\right) + cos\left(\omega t - \frac{4\pi}{3}\right) = \cos(\omega t)\cos\left(\frac{2\pi}{3}\right) \quad 4.11$$

Subtitling in…. yield

$$f_\alpha = Fcos(\omega t) - F\cos(\omega t)\cos\left(\frac{2\pi}{3}\right) = \frac{3}{2}Fcos(\omega t) \quad 4.12$$

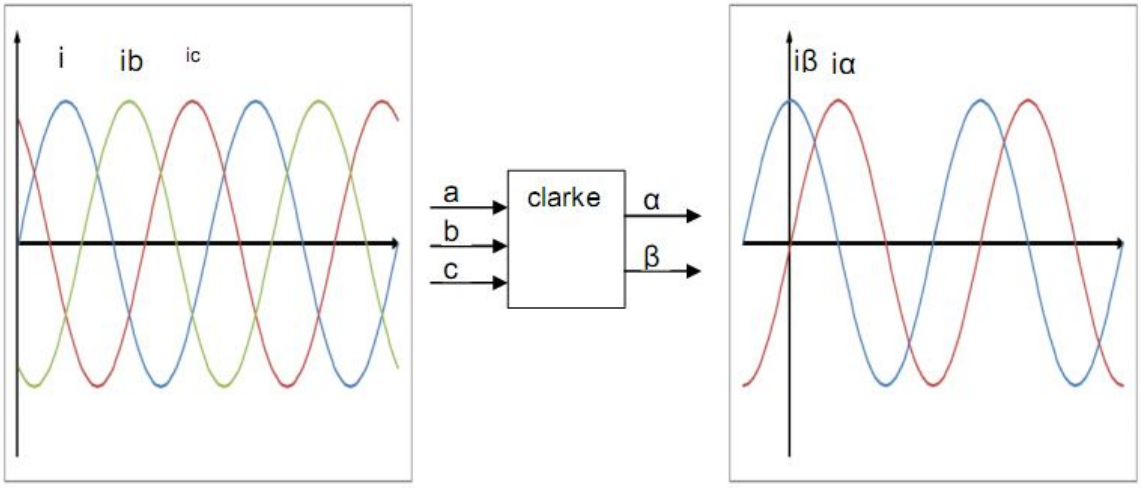


Figure 4.3 αβ Componenet wave form

The inverse transformation $\alpha\beta/abc$ is given by

$$\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix} = T^{-1}_{abc/\alpha\beta} \begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \\ f_0(t) \end{bmatrix} \quad 4.13$$

$$T^{-1}_{abc/\alpha\beta} = T_{\alpha\beta/abc} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \quad 4.14$$

For a balanced three phase system, the zero sequence component is always zero

4.4 Park transform

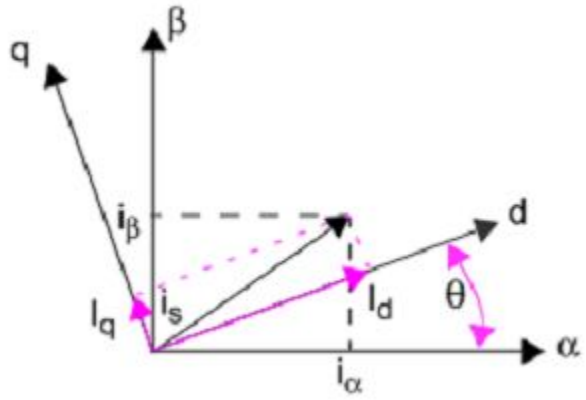


Figure 4.4 dq Componenet representation

$$f_{dq}(t) = [f_d(t) + jf_q(t)] \quad 4.15$$

The transformation matrix between three phase and Park frame (abc/dq) is given by

$$\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{4\pi}{3}) \end{bmatrix}\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix} \quad 4.16$$

Hence the inverse transformation is given by the following matrix.

$$\begin{bmatrix} f_a(t) \\ f_b(t) \\ f_c(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) \\ \cos(\theta - \frac{4\pi}{3}) & -\sin(\theta - \frac{4\pi}{3}) \end{bmatrix}\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} \quad 4.17$$

4.5 Transformation between Clark and Park Frames

The two matrices below give the direct and inverse transformation between the fixed frame and the rotating frame

$$\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \end{bmatrix} \quad 4.18$$

$$\begin{bmatrix} f_\alpha(t) \\ f_\beta(t) \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}\begin{bmatrix} f_d(t) \\ f_q(t) \end{bmatrix} \quad 4.19$$

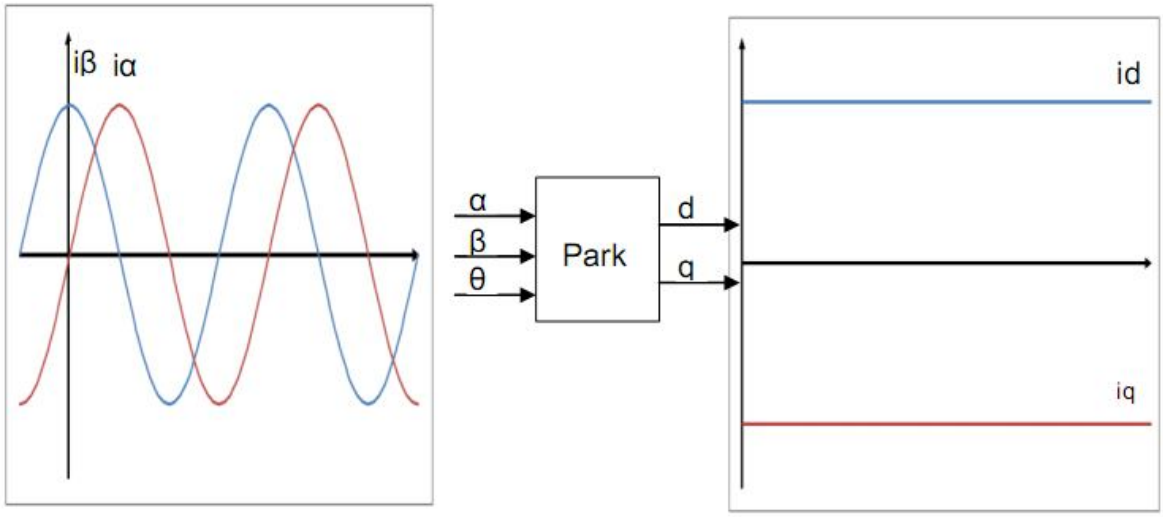


Figure 4.5 dq Componenet wave form

4.6 Reduced sensor Clark transformation

In order to reduce the number of sensors in electrical drive system, we introduce another clark transform matrix using line to line voltages instead of line to neutral.

The following matrix can be deduced:

$$\begin{bmatrix} E_\alpha \\ E_\beta \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ -1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} e_{ab} \\ e_{ca} \end{bmatrix} \quad 4.20$$

The inverse Clarke transform becomes:

$$\begin{bmatrix} e_{ab} \\ e_{ca} \end{bmatrix} = \begin{bmatrix} 3/2 & -\sqrt{3}/2 \\ -3/2 & -\sqrt{3}/2 \end{bmatrix} \quad 4.21$$

Under the particular case the neutral and is not connected the sum of the three currents is zero, the Clarke transform of the currents turns out:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/\sqrt{3} & 2/\sqrt{3} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \quad 4.22$$

4.7 Unbalanced three phase system analysis

As shown in equation (4.33) an unbalanced three-phase system can be written as three equations each different from the other in phase and amplitude.

$$\begin{cases} f_a = F_{amax} \sin(\omega t + \theta_a) \\ f_b = F_{bmax} \sin(\omega t + \theta_b) \\ f_c = F_{cmax} \sin(\omega t + \theta_c) \end{cases} \quad 4.23$$

4.7.1 Symmetrical component

The theory of symmetric components (superposition theory) says that an unbalanced polyphase system can be considered as a sum of a set of balanced systems. The appearance of an imbalance (in amplitude or phase) on a three-phase system results in the appearance of an inverse balanced three-phase component superposed at the direct component.

Using the theory of symmetric components, the three-phase system in (4.23) can be decomposed into three systems: direct sequence, inverse sequence and homo-polar sequence.

In the complex plane the relationship between the direct, inverse and zero sequence and the instantaneous values of the abc system is as follows.

$$\begin{bmatrix} f^{+} \\ f_b^{-} \\ f^{o} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ ef_c \end{bmatrix} \quad 4.24$$

$$f_{abc}^{+} = [T_{+}]e_{abc}\begin{bmatrix} f_a^{+} \\ f_b^{+} \\ f_c^{+} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad 4.25$$

$$f_{abc}^{-} = [T_{-}]e_{abc}\begin{bmatrix} f_a^{-} \\ f_b^{-} \\ f_c^{-} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a^2 & a \\ a & 1 & a^2 \\ a^2 & a & 1 \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad 4.26$$

$$ef_{abc}^{0} = [T_0]e_{abc}\begin{bmatrix} f \\ f_b^{0} \\ f_c^{0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad 4.27$$

$a = e^{j(\frac{2\pi}{3})}$.

4.7.2 Clark transformation of unbalanced system

In most electrical systems, the neutral is not connected, so the zero sequence component is zero. three-phase quantities of the positive and negative components can be transformed to the two-phase system in the fixed frame $\alpha\beta$ as:

$$f_{\alpha\beta} = [T_{\alpha\beta}]f_{abc}[T_{\alpha\beta}] = \frac{2}{3}\begin{bmatrix} 1 & \frac{-1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad 4.28$$

$$f_{\alpha\beta}^{+} = [T_{\alpha\beta}]f_{abc}^{+} = [T_{\alpha\beta}][T_{+}]f_{abc}$$

$$= [T_{\alpha\beta}][T_{+}][T_{\alpha\beta}]^{t}f_{\alpha\beta} = \frac{1}{2}\begin{bmatrix} 1 & -q \\ q & 1 \end{bmatrix}f_{\alpha\beta} \quad 4.29$$

$$f_{\alpha\beta}^{-} = [T_{\alpha\beta}]f_{abc}^{-} = [T_{\alpha\beta}][T_{-}]f_{abc}$$

$$= [T_{\alpha\beta}][T_{-}][T_{\alpha\beta}]^{t}f_{\alpha\beta} = \frac{1}{2}\begin{bmatrix} 1 & q \\ -q & 1 \end{bmatrix}f_{\alpha\beta} \quad 4.30$$

$q = e^{-j(\frac{\pi}{2})}$.

4.7.3Park transformation of unbalanced system

The transformation into the rotating frame of the direct balanced component of equation (4.29) is obtained by the use of the Park transform which makes it possible to pass through a rotation of *(ωt)* of the system in the Clarke frame at two-phase system in the Park rotating frame, in which the quantities of the direct balanced component have the property of being constant:

$$f_{dq}^{+} = e^{j\omega t} f_{\alpha\beta}^{+} = f_d^{+} + j f_q^{+} \qquad 4.31$$

Likewise, if we consider the inverse balanced three-phase component, the transformation into the two-phase frame is carried out using the Park transform in order to obtain constant two-phase coordinates. The two-phase frame in which an inverse balanced three-phase system has constant coordinates is obtained by rotation in the opposite direction (-ωt).

$$f_{dq}^{-} = e^{-j\omega t} f_{\alpha\beta}^{-} = f_d^{-} + j f_q^{-} \qquad 4.32$$

The space vector of an unbalanced system can be expressed from the direct and inverse components in the direct and inverse Park frame as follows:

$$f_{\alpha\beta} = e^{j\omega t} f_{\alpha\beta}^{+} + e^{-j\omega t} f_{\alpha\beta}^{-} = e^{j\omega t}\left(f_d^{+} + j f_q^{+}\right) + e^{-j\omega t}\left(f_d^{-} + j f_q^{-}\right) \qquad 4.33$$

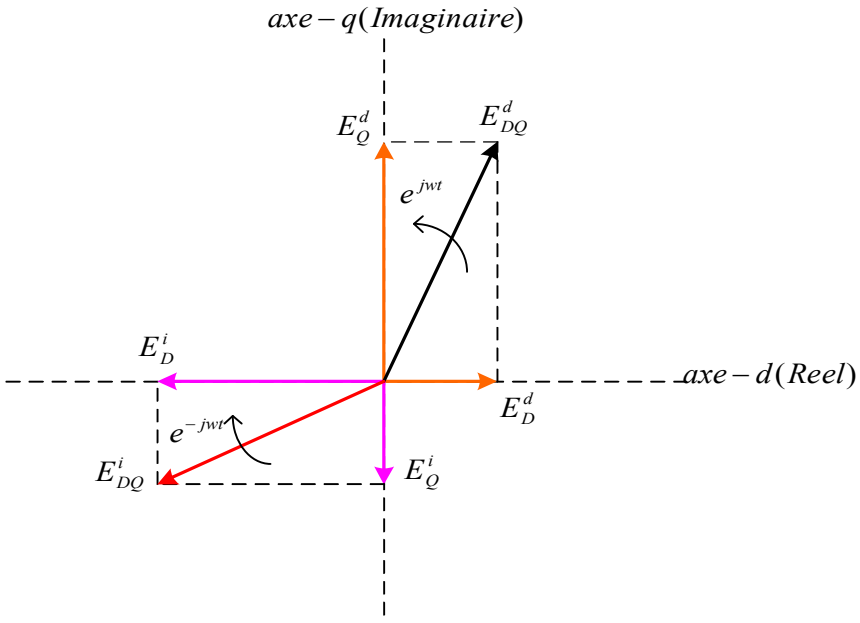


Figure 2.1 space vector diagram of symmetric Park frame

Transformation Matrices

For a better understanding we introduce the matrix notation to determine the relationship between the direct and inverse Park frame and the positive component of abc frame.

The matrix equation 4.44 relates the positive sequence of tensions in the frame to that in the Clarke frame

$$\begin{bmatrix} f_\alpha^+ \\ f_\beta^+ \\ f_0^+ \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} f_a^+ \\ f_b^+ \\ f_c^+ \end{bmatrix} \quad 4.44$$

The in the Clarke frame of 4.44 are transformed to the direct Park frame by multiplying them by counterclockwise rotation matrix.

$$\begin{bmatrix} f_d^+ \\ f_q^+ \\ f_0^+ \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & -0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} f_\alpha^+ \\ f_\beta^+ \\ f_0^+ \end{bmatrix} \quad 4.45$$

Therefore the matrix equation which connects the direct sequence in the direct abcframe and that in the direct Park frame is :

$$\begin{bmatrix} f_d^+ \\ f_q^+ \\ f_0^+ \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & -0 \\ 0 & 0 & 1 \end{bmatrix}\frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} f_a^+ \\ f_b^+ \\ f_c^+ \end{bmatrix} \quad 4.46$$

After simplification :

$$\begin{bmatrix} f_d^+ \\ f_q^+ \\ f_0^+ \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta & cos(\theta - 2\pi/3) & cos(\theta + 2\pi/3) \\ -sin\theta & -sin(\theta - 2\pi/3) & -sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} f_a^+ \\ f_b^+ \\ f_c^+ \end{bmatrix} \quad 4.47$$

The inverse of the Park transform matrix in (4.47) is as follows:

$$\begin{bmatrix} f_d^+ \\ f_q^+ \\ f_0^+ \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta & -sin\theta & 1 \\ cos(\theta - 2\pi/3) & -sin(\theta - 2\pi/3) & 1 \\ cos(\theta + 2\pi/3) & -sin(\theta + 2\pi/3) & 1 \end{bmatrix}\begin{bmatrix} f_a^+ \\ f_b^+ \\ f_c^+ \end{bmatrix} \quad 4.48$$

Same procedure for the inverse (negative) component.

$$\begin{bmatrix} f_\alpha^- \\ f_\beta^- \\ f_0^- \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}\begin{bmatrix} f_a^- \\ f_b^- \\ f_c^- \end{bmatrix} \quad 4.49$$

$$\begin{bmatrix} f_d^- \\ f_q^- \\ f_0^- \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & -0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_\alpha^- \\ f_\beta^- \\ f_0^- \end{bmatrix} \quad 4.50$$

By replacing 4.49) into 4.50 yield.

$$\begin{bmatrix} f_d^- \\ f_q^- \\ f_0^- \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & -0 \\ 0 & 0 & 1 \end{bmatrix} \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \quad 4.51$$

After multiplication of the two matrices we find.

$$\begin{bmatrix} f_d^+ \\ f_q^+ \\ f_0^+ \end{bmatrix} = \frac{2}{3} \begin{bmatrix} cos\theta & cos(\theta + 2\pi/3) & cos(\theta - 2\pi/3) \\ sin\theta & sin(\theta + 2\pi/3) & sin(\theta - 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} f_a^+ \\ f_b^+ \\ f_c^+ \end{bmatrix} \quad 4.52$$

The inverse of the Park transform matrix in (4.52) is as follows:

$$\begin{bmatrix} f_a^- \\ f_b^- \\ f_0^- \end{bmatrix} = \frac{2}{3} \begin{bmatrix} cos\theta & sin(\theta) & 1 \\ sin\theta & sin(\theta + 2\pi/3) & 1 \\ cos(\theta - 2\pi/3) & sin(\theta - 2\pi/3) & 1 \end{bmatrix} \begin{bmatrix} f_d^- \\ f_q^- \\ f_0^- \end{bmatrix} \quad 4.53$$

4.8 n phase system

The vector of n phase system in stationary coordinate system is given by:

$$F_n = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix} = \begin{bmatrix} Fcos(\omega t + \theta_0) \\ Fcos\left(\omega t + \theta_0 + \frac{2\pi}{n}\right) \\ \vdots \\ Fcos\left(\omega t + \theta_0 + \frac{2(n-1)\pi}{n}\right) \end{bmatrix} \quad 4.54$$

where: $f_1(t)$, $f_2(t)$ .. $f_n(t)$: are the instantaneous values of the phase quantities.

n: The number of phases.

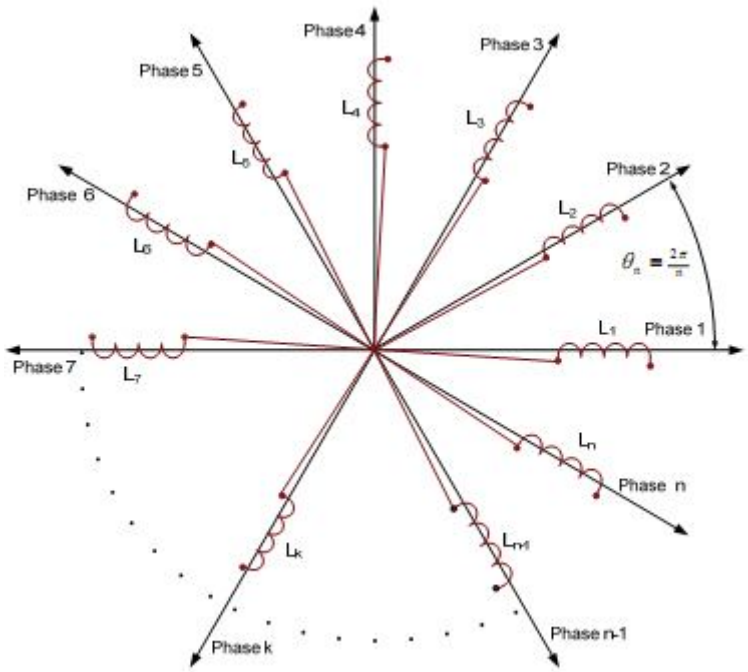


Fig. 2. Simplified diagram of the stator windings of an n-phase motor

4.8.1 Direct Clark transformation of

The transformation of vector Xn from the stationary n-phase coordinate system to a stationary orthogonal coordinate system is performed by the matrix:

$$T_{\alpha\beta/n} = \frac{2}{n}.\begin{bmatrix} 1 & cos\left(\frac{2\pi}{n}\right) & cos\left(\frac{4\pi}{n}\right) & \cdots & cos\left(\frac{2(n-1)\pi}{n}\right) \\ 0 & -sin\left(\frac{2\pi}{n}\right) & -sin\left(\frac{4\pi}{n}\right) & \cdots & -sin\left(\frac{2(n-1)\pi}{n}\right) \end{bmatrix} \quad 4.55$$

Combining 4.49 and 4.50 we find:

$$f_{\alpha\beta} = \frac{2}{n}.\begin{bmatrix} 1 & cos\left(\frac{2\pi}{n}\right) & cos\left(\frac{4\pi}{n}\right) & \cdots & cos\left(\frac{2(n-1)\pi}{n}\right) \\ 0 & -sin\left(\frac{2\pi}{n}\right) & -sin\left(\frac{4\pi}{n}\right) & \cdots & -sin\left(\frac{2(n-1)\pi}{n}\right) \end{bmatrix}\begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix} \quad 4.56$$

a) Four phase system

$$T_{\alpha\beta/4} = \frac{1}{2}.\begin{bmatrix} 1 & cos\left(\frac{\pi}{2}\right) & cos(\pi) & cos\left(\frac{3\pi}{2}\right) \\ 0 & -sin\left(\frac{\pi}{2}\right) & -sin(\pi) & -sin\left(\frac{3\pi}{2}\right) \end{bmatrix} \quad 4.57$$

$$=.\frac{1}{2}.\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}$$

b) Five phase system

$$T_{\alpha\beta/5}=\frac{1}{2}\begin{bmatrix}1 & cos\left(\frac{2\pi}{5}\right) & cos\left(\frac{4\pi}{5}\right) & cos\left(\frac{6\pi}{5}\right) & cos\left(\frac{8\pi}{5}\right)\\ 0 & -sin\left(\frac{2\pi}{5}\right) & -sin\left(\frac{4\pi}{5}\right) & -sin\left(\frac{6\pi}{5}\right) & -sin\left(\frac{8\pi}{5}\right)\end{bmatrix} \quad 4.58$$

c Six phase system

$$T_{\alpha\beta/6}=\frac{1}{2}\begin{bmatrix}1 & cos\left(\frac{\pi}{3}\right) & cos\left(\frac{2\pi}{3}\right) & cos(\pi) & cos\left(\frac{4\pi}{3}\right) & cos\left(\frac{5\pi}{3}\right)\\ 0 & -sin\left(\frac{\pi}{3}\right) & -sin\left(\frac{2\pi}{3}\right) & -sin(\pi) & -sin\left(\frac{4\pi}{3}\right) & -sin\left(\frac{5\pi}{3}\right)\end{bmatrix} \quad 4.59$$

4.8.2 Inverse Clark Transformation

$$T_{n/\alpha\beta}=\begin{bmatrix}1 & 0\\ cos\left(\frac{2\pi}{n}\right) & -sin\left(\frac{2\pi}{n}\right)\\ cos\left(\frac{4\pi}{n}\right) & -sin\left(\frac{4\pi}{n}\right)\\ \vdots & \vdots\\ cos\left(\frac{2(n-1)\pi}{n}\right) & -sin\left(\frac{2(n-1)\pi}{n}\right)\end{bmatrix} \quad 4.60$$

a) Four phase system

$$T_{4/\alpha\beta}=\begin{bmatrix}1 & 0\\ cos\left(\frac{\pi}{2}\right) & -sin\left(\frac{\pi}{2}\right)\\ cos(\pi) & -sin(\pi)\\ cos\left(\frac{3\pi}{2}\right) & -sin\left(\frac{3\pi}{2}\right)\end{bmatrix} \quad 4.61$$

b) Five phase system

$$T_{5/\alpha\beta}=\begin{bmatrix}1 & 0\\ cos\left(\frac{2\pi}{5}\right) & -sin\left(\frac{2\pi}{5}\right)\\ cos\left(\frac{4\pi}{5}\right) & -sin\left(\frac{4\pi}{5}\right)\\ cos\left(\frac{6\pi}{5}\right) & -sin\left(\frac{6\pi}{5}\right)\\ cos\left(\frac{8\pi}{5}\right) & -sin\left(\frac{8\pi}{5}\right)\end{bmatrix} \quad 4.62$$

c) Six phase system

$$T_{5/\alpha\beta} = \begin{bmatrix} 1 & 0 \\ cos\left(\frac{\pi}{3}\right) & -sin\left(\frac{\pi}{3}\right) \\ cos\left(\frac{2\pi}{3}\right) & -sin\left(\frac{2\pi}{3}\right) \\ cos(\pi) & -sin(\pi) \\ cos\left(\frac{4\pi}{3}\right) & -sin\left(\frac{4\pi}{3}\right) \\ cos\left(\frac{5\pi}{3}\right) & -sin\left(\frac{5\pi}{3}\right) \end{bmatrix} \quad 4.63$$

4.8.3 Park transformation

Decoupling Transformation matrix for an arbitrary phase number n can be given in power invariant form shown in (1), where α=2π/n. The first two rows of the matrix define variables that will lead to fundamental flux and torque production (α–β components; stator to rotor coupling appears only in the equations for α–β components). The last two rows define the two zero sequence components are omitted for all odd phase numbers n. In between, there are x–y components

$$\begin{bmatrix} v_q \\ v_d \\ v_x \\ v_y \\ \vdots \\ 0+ \\ 0- \end{bmatrix} = \sqrt{\frac{2}{n}} \begin{bmatrix} 1 & cos\theta & cos2\theta & cos3\theta & \dots & cosn\theta \\ 0 & sin\theta & sin\theta & sin\theta & \dots & sinn\theta \\ 1 & cos2\theta & cos4\theta & cos6\theta & \dots & cosn\theta \\ 0 & sin2\theta & sin4\theta & sin6\theta & \dots & sinn\theta \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \dots & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \dots & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \\ \vdots \\ v_n \end{bmatrix} \quad 4.64$$

A) Five phase system

$$T = \sqrt{\frac{2}{5}} \begin{bmatrix} 1 & cos\left(\frac{2\pi}{5}\right) & cos\left(\frac{4\pi}{5}\right) & cos\left(\frac{4\pi}{5}\right) & cos\left(\frac{2\pi}{5}\right) \\ 0 & sin\left(\frac{2\pi}{5}\right) & sin\left(\frac{4\pi}{5}\right) & sin\left(\frac{4\pi}{5}\right) & sin\left(\frac{2\pi}{5}\right) \\ 1 & cos\left(\frac{4\pi}{5}\right) & cos\left(\frac{8\pi}{5}\right) & cos\left(\frac{8\pi}{5}\right) & cos\left(\frac{4\pi}{5}\right) \\ 0 & sin\left(\frac{4\pi}{5}\right) & sin\left(\frac{8\pi}{5}\right) & -sin\left(\frac{8\pi}{5}\right) & sin\left(\frac{4\pi}{5}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad 4.65$$

B) Six phase system

$$T = \begin{bmatrix} [T_1] & & \\ & [T_2] & \\ & & [I_{3x3}] \end{bmatrix} \quad 4.66$$

Where

$$[T_1] = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} 1 & cos\left(-\frac{2\pi}{3}\right) & cos\left(\frac{2\pi}{3}\right) \\ 0 & sin\left(-\frac{2\pi}{3}\right) & sin\left(\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad 4.67$$

$$[T_2] = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} cos\left(-\frac{\pi}{6}\right) & cos\left(-\frac{5\pi}{6}\right) & cos\left(\frac{\pi}{2}\right) \\ -sin\left(-\frac{\pi}{6}\right) & -sin\left(-\frac{5\pi}{3}\right) & sin\left(\frac{\pi}{2}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad 4.86$$

4.9 . Conclusion

Clarck and Park transformations simplify the analysis of three phase machines. Clarck convert three phase quantities on a two axis stationary frame, reducing complexity. Park rotate this frame into a rotating dq-frame, making AC quantities appear as DC, which simplify control of torque and flux. Together are essential for modern AC machine modeling and vector control.

Chapter 5

Synchronous Machine Modeling

5.1 Introduction

The synchronous machine is a crucial element in electric power systems. It is necessary to model synchronous machines for power systems analysis. In many cases, electric machines are connected with power electronic devices. This section introduces a sophisticated modeling approach for synchronous machines that focuses on systems with a combination of synchronous machines and power electronic components. These types of systems are commonly seen in electric drive systems, DC power systems, renewable energy, and traditional synchronous machine excitation. Various models and equations have been employed to examine synchronous machines in various scenarios.

5.2 Modeling of synchronous machine

The mathematical model of the electric synchronous machine requires certain simplifying assumptions. The absence of saturation in the magnetic circuit.

- ✓ The machine is constituted by identical windings equally distributed around the stator and rotor.
- ✓ The rotor is a squirrel cage topology;
- ✓ The air gap of the machines is considered uniform;
- ✓ The core losses are negligible;
- ✓ The sinusoidal distribution in the field;
- ✓ The magnetic circuit is linear;
- ✓ The mutual leakage are neglected.

5.3 Modeling of saliant pole wound rotor synchronous machine

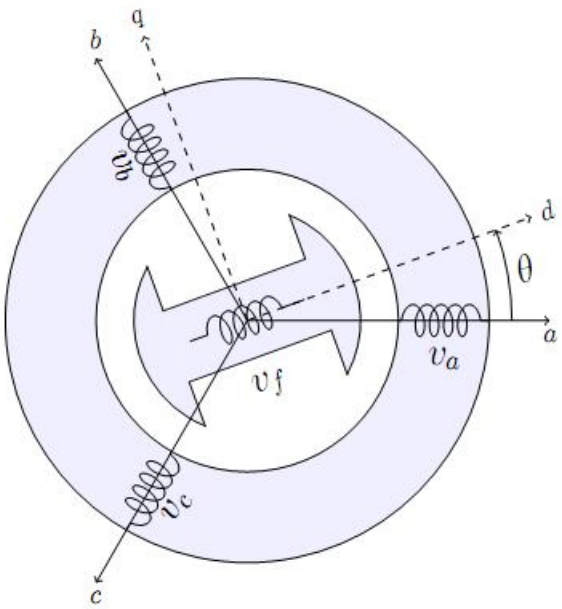


Figure 5.1. Representation of salient pole wound rotor synchronous machine

5.3.1 natural frame modeling

The machine flux are written as follows:

$$\varphi_{sabc} = L_s i_{sabc} + M_{sf} i_f \quad 5.1$$

$$\varphi_f = M_{sf}^T i_{sabc} + L_f i_f \quad 5.2$$

The inductance matrix contains two terms; a constant term, and a variable term in function of the electric angle θ:

$$L_s = L_{s0} + L_{s2}(\theta) \quad 5.3$$

$$L_{s0} = \begin{bmatrix} L_{s0} & M_{s0} & M_{s0} \\ M_{s0} & L_{s0} & M_{s0} \\ M_{s0} & M_{s0} & L_{s0} \end{bmatrix} \quad 5.4$$

$$L_{s2}(\theta) = L_{s2} \begin{bmatrix} cos(2\theta) & cos(2\theta - 2\pi/3) & cos(2\theta + 2\pi/3) \\ cos(2\theta - 2\pi/3) & cos(2\theta + 2\pi/3) & cos(2\theta) \\ cos(2\theta + 2\pi/3) & cos(2\theta) & cos(2\theta - 2\pi/3) \end{bmatrix} \quad 5.5$$

$$M_{sf} = M_0 \begin{bmatrix} \cos(\theta) \\ \cos(\theta - \frac{2\pi}{3}) \\ \cos(\theta - \frac{4\pi}{3}) \end{bmatrix} \quad 5.6$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{as} \\ \varphi_{bs} \\ \varphi_{cs} \end{bmatrix} \quad 5.7$$

$$v_{sabc} = R_s i_{sabc} + \frac{d\varphi_{sabc}}{dt} \quad 5.8$$

$$v_{sabc} = R_s i_{sabc} + L_s \frac{di_{sabc}}{dt} + M_{sf}^T \frac{di_f}{dt} + \omega \frac{dL_s}{d\theta} i_{sabc} + \omega \frac{dM_{sf}^T}{d\theta} i_f \quad 5.9$$

$$v_f = R_f i_f + \frac{d\varphi_f}{dt} \quad 5.10$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} + M_{sf}^T \frac{di_{sabc}}{dt} + \omega \frac{dM_{sf}^T}{d\theta} i_{sabc} \quad 5.11$$

5.3.2αβ frame modeling

The model in fixe frame can be obtained by applying Clark transformation for all magnetic and electric variables.

$$f_{\alpha\beta} = T_{abc/\alpha\beta} f_{abc} \quad 5.12$$

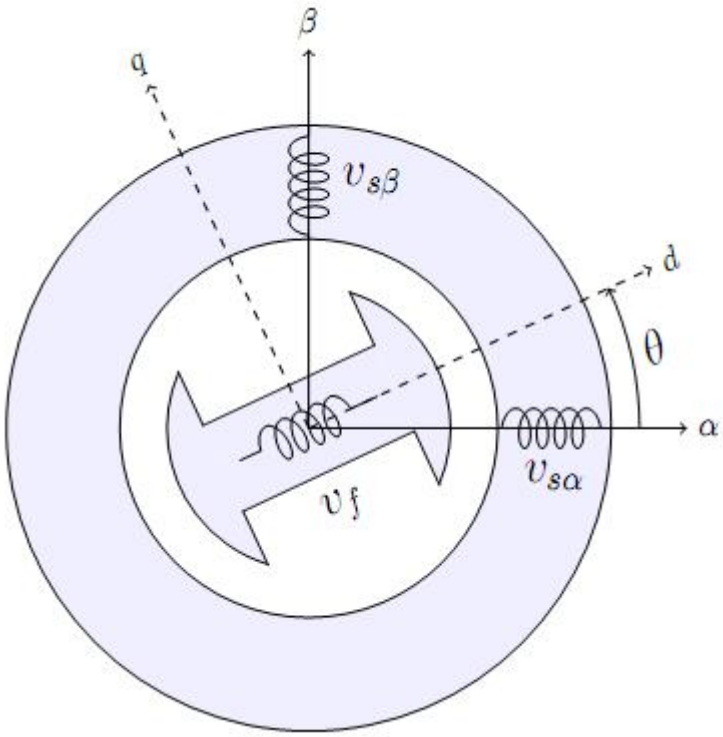


Figure 5.2. αβ Representation of salient pole wound rotor synchronous machine

a) Magnetic equation

$$\varphi_{sabc} = L_s i_{sabc} + M_{sf} i_f \quad 5.13$$

$$\varphi_f = M_{sf}^T i_{sabc} + L_f i_f \quad 5.14$$

$$\varphi_{s\alpha\beta} = L_{s\alpha\beta} i_{\alpha\beta} + M_f i_{f\alpha\beta} \quad 5.15$$

$$L_{s\alpha\beta} = \begin{bmatrix} L_0 + L_2 cos\theta & L_2 sin\theta \\ L_2 sin\theta & L_0 - L_2 cos\theta \end{bmatrix} \quad 5.16$$

$$M_f = \sqrt{\frac{3}{2}} M_0 \quad 5.17$$

$$i_{f\alpha\beta} = i_f \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} \quad 5.18$$

$$\varphi_{f\alpha\beta} = \varphi_f \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} = M_{sf}^T i_{sabc} \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} + L_f i_{f\alpha\beta} \quad 5.19$$

$$\varphi_{f\alpha\beta} = M_{f\alpha\beta} i_{s\alpha\beta} + L_f i_{f\alpha\beta} \quad 5.20$$

$$M_{f\alpha\beta}(\theta) = M_f \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ cos\theta sin\theta & sin^2\theta \end{bmatrix} \quad 5.21$$

$$M_{f\alpha\beta} = \frac{M_f}{2} \begin{bmatrix} 1 + cos2\theta & sin2\theta \\ sin2\theta & 1 - cos2\theta \end{bmatrix} \quad 5.22$$

$$\varphi_{s\alpha} = (L_0 + L_2 cos2\theta) i_{s\alpha} + L_2 i_{s\beta} sin2\theta + M_f i_{f\alpha} \quad 5.23$$

$$\varphi_{s\beta} = (L_0 - L_2 cos2\theta) i_{s\beta} + L_2 i_{s\alpha} sin2\theta + M_f i_{f\beta} \quad 5.24$$

$$\varphi_{f\alpha} = L_f i_{f\alpha} + M_f i_{s\alpha} cos^2\theta + M_f i_{s\beta} sin\theta cos\theta \quad 5.25$$

$$\varphi_{f\beta} = L_f i_{f\beta} + M_f i_{s\alpha} cos\theta sin\theta + M_f i_{s\beta} sin^2\theta \quad 5.26$$

Flux equations can be rewriten in th ematrix form as

$$\begin{bmatrix} \varphi_{s\alpha\beta} \\ \varphi_{f\alpha\beta} \end{bmatrix} = \begin{bmatrix} L_{s\alpha\beta} & M_f \mathrm{I}_2 \\ M_{s\alpha\beta} & L_f \mathrm{I}_2 \end{bmatrix} \begin{bmatrix} i_{s\alpha\beta} \\ i_{f\alpha\beta} \end{bmatrix} \quad 5.27$$

b) Electric equation

The final electric equation will be

$$v_{s\alpha\beta} = R_s i_{s\alpha\beta} + \frac{d\varphi_{s\alpha\beta}}{dt} \quad 5.28$$

$$v_{f\alpha\beta} = R_f i_{f\alpha\beta} + \frac{d\varphi_{f\alpha\beta}}{dt} \quad 5.29$$

$$v_{s\alpha\beta} = R_s i_{s\alpha\beta} + L_{s\alpha\beta} \frac{di_{s\alpha\beta}}{dt} + M_f \frac{di_{f\alpha\beta}}{dt} + \omega \frac{dL_{s\alpha\beta}}{d\theta} i_{s\alpha\beta} \quad 5.30$$

$$v_{f\alpha\beta} = R_f i_{f\alpha\beta} + M_{f\alpha\beta} \frac{di_{s\alpha\beta}}{dt} + L_f \frac{di_{f\alpha\beta}}{dt} + \omega \frac{dM_{f\alpha\beta}}{d\theta} i_{s\alpha\beta} \quad 5.31$$

C) Electromagnetique torque

$$T_e = \frac{p}{2} i_{s\alpha\beta}^T \frac{dL_{s\alpha\beta}}{d\theta} i_{s\alpha\beta} + \frac{p}{2} i_{f\alpha\beta}^T \frac{dM_{f\alpha\beta}}{d\theta} i_{f\alpha\beta} \quad 5.32$$

$$T_e = pM_f i_f \left(i_{s\beta} cos\theta - i_{s\alpha} sin\theta\right) - pL_2 \left[(i_{s\alpha}^2 - i_{sb}^2) sin2\theta - 2 i_{s\alpha} i_{s\beta} cos2\theta\right] \quad 5.33$$

5.3.3 Park frame model

a) Magnetic equation

$$\varphi_{sdq} = L_{sdq} i_{\alpha\beta} + M_f i_{fdq} \quad 5.34$$

The quadrature component of the rotor current vector in the frame dq is zero ($i_{fq} = 0$), whilethe direct axis component is equal toexcitation current $i_{fd} = i_f$

$$\varphi_{fd} = M_f i_{sd} + L_f i_f \quad 5.35$$

$$\varphi_{fq} = 0 \quad 5.36$$

The stator and rotor flux of the synchronous machine in the Park frame are written in the matrix form:

$$\begin{bmatrix} \varphi_{sd} \\ \varphi_{sq} \\ \varphi_f \end{bmatrix} = \begin{bmatrix} L_d & 0 & M_f \\ 0 & L_q & 0 \\ M_f & 0 & L_f \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_f \end{bmatrix} \quad 5.37$$

b) Electric equation

Appling park transformation on ….and subtitling the flux in equation …we will finally have the voltage equations park frame which are written as follows:

$$v_{sd} = R_s i_{sd} + L_d \frac{di_{sd}}{dt} + M_f \frac{di_f}{dt} - \omega L_q i_{sq} \quad 5.38$$

$$v_{sq} = R_s i_{sq} + L_q \frac{di_{sq}}{dt} + \omega\left(L_d i_{sd} + M_f i_f\right) \quad 5.39$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt} + M_f \frac{di_{sd}}{dt} \quad 5.40$$

c) Electromagnetic torque

The electrical power supplied to the terminals of the synchronous machine is:

$$P_{in} = \left(v_{sd} i_{sd} + v_{sq} i_{sq} + v_f i_f\right) \quad 5.41$$

$$P_{in} = R_s\left(i_{sd}^2 + i_{sq}^2\right) + R_f i_f^2 + i_{sd} \frac{d\varphi_{sd}}{dt} + i_{sq} \frac{d\varphi_{sq}}{+dt} + i_f \frac{d\varphi_f}{dt} + p\left(\varphi_{sd} i_{sq} - \varphi_{sq} i_{sq}\right)\Omega \quad 5.42$$

The last term in the equation above represents the mechanical power transmitted to the shaft of the machine

$$T_e = \frac{P_{mec}}{\Omega} = p(\varphi_{sd} i_{sq} - \varphi_{sq} i_{sq}) \quad 5.43$$

$$T_e = [(L_d - L_q) i_{sd} + M_f i_f] i_{sq} \quad 5.44$$

d) State space representation

$$\frac{di_{sd}}{dt} = \frac{1}{\sigma_d L_d}[v_{sd} - R_s i_{sd} + \omega L_q i_{sq} - k_f(v_f - R_f i_f)]$$

$$\frac{di_{sq}}{dt} = \frac{1}{L_q}[v_{sq} - R_s i_{sq} - (L_d i_{sd} + M_f i_f)\omega] \quad 5.45$$

$$\frac{di_{sd}}{dt} = \frac{1}{\sigma_d L_f}[v_f - R_f i_f - k_d(v_{sd} - R_s i_{sd} + \omega L_q i_{sq})]$$

$$\sigma_d = 1 - k_d k_f \; ; k_d = \frac{M_f}{L_d} \; ; k_d = \frac{M_f}{L_f}.$$

5.4 Permanent magnet synchronous machine modeling

The structure of the permanent magnet synchronous machine has a three-phase winding at the stator. Rotor excitation is created by permanent magnets at the rotor.

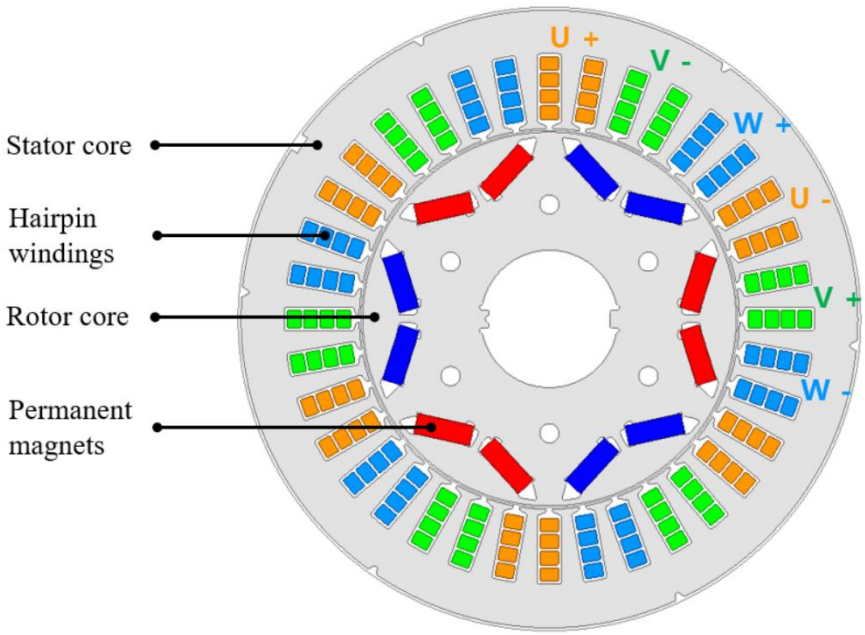


Figure 5-3Presentation of PMSM

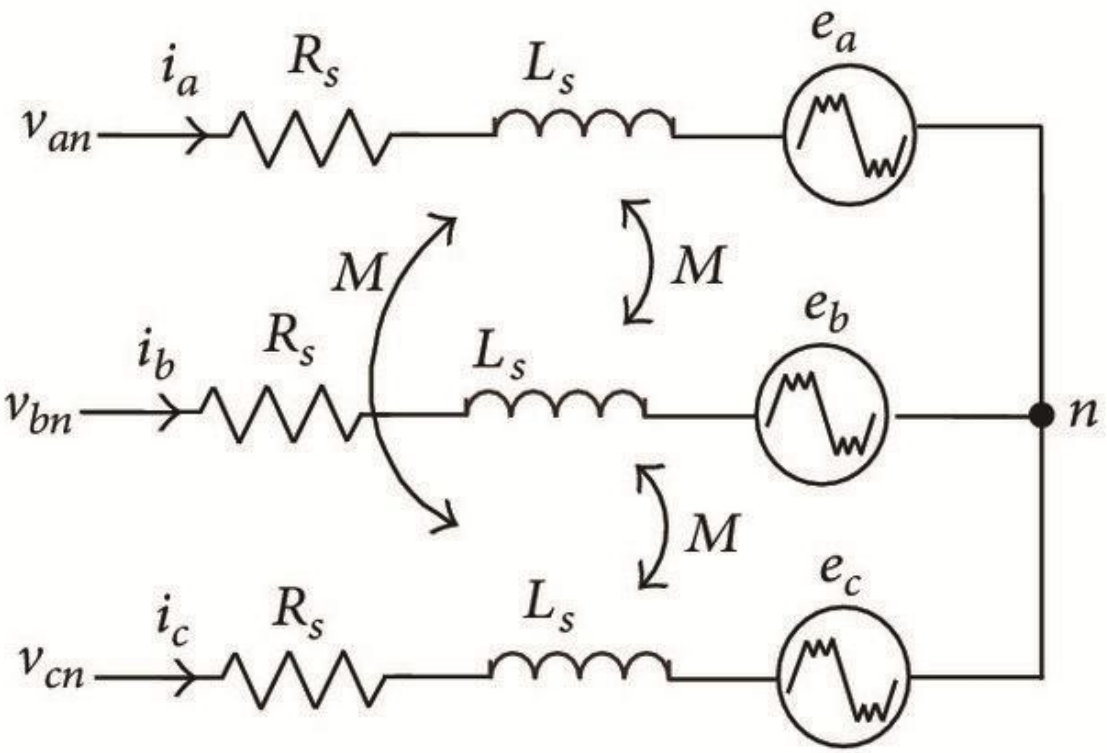


Figure 5.4 Equivalent PMSM in natural frame

We can derive the models of other synchronous machines from that of the wound rotor synchronous machine with salient poles discussed above with the following suggestions:

- ✓ we consider that the rotor flux and current are constant. The permanent magnet flux φr is equivalent to the rotor flux induced in the winding of the stator:

$$\frac{d\varphi_f}{dt} = 0, \varphi_r = M_f i_f.$$

5.4.1Natural frame model

The stator voltages are given as a function of the current and the flux as follows.

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{as} \\ \varphi_{bs} \\ \varphi_{cs} \end{bmatrix} \quad 5.46$$

$v_{a,b,c}$Stator voltage

$i_{a,b,c}$Stator current

$\varphi_{a,b,c}$ flux

$R_s$stator winding resistance

So the stator flux

$$\varphi_{sabc} = L_s i_{sabc} + \begin{bmatrix} \varphi_{ra} \\ \varphi_{rb} \\ \varphi_c \end{bmatrix} \quad 5.47$$

$$L_s = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} \\ M_{ab} & L_{bb} & M_{bc} \\ M_{ac} & M_{bc} & L_{bb} \end{bmatrix} \quad 5.48$$

$$\begin{bmatrix} \varphi_{as} \\ \varphi_{bs} \\ \varphi_{cs} \end{bmatrix} = \begin{bmatrix} L_{aa} & M_{ab} & M_{ac} \\ M_{ab} & L_{bb} & M_{bc} \\ M_{ac} & M_{bc} & L_{bb} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} \varphi_r \cos(\theta_r) \\ \varphi_r \cos(\theta_r - \frac{2\pi}{3}) \\ \varphi_r \cos(\theta_r - \frac{4\pi}{3}) \end{bmatrix} \quad 5.49$$

LRepresents self inductance

MIs the mutual between the stator winding

φIs the flow generated by the rotor.

θ_r Rotor phase shift

5.4.2Model in Park's coordinate system

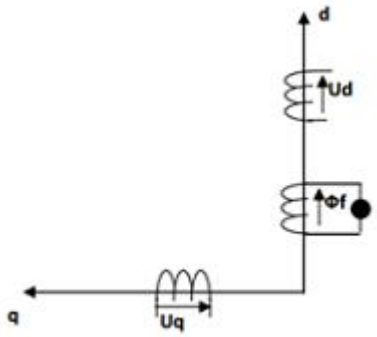


Figure 5.5equivalent scheme of PMSM in rotating frame

The stator voltages in the Park reference frame can be written in the following matrix form

$$\begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix} = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{ds} \\ \varphi_{qs} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_r \\ \omega_r & 0 \end{bmatrix} \begin{bmatrix} \varphi_{ds} \\ \varphi_{qs} \end{bmatrix} \quad 5.50$$

Therefore the flux equation will be:

$$\begin{bmatrix} \varphi_{ds} \\ \varphi_{qs} \end{bmatrix} = \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} \varphi_r \\ 0 \end{bmatrix} \quad 5.51$$

By combining equation (5.50) and (5.51) we obtain the dynamics of the stator voltage in the Park frame

$$\begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix} = \begin{bmatrix} R_s & -\omega_r L_{qs} \\ \omega_r L_{ds} & R_s \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} L_{ds} & 0 \\ 0 & L_{qs} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_r \varphi_r \end{bmatrix} \quad 5.52$$

From equation (4-52) we can derive the dynamics of the stator current as a result.

$$\frac{d}{dt} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_{ds}} & \frac{\omega_r L_{qs}}{L_{ds}} \\ -\frac{\omega_r L_{ds}}{L_{qs}} & -\frac{R_s}{L_{qs}} \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{ds}} & 0 \\ 0 & \frac{1}{L_{qs}} \end{bmatrix} \begin{bmatrix} v_{ds} \\ v_{qs} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\omega_r \varphi_r}{L_{qs}} \end{bmatrix} \quad 5.53$$

5.4.3Electromagnetic Torque

The relationship of power as a function of voltages and stator currents is given by

$$S_s = P_s + jQ_s = \frac{3}{2}\left(v_{ds} + jv_{qs}\right)(i_{ds} - ji_{qs}) \quad 5.54$$

After developing (5.57) and separation of the real and imaginary part, the active and reactive stator power is written in the form.

$$P_s = \frac{3}{2}(v_{ds} i_{ds} + v_{qs} i_{qs}) \quad 5.56$$

$$Q_s = \frac{3}{2}(v_{qs} i_{ds} - v_{ds} i_{qs}) \quad 5.57$$

The relationship of the mechanical power of the PMSM as a function of speed and torque is given by

$$P_m = T_m \omega_m = \frac{T_e \omega_r}{p} \quad 5.58$$

Hence

$$T_e = \frac{P_m p}{\omega_r} \quad 5.59$$

Electromagnetic torque as a function of current and flux

$$T_e = \frac{3p}{2}(\varphi_{ds} i_{qs} - \varphi_{qs} i_{ds}) \quad 5.60$$

Finaly the torque can be rewritten as:

$$T_e = \frac{3p}{2}\left[\varphi_r i_{qs} + (L_{ds} - L_{qs}) i_{ds} i_{qs}\right] \quad 5.61$$

5.5mecanique Equation

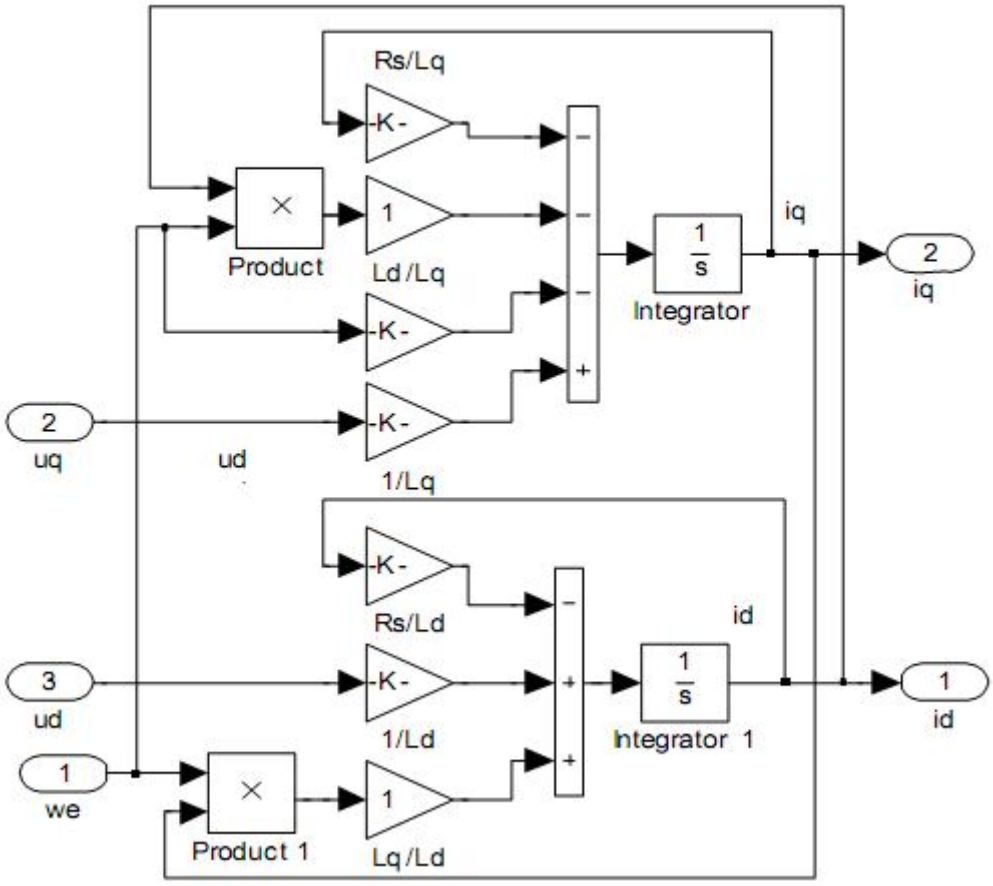
The dynamics of the rotation speed is given by

$$J_m \frac{d}{dt} \omega_m + B_m \omega_m = T_e - T_m \quad 5.62$$

$$\omega_m = \frac{1}{p}\omega_r \qquad 5.63$$

Hence

$$\frac{d}{dt}\omega_r = \frac{p}{J_m}(T_e - T_m) - \frac{B_m}{J_m}\omega_r \qquad 5.64$$



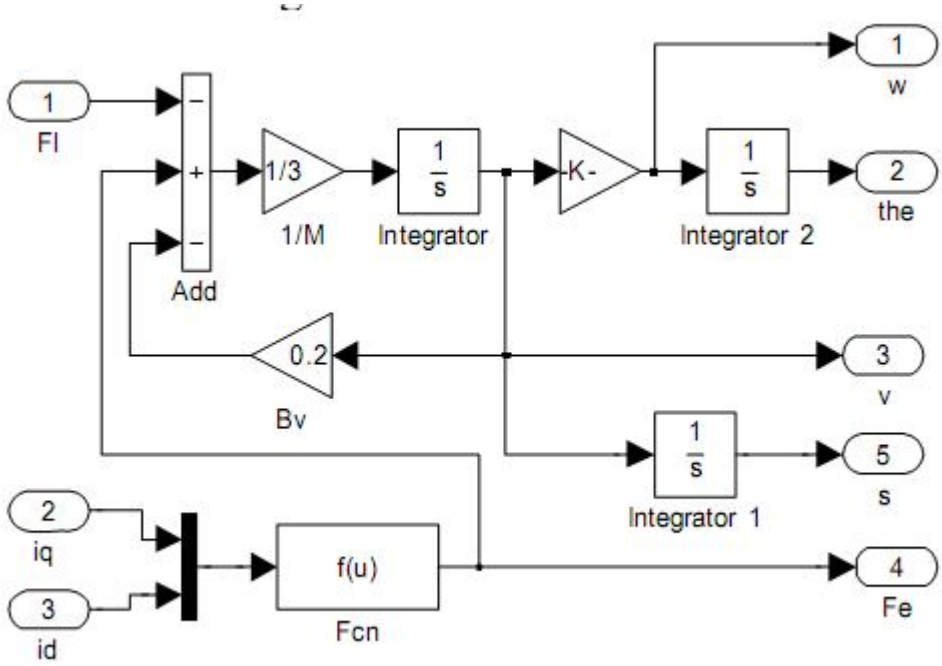


Figure 5.8 simulation scheme of PMSM

5.6 Conclusion

Synchronous machines find applications in various fields, such as motors and generators. The reasons for modeling a synchronous machine can be categorized into two main objectives: gaining a deeper understanding of its complex electromagnetic behavior and for simulation or control purposes. The modeling and analysis of synchronous machines have posed a consistent challenge. There is a significant body of literature available on synchronous machines within the field of power engineering. Research on synchronous machines typically relies on intuition, experience, and years of practical knowledge. Numerous software tools are available for analyzing, planning, optimizing, and simulating steady states and transitional conditions, featuring pre-programmed components and user interfaces that facilitate the modeling of synchronous machines and other power system elements. These tools have been utilized for exploring synchronous machines in various applications.

Chapter 6

Induction Machine Modeling

6.1 Introduction

The three-phase Induction Motor (IM) is the predominant machine in the industrial sector, comprising 85% of all motors utilized. Known for its cost-effectiveness, lightweight construction, and durability compared to DC motors, the IM is widely favored in various industrial applications. Referred to as an "asynchronous machine," this chapter focuses on detailing the model of the IM. Operating efficiently when directly connected to the electric network, the IM is valued for its affordability, reliability, and resilience. The mathematical model of the IM will be elucidated in this chapter, with a specific emphasis on deriving a simplified two-phase model. Two reference frames will be introduced for further analysis.

- the fixed stator reference frame (αβ);
- the rotating synchronous reference frame with the rotor flux (dq).

6.2 Natural frame Model of induction machine

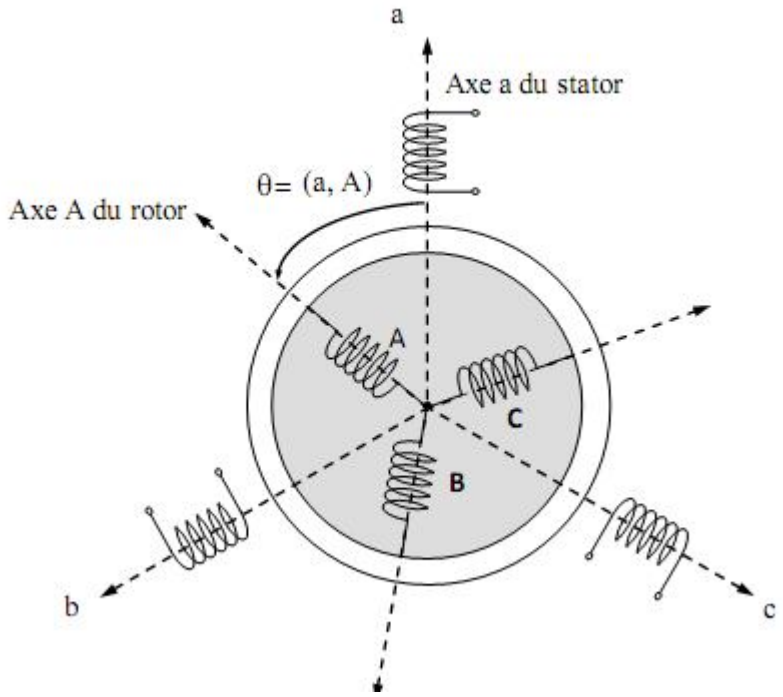


Figure 6.1. Representation of three phase induction machine

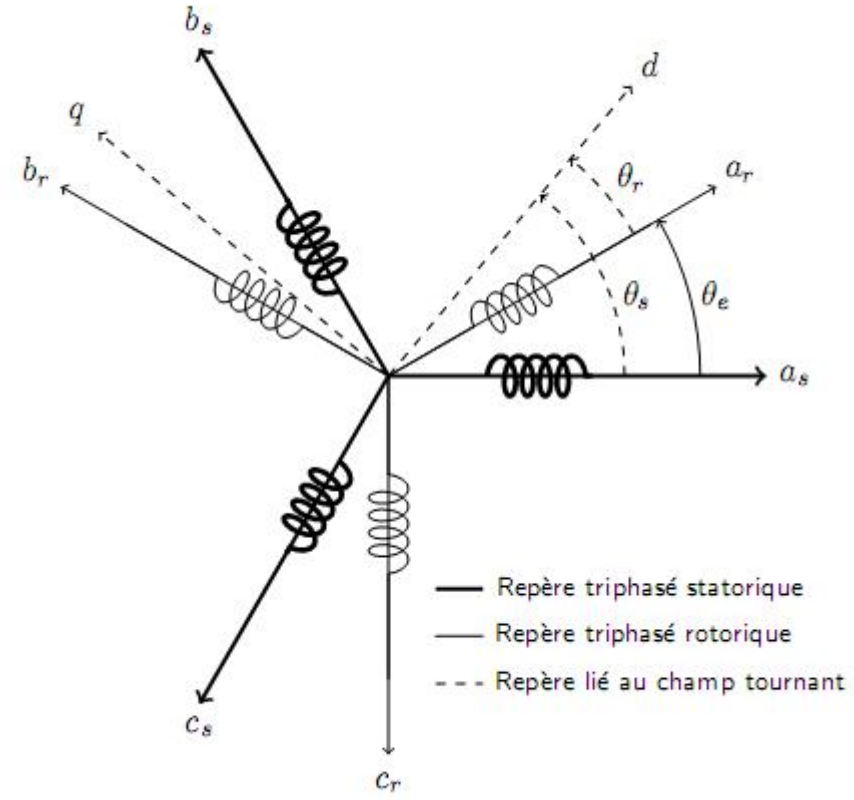


Figure 6.2. Electric angle definition

The voltage equations of the stator phases and the rotor phases are:

$$\begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{sa} \\ \varphi_{sb} \\ \varphi_{sc} \end{bmatrix} \quad 6.1$$

$$\begin{bmatrix} v_{ra} \\ v_{rb} \\ v_{rc} \end{bmatrix} = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_{ra} \\ \varphi_{rb} \\ \varphi_{rc} \end{bmatrix} \quad 6.2$$

Vsa, Vsb, Vsc : Stator voltages.

i_{sa}, i_{sb}, i_{sc}: . stator current

φ_{sa}, φ_{sb}, φ_{sc} : total stator flux.

v_{ra}, v_{rb}, v_{rc}:Tensions rotoriques

i_{ra}, i_{rb}, i_{ra}: Rotor Courants

φ_{ra}, φ_{rb}, φ_{rc} : Rotor Flux

R_s: per phase stator resistance.

R_r: per phase rotor resistance.

The simplified assumptions cited previously lead to linear relationships between the flows and the currents of the asynchronous machine, these relationships are written as follows:

$$\begin{bmatrix}\varphi_{sa}\\ \varphi_{sb}\\ \varphi_{sc}\end{bmatrix} = [L_s]\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} + [M_{sr}]\begin{bmatrix}i_{ra}\\ i_{rb}\\ i_{rc}\end{bmatrix} \quad 6.3$$

$$\begin{bmatrix}\varphi_{ra}\\ \varphi_{rb}\\ \varphi_{rc}\end{bmatrix} = [L_r]\begin{bmatrix}i_{ra}\\ i_{rb}\\ i_{rc}\end{bmatrix} + [M_{rs}]\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{sc}\end{bmatrix} \quad 6.4$$

$$[M_{sr}] = [M_{rs}]^T$$

Where

$$[L_s] = \begin{bmatrix}l_s & M_s & M_s\\ M_s & l_s & M_s\\ M_s & M_s & l_s\end{bmatrix} \quad 6.5$$

$$[L_r] = \begin{bmatrix}l_r & M_r & M_r\\ M_r & l_r & M_r\\ M_r & M_r & l_r\end{bmatrix} \quad 6.6$$

$$[M_{sr}] = M_0\begin{bmatrix}\cos(\theta) & \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3})\\ \cos(\theta+\frac{2\pi}{3}) & \cos(\theta) & \cos(\theta-\frac{2\pi}{3})\\ \cos(\theta-\frac{2\pi}{3}) & \cos(\theta+\frac{2\pi}{3}) & \cos(\theta)\end{bmatrix} \quad 6.7$$

l_s: Stator self Inductance

l_r : rotor self inductance.

Ms: mutual Inductance between stator phases .

Mr : mutual Inductance between rotor phases .

θ :Electrique angle

$$v_{sabc} = R_s i_{sabc} + L_s\frac{di_{sabc}}{dt} + \frac{d}{dt}(M_{sr}i_{rabc}) \quad 6.8$$

$$v_{sabc} = R_s i_{sabc} + L_s\frac{di_{sabc}}{dt} + M_{sr}\frac{di_{rabc}}{dt} + \omega\frac{dM_{sr}}{d\theta}i_{rabc} \quad 6.9$$

$$v_{rabc} = R_r i_{rabc} + L_r\frac{di_{rabc}}{dt} + \frac{d}{dt}(M_{sr}^T i_{sabc}) \quad 6.10$$

$$v_{rabc} = R_s i_{rabc} + L_r \frac{di_{rabc}}{dt} + M_{sr} \frac{di_{rabc}}{dt} + \omega \frac{dM_{sr}^T}{dt} i_{sabc} = 0 \quad 6.11$$

6.3 Arbitrary reference frame Model

$$v_s = R_s i_s + \frac{d}{dt}\varphi_s + \omega_a J_2 \varphi_s \quad 6.12$$

$$0 = R_r i_r + \frac{d}{dt}\varphi_r + (\omega_a - \omega_e) J_2 \varphi_r \quad 6.13$$

J_2 : (π/2) Rotation matrix

$$J_2 = P\left(\frac{\pi}{2}\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad 6.14$$

6.3.1 Model in αβ frame

The stationary frame does not rotate inspace. Therefore, the equations of the machine in this frame can be deduced by replacing.

$$\omega_a = 0$$

That gives the model Bellow :

$$v_s = R_s i_s + \frac{d}{dt}\varphi_s \quad 6.15$$

$$0 = R_r i_r + \frac{d}{dt}\varphi_r - \omega_e J_2 \varphi_r \text{v} \quad 6.16$$

6.3.2 Model in dq frame

The synchronous reference is linked to the rotating field, its pulsation is equal to the electrical pulsation of the stator voltages:

$$\omega_a = \omega_s$$

The model of the machine in this reference is written as follows:

$$v_s = R_s i_s + \frac{d}{dt}\varphi_s + \omega_s J_2 \varphi_s \quad 6.17$$

$$0 = R_r i_r + \frac{d}{dt}\varphi_r - \omega_r J_2 \varphi_r \quad 6.18$$

$$\omega_r = \omega_s - \omega_e \quad 6.19$$

6.4 rotor current free model

$$i_r = \frac{1}{L_r}\varphi_r - \frac{M}{L_r} i_s \quad 6.20$$

Therefore we can write the stator flux as follows:

$$\varphi_s = \sigma L_s i_s + k_r \varphi_r \quad 6.21$$

k_r: Rotor coupling coefficient

$$k_r = \frac{L_m}{L_r} \quad 6.22$$

k_s: Stator coupling coefficient

$$k_r = \frac{L_m}{s} \quad 6.23$$

σ: Total leakage coefficient

$$\sigma = 1 - k_s k_r \quad 6.24$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \quad 6.25$$

$$\frac{d}{dt}\varphi_r = \frac{L_m}{\tau_r} i_s - \left[\frac{1}{\tau_r} + j(\omega_a - \omega_r)\right] \varphi_r + v_r \quad 6.26$$

τ_r: The rotor time constant

$$\tau_r = \frac{L_r}{L_r} \quad 6.27$$

From the equations above, the current dynamics can be deduced as:

$$\frac{d}{dt} i_s = \frac{1}{\sigma L_s}\left[-(R_s + j\omega_k \sigma L_s) i_s - j\omega_k \varphi_r - k_r \frac{d}{dt}\varphi_r + v_s\right] \quad 6.28$$

Replacing $\frac{d}{dt}\varphi_r$ by its value in 6.26 yield

$$\frac{d}{dt} i_s = -\left(\frac{1}{\tau_\sigma} + j\omega_k\right) i_s + \frac{k_r}{\sigma L_s}\left(\frac{1}{\tau_r} - j\omega_r\right) \varphi_r + \frac{1}{\sigma L_s} v_s - \frac{k_r}{\sigma L_s} v_r \quad 6.29$$

τ_σ : Stator transient constant time

$$\sigma_\tau = \frac{\sigma L_s}{r_\sigma} \quad 6.30$$

r_σ: Equivalent resistance.

$$r_\sigma = R_s + k_r^2 R_r \quad 6.31$$

The final model is given by

$$\frac{d}{dt}\begin{bmatrix} i_s \\ \varphi_r \end{bmatrix} = A \begin{bmatrix} i_s \\ \varphi_r \end{bmatrix} + B \begin{bmatrix} v_s \\ v_r \end{bmatrix} \text{ et } \varphi_s = C \begin{bmatrix} i_s \\ \varphi_r \end{bmatrix} \quad 6.32$$

$$A = \begin{bmatrix} -\left(\frac{1}{\sigma_\tau} + j\omega_k\right) & \frac{k_r}{\sigma L_s}\left(\frac{1}{\tau_r} - j\omega_r\right) \\ \frac{L_m}{\tau_r} & -\left(\frac{1}{\tau_r} + j(\omega_k - \omega_r)\right) \end{bmatrix}, B = \begin{bmatrix} \frac{1}{\sigma L_s} & -\frac{k_r}{\sigma L_s} \\ 0 & 1 \end{bmatrix}, C = [\sigma L_s \quad k_r].$$

6.4.1 Park reference frame model

Frome equation 6.29 we have :

$$\frac{d}{dt}(i_{ds} + ji_{qs}) = -\left(\frac{1}{\tau_\sigma} + j\omega_s\right)(i_{ds} + ji_{qs}) + \frac{k_r}{\sigma L_s}\left(\frac{1}{\tau_r} - j\omega_r\right)(\varphi_{dr} + j\varphi_{qr}) + \frac{1}{\sigma L_s}(v_{ds} + jv_{qs}) - \frac{k_r}{\sigma L_s}(v_{dr} + jv_{qr}) \quad 6.33$$

By separating real and imaginary components from the above expression, we have

$$\frac{di_{ds}}{dt} = -\frac{1}{\tau_\sigma} i_{ds} + \omega_s i_{qs} + \frac{k_r}{\sigma L_s \tau_r}\varphi_{dr} + \frac{k_r \omega_r}{\sigma L_s}\varphi_{qr} + \frac{1}{\sigma L_s} v_{ds} - \frac{k_r}{\sigma L_s} v_{dr} \quad 6.34$$

$$\frac{di_{qs}}{dt} = -\frac{1}{\tau_\sigma} i_{qs} - \omega_s i_{ds} + \frac{k_r}{\sigma L_s \tau_r}\varphi_{qr} - \frac{k_r \omega_r}{\sigma L_s}\varphi_{dr} + \frac{1}{\sigma L_s} v_{qs} - \frac{k_r}{\sigma L_s} v_{qr} \quad 6.35$$

Similarly, the dq-frame rotor flux dynamics are obtained below by solving

$$\frac{d\varphi_{dr}}{dt} = \frac{L_m}{\tau_r} i_{ds} - \frac{1}{\tau_r}\varphi_{dr} + (\omega_s - \omega_r)\varphi_{qr} + v_{dr} \quad 6.36$$

$$\frac{d\varphi_{qr}}{dt} = \frac{L_m}{\tau_r} i_{qs} - \frac{1}{\tau_r}\varphi_{qr} - (\omega_s - \omega_r)\varphi_{dr} + v_{qr} \quad 6.37$$

Finally, dq-frame stator flux-linkages are derived:

$$\varphi_{ds} = \sigma L_s i_{ds} + k_r \varphi_{dr} \quad 6.38$$

$$\varphi_{qs} = \sigma L_s i_{qs} + k_r \varphi_{qr} \quad 6.39$$

$$\frac{d}{dt}\begin{bmatrix} i_{ds} \\ i_{qs} \\ \varphi_{dr} \\ \varphi_{qs} \end{bmatrix} = A \begin{bmatrix} i_{ds} \\ i_{qs} \\ \varphi_{dr} \\ \varphi_{qs} \end{bmatrix} + B \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qs} \end{bmatrix} \text{ et } \begin{bmatrix} \varphi_{ds} \\ \varphi_{qs} \end{bmatrix} = C \begin{bmatrix} i_{ds} \\ i_{qs} \\ \varphi_{dr} \\ \varphi_{qs} \end{bmatrix} \quad 6.40$$

$$A = \begin{bmatrix} \frac{-1}{\tau_\sigma} & \omega_s & \frac{k_r \omega_r}{\sigma L_s \tau_r} & \frac{k_r \omega_r}{\sigma L_s} \\ -\omega_s & \frac{-1}{\tau_\sigma} & -\frac{k_r \omega_r}{\sigma L_s} & \frac{k_r}{\sigma L_s \tau_r} \\ \frac{L_m}{\tau_r} & 0 & \frac{-1}{\tau_r} & (\omega_s - \omega_r) \\ 0 & \frac{L_m}{\tau_r} & -(\omega_s - \omega_r) & \frac{-1}{\tau_r} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & -\frac{k_r}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & -\frac{k_r}{\sigma L_s} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} \sigma L_s & 0 & k_r & 0 \\ 0 & \sigma L_s & 0 & k_r \end{bmatrix}$$

6.5 Doubly fed induction machine modeling

6.5.1 Natural frame Model

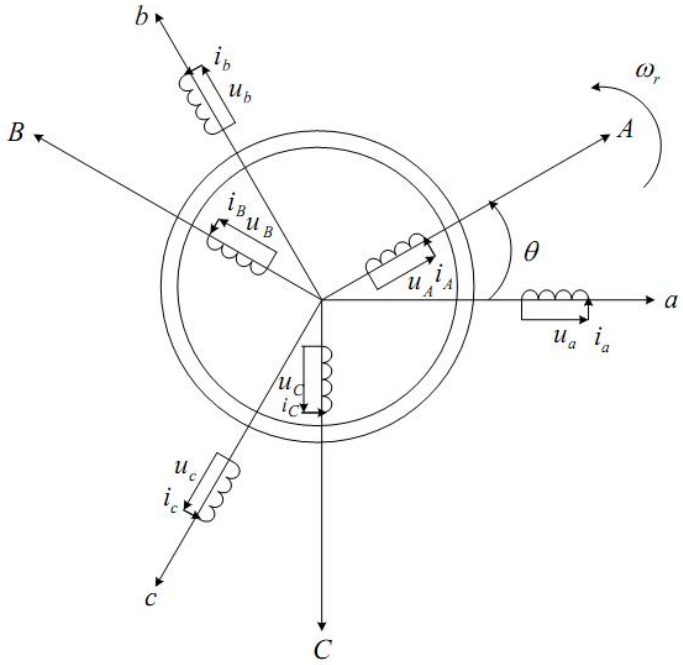


Figure 6.3 Simplified circuit of DFIM

The stator voltages can be expressed by:

$$\begin{cases} v_{sa} = \frac{d\psi_{sa}}{dt} + i_{sa}R_s \\ v_{sb} = \frac{d\psi_{sb}}{dt} + i_{sb}R_s \\ v_{sc} = \frac{d\psi_{sc}}{dt} + i_{sc}R_s \end{cases} \quad 6.41$$

And the rotor voltages by:

$$\begin{cases} v_{ra} = \frac{d\psi_{ra}}{dt} + i_{ra}R_r \\ v_{rb} = \frac{d\psi_{rb}}{dt} + i_{rb}R_r \\ v_{rc} = \frac{d\psi_{rc}}{dt} + i_{rc}R_r \end{cases} \quad 6.42$$

6.5.2 dq frame model of DFIM

$$\begin{cases} v_{sd} = R_s i_{sd} + \frac{d\psi_{sd}}{dt} - \omega_s \psi_{sq} \\ v_{sq} = R_s i_{sq} + \frac{d\psi_{sq}}{dt} - \omega_s \psi_{sd} \end{cases} \quad 6.43$$

$$\begin{cases} v_{rd} = R_r i_{rd} + \frac{d\psi_{rd}}{dt} - (\omega_s - \omega_r)\psi_{sq} \\ v_{rq} = R_r i_{rq} + \frac{d\psi_{rq}}{dt} - (\omega_s - \omega_r)\psi_{sd} \end{cases} \quad 6.44$$

$$\begin{cases}\psi_{sd} = L_s I_{sd} + L_m I_{rd} \\ \psi_{sq} = L_s I_{sq} + L_m I_{rq} \\ \psi_{rd} = L_r I_{rd} + L_m I_{sd} \\ \psi_{rq} = L_r I_{rq} + L_m I_{sq}\end{cases} \quad 6.45$$

The electromagnetic torque can be deduced as:

$$T_{em} = p(\psi_{sd} i_{sq} - \psi_{sq} i_{sd}) = L_m(i_{qs} i_{dr} - i_{ds} i_{qr}) = p(\psi_{dr} i_{qr} - \psi_{qr} i_{dr}). \quad 6.46$$

The stator active and reactive powers are:

$$\begin{cases}P_s = (v_{sd} i_{sd} - v_{sq} i_{sq}) \\ Q_s = (v_{sq} i_{sd} - v_{sd} i_{sq})\end{cases} \quad 6.47$$

The rotor active and reactive powers are:

$$\begin{cases}P_r = (v_{rd} I_{rd} - v_{rq} I_{rq}) \\ Q_r = (v_{rq} i_{rd} - v_{rd} i_{rq})\end{cases} \quad 6.48$$

Supposing that the stator resistance Rs is negligible and that the stator flux is constant and oriented along the d axis, we deduce:

$$\begin{cases}\psi_{sd} = \psi_s \\ \psi_{sq} = 0\end{cases} \quad 6.49$$

Hence

$$\begin{cases}\frac{d\psi_{sd}}{dt} = 0 \\ \frac{d\psi_{sq}}{dt} = 0\end{cases} \quad 6.50$$

$$\begin{cases}\emptyset_s = L_s I_{sd} + L_m I_{rd} \\ 0 = L_s I_{sq} + L_m I_{rq}\end{cases} \quad 6.51$$

$$\begin{cases}\psi_{rd} = L_r I_{rd} + \frac{L_m}{L_s}\psi_{sd} \\ \psi_{rq} = L_r I_{rq}\end{cases} \quad 6.52$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}$$

the direct and quadrature components of the voltages are written as:

$$\begin{cases}v_{sd} = R_s I_{sd} + \frac{d\emptyset_{sd}}{dt} = 0 \\ v_{sq} = R_s I_{sq} + \omega_s \emptyset_{sd} = \omega_s \emptyset_s\end{cases} \quad 6.53$$

$$\begin{cases} v_{rd} = R_r I_{rd} + \sigma L_r \frac{dI_{rd}}{dt} - v_q \\ v_{rq} = R_r I_{rq} + \sigma L_r \frac{dI_{rq}}{dt} + v_d + e_\emptyset \end{cases} \quad 6.54$$

$$\begin{cases} v_q = -\omega_r \sigma L_r I_{rq} \\ v_d = \omega_r \sigma L_r I_{rd} \end{cases} \quad 6.55$$

$$e_\emptyset = \omega_r \frac{L_m}{L_s} \emptyset_s \quad 6.56$$

From the equations(6.49)and (6.50), stator currents can be expressed in terms of rotor currents as follows:

$$\begin{cases} i_{sd} = \frac{\emptyset_s}{L_s} - \frac{L_m}{L_s} i_{rd} \\ i_{sq} = -\frac{L_m}{L_s} i_{rq} \end{cases} \quad 6.57$$

L'équation du couple devient

$$T_{em} = -p\emptyset_s \frac{L_m}{L_s} i_{rq} \quad 6.58$$

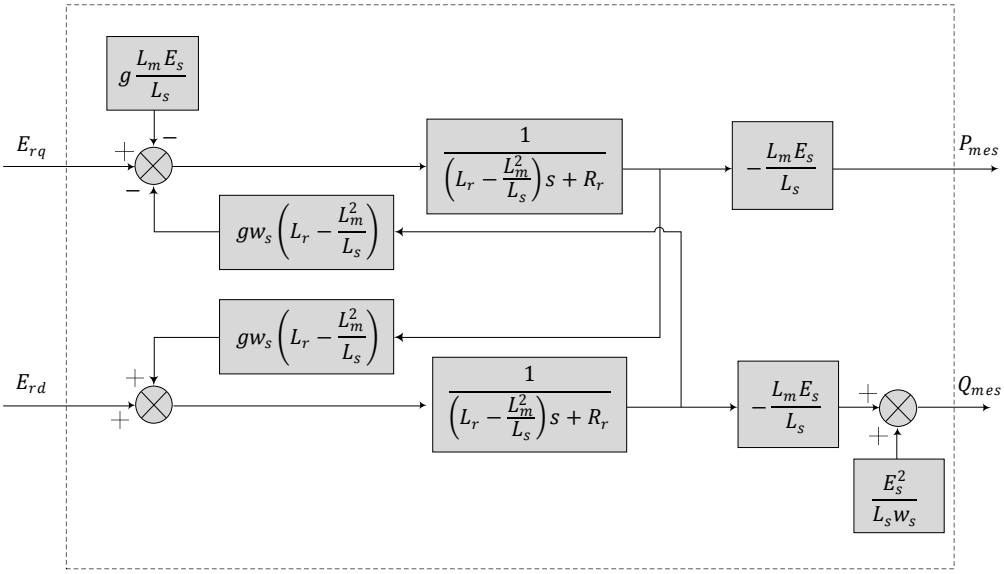


Figure 6.4 Schéma bloc du modèle simplifié de la MADA.

The expressions of stator and rotor powers are:

$$\begin{cases} P_s = \frac{3}{2}(v_{sd}i_{sd} + v_{sq}i_{sq}) \\ Q_s = \frac{3}{2}(v_{sq}i_{sd} - v_{sd}i_{sq}) \end{cases} \quad 6.59$$

$$\begin{cases} P_r = \frac{3}{2}(v_{rd}i_{rd} + v_{rq}i_{rq}) \\ Q_r = \frac{3}{2}(v_{rq}i_{rd} - v_{rd}i_{rq}) \end{cases} \quad 6.60$$

Taking into account equations (6.49) and (6.50) we obtain:

$$\begin{cases} P_s = \frac{3}{2}v_{sq}i_{sq} \\ Q_s = \frac{3}{2}v_{sq}i_{sd} \end{cases} \quad 6.61$$

By replacing the stator currents by their expressions as a function of the rotor currents, we obtain the expressions of the stator powers as a function of the rotor currents:

$$\begin{cases} P_s = \frac{3L_m}{2L_s}v_{sq}i_{rq} \\ Q_s = -(\frac{3L_m}{2L_s}v_{sq}i_{rd} + v_{sq}\frac{3v_{sq}}{2l_s w_s}) \end{cases} \quad 6.62$$

$$\begin{cases} P_s = -v_s\frac{L_m}{L_s}I_{rq} \\ Q_s = \frac{v_s\emptyset_s}{L_s} - \frac{v_s L_m}{L_s}i_{rd} \end{cases} \quad 6.63$$

6.6 conclusion

The benefits of induction machines are widely recognized when compared to other types of machines. Therefore, it is crucial to study induction machines using precise models and to substitute costly test benches and equipment with more efficient and cost-effective testing procedures.

This chapter introduced a robust mathematical model for induction machines in the abc reference frame, and subsequently, through the application of Park's transformation, the machine model was transformed into the dq reference frame to streamline calculations during simulation.

Refrences

Yildiz, A. B. (2012). *Electrical equivalent circuit based modeling and analysis of direct current motors. International Journal of Electrical Power & Energy Systems, 43(1), 1043–1047.* doi:10.1016/j.ijepes.2012.06.063.

The Research on Fuzzy PID Control of the Permanent Magnet Linear Synchronous MotorDecember 2012 Physics Procedia 24:1311–1318 DOI: 10.1016/j.phpro.2012.02.196

G. Renukadevi, K. Rajambal" Generalized dq Model of nPhase Induction Motor Drive" World Academy of Science, Engineering and Technology International Journal of Electrical and Computer Engineering Vol:6, No:9, 2012.

Michaá "JANASZEK EXTENDED CLARKE TRANSFORMATION FOR n-PHASE SYSTEMS" PROCEEDINGS OF THE ELECTROTECHNICAL INSTITUTE, ISSN-0032-6216, LXIII, Issue 274, 2016.

Book " Electromechanical Energy Conversion" By Zeki Uğurata Kocabiyikoğlu ISBN 978036752402 296 Pages 329 B/W Illustrations Published March 1, 2022 by CRC Press

Book ELECTRIC MACHINES and DRIVES Principles, Control, Modeling, and Simulation by Shaahin Filizadeh Date: 20130225 International Standard Book Number-13: 978-1-4665-9942-0 (eBook - PDF)

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Book Model Predictive Control of Wind Energy Conversion Systems 1st Ed. | Dec 2016 Venkata Yaramasu, Bin Wu

Lesics Engineers Pvt. Ltd. (2019, April 4). *Slip ring induction motor – how it works*. Lesics. Retrieved [insert date of access], from https://www.lesics.com/slip-ring-induction-motor-how-it-works