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Ministry of Higher Education and Scientific Research

Ziane Achour University of Djelfa
Faculty of Sciences and Technology
Department of Mechanical Engineering
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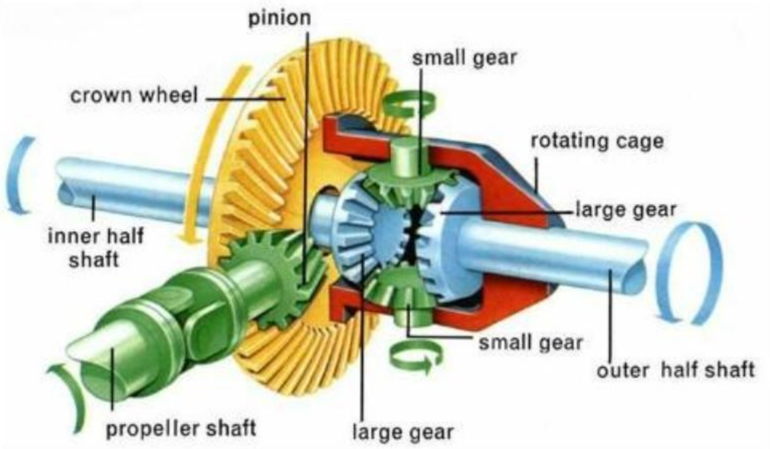
جامعـة زيــان عاشــور الجلفــة
كليـــــة العلــــوم والتكنولوجيــا
قســـم الهندســة الميكـانيكيـــة
الرقم :......../......../2026

Polycopy of courses

Mechanical Construction II

Intended for 3rd Year Undergraduate Students.

Option: Mechanical Construction



Developed by:

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Academic year: 2025-2026

Preface

This mechanical construction course is intended for third-year undergraduate students who have chosen the mechanical engineering track with a specialization in mechanical construction. It corresponds to the Construction Mechanical (II) module:

Semester: 6
Subject: Mechanical construction II
Course Unit: UEF
Hours: 67 (Lectures: 3 hours, Tutorials: 1.5 hours)
Credits: 6
Coefficient: 3

This course is a continuation of the CM1 curriculum and focuses primarily on the sizing calculations for the main motion transmission components of machines (gears, bearings, shafts, etc.), as well as the general technological study of mechanisms (reducers, gearboxes, etc.). It is written in a simplified style, with exercises at the end of each chapter that I believe are helpful for understanding the course content. This course equips students with the ability to analyze and calculate the various components of a machine. The content of this course material is structured into five chapters.

Chapter 1 is devoted to gears, studying geometric characteristics of gears for different types, namely cylindrical gears with straight teeth, helical gears, bevel gears with straight teeth and helical teeth, and finally the worm and wheel. The processes for obtaining them and the cutting of gears are also part of this chapter.

Chapter 2 is devoted to the dynamic study of surface pressure and breaking strength for cylindrical gears (straight and helical teeth).

The study of shafts and axles under various stresses, including traction and compression, shear, torsion, and bending, occupies **Chapter 3**. The calculation of the preliminary diameter of the axles and shafts, as well as the verification of the shafts and axles for fatigue, is studied.

Chapter 4 is dedicated to the study of bearings and thrust bearings as well as belt and chain transmission systems. We are looking for their essential characteristics such as length, transmission ratio and center distance already studied.

Chapter 5 is devoted to the technological applications of the elements studied previously, namely: reducers, gearboxes and epicyclic gear trains, their description and operation, as well as their kinematic diagrams.

Chapter 6 is devoted to general concepts of couplings. Their types and functions are examined. The different types of clutches and brakes are discussed. Finally, this course conforms to the curriculum adopted by the Ministry of Higher Education and Scientific Research; it contributes to the improvement of mechanical engineering education.

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Gears

I. Introduction

Gears are essential mechanical components. They are among the most widely used, strongest, and most durable systems for transmitting motion and power. We call gear the assembly of two toothed wheels meshing with each other.

I. 1. Cylindrical gears with straight teeth (Spur gears)

Spur gears, as depicted in Fig. I.1, have teeth that are parallel to the axis of rotation and are employed to transfer motion from one shaft to another parallel shaft. To establish the primary kinematic relationships of the tooth form, the spur gear is chosen because it is the most basic among all types.

Typical gears are pinion/gear (Fig.I.1), internal pinion/crown (Fig.I.2) and rack and pinion (Fig. I.3). The pinion is the smaller of the two wheels; it is often the driving wheel. The wheels have different shapes depending on their dimensions; it is often called the gear.

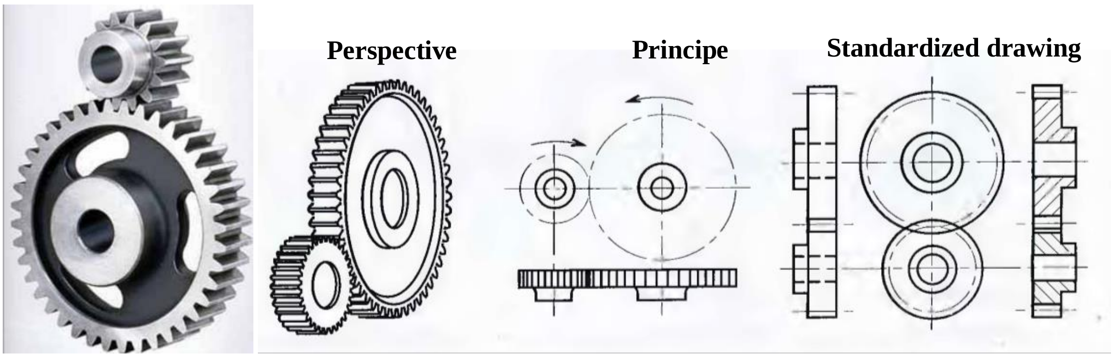


Fig. I. 1: Pinion and gear

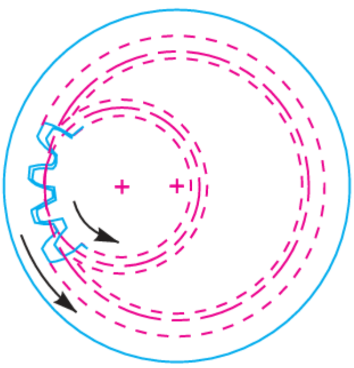


Fig. I. 2: Annular wheel and pinion

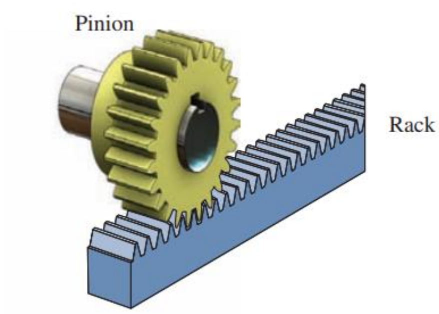


Fig. I. 3: Rack and pinion

I. 1.1. Definitions, terminology and standardization symbols ISO

The terminology of spur-gear teeth is illustrated in figure I.4. The pitch circle is a theoretical circle upon which all calculations are usually based; its diameter is the **pitch diameter**. The pitch circles of a pair of mating gears are tangent to each other. The following table shows the geometric characteristics of spur gears, and their standardized symbols.

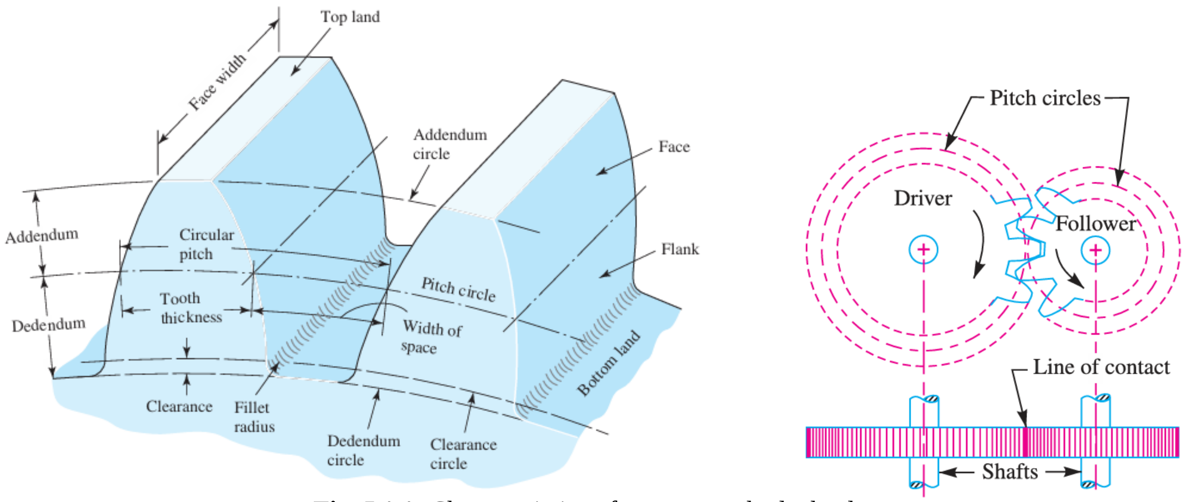


Fig. I.1.1: Characteristics of gear or toothed wheel.

Table I. 1: Geometrical characteristics of spur gear

Description	***Pinion***	***Gear***
Number of teeth	Z_1	Z_2
Module	m	
Pitch circle diameter	$d_1 = mZ_1$	$d_2 = mZ_2$
Pitch	$p = \pi m, \quad p_1 = p_2 = p$	
Face width	$b = km, \quad (7 \leq k \leq 12)$	
Addendum	$ha = m$	
Dedendum	$hf = 1.25m$	
Whole depth of tooth	$h = ha + hf = 2.25m$	
Tip circle diameter	$da = d + 2m$	
Root circle diameter	$df = d - 2.5m$	
Center distance	$a = \frac{d_1 + d_2}{2} = \frac{m(Z_1 + Z_2)}{2}$	
Pressure angle	α=20°	

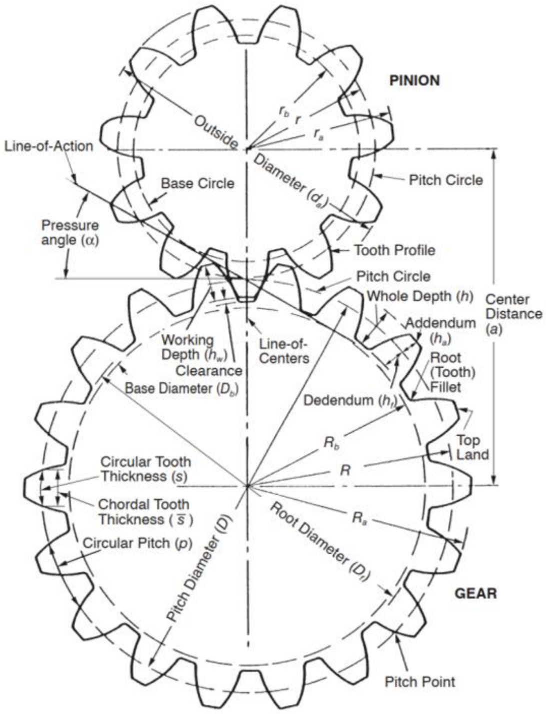


Fig. I. 1. 2: geometrical characteristics of gear

I. 1. 1. a. Primitive circumference of perimeter (π *d)*, it must imperatively include an integer number of teeth (Z) all placed at successive intervals equal to the primitive pitch (p), it results that:

$$\pi . d = p . z$$

we set *: m=p/ π*.

The expression simplifies and becomes: $d = m . Z$

I. 1. 1. b. Module m

Regardless of the number of teeth, all gears with the same module and pressure angle can be manufactured from the same tool.

To limit the number of tools and measuring systems, a series of modules has been standardized.

The thickness of the tooth and its resistance depend on the choice of module

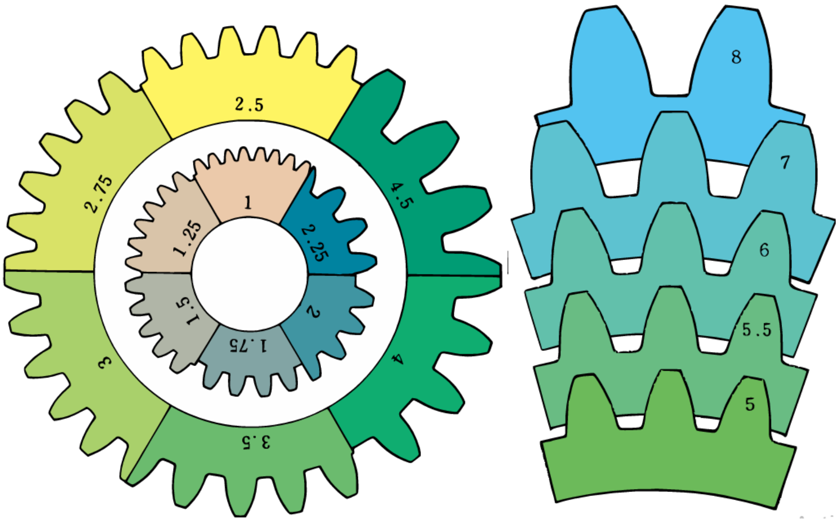


Fig. I. 1.3: Tooth profile shape according to the module

Table I. 2: Normalized values of the modul

Normalized values of the modul m (mm)									
Main values (Preferred)					**Secondary values** (Next choice)				
0.06	**0.25**	**1.25**	**5**	**20**	**0.07**	**0.28**	**1.25**	**5.5**	**22**
0.08	**0.30**	**1.50**	**6**	**25**	**0.09**	**0.35**	**1.375**	**7**	**28**
0.10	**0.40**	**2**	**8**	**32**	**0.11**	**0.45**	**1.75**	**9**	**36**
0.12	**(0.50)**	**2.5**	**10**	**40**	**0.14**	**(0.55)**	**2.75**	**11**	**45**
0.15	**(0.80)**	**3**	**12**	**50**	**0.18**	**(0.7)**	**3.5**	**14**	**55**
0.20	**1.0**	**4**	**16**	**60**	**0.22**	**(0.9)**	**4.5**	**18**	**70**

I. 1. 2. Kinematic study

When pinion (1) meshes with gear (2), the pitch circles of the two wheels roll over each other without slipping at point I (no spinning). Two wheels must have the same module to be able to mesh together.

The pitch line velocity at point I is:

$$V_I = \omega_1 R_1 = \omega_2 R_2$$

$$\omega = \frac{\pi N}{30} ; R = \frac{d}{2}$$

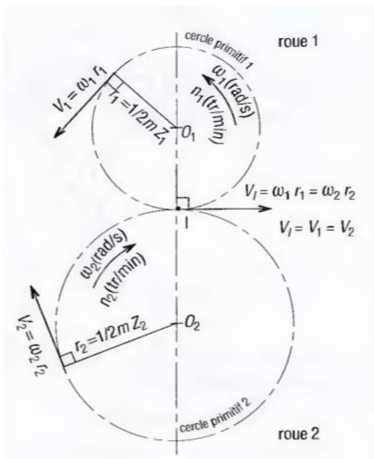


Fig. I.1.4: Simple Gear

I. 1. 2. 1. Simple gear

the ratio of power transmission is:

$$r_{1/2} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{Z_1}{Z_2}$$

I. 1. 2. 2. Simple Gear Train

The transmission ratio in the case of a gear train is calculated according to the following general formula:

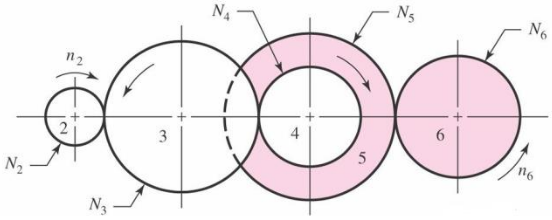


Fig. I. 1. 5: Simple Gear Trains

$$r_{2/6} = \frac{N_6}{N_2} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$r_{2/6} = \frac{N_6}{N_2} = (-1)^3 \frac{Z_2 \times Z_3 \times Z_5}{Z_3 \times Z_4 \times Z_6}$$

With (n) represents the total number of external contacts between wheels, $(-1)^n$ allows us to know whether or not there is inversion of the direction of rotation between the input and the output.

Example

Calculate the overall transmission ratio of the reducer shown in Figure I.9.

Solution

$$r_{1/4} = \frac{N_4}{N_1} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$r_{1/4} = \frac{N_4}{N_1} = (-1)^2 \frac{Z_1 \times Z_3}{Z_2 \times Z_4}$$

$$r_{1/4} = \frac{N_4}{N_1} = \frac{25 \times 28}{79 \times 76} = 0.11$$

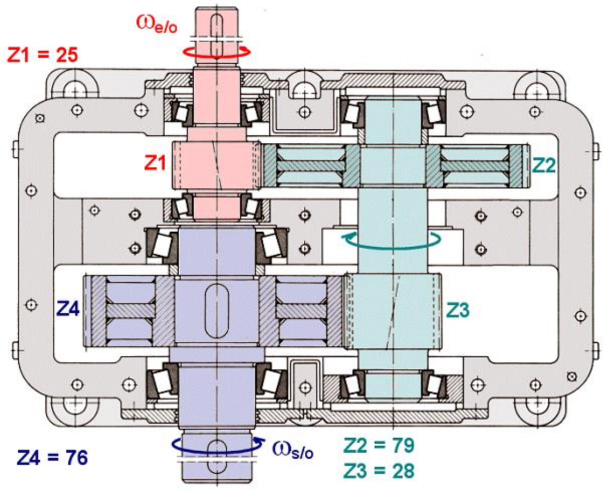


Fig. I. 1.6: A reducer consisting of two gear trains

I. 1. 3. Dynamic study

- ✓ We will assume: that only one tooth of the wheel receives the tangential force $\mathbf{F_t}$.
- ✓ That this force acts at the end of the tooth perpendicular to its axis *xx'*.

The maximum bending moment is given by:

$$Mf_{\max} = F_t.h = 2.25m.F_t$$

$$\sigma_{\max} = \frac{Mf_{\max}}{I/\upsilon} \leq Rp$$

$$I = \frac{b.e^3}{12}; \upsilon = \frac{e}{2}; \frac{I}{\upsilon} = \frac{be^2}{6}$$

We have $b = k.m$, tooth width, and $e = \dfrac{\pi.m}{2}$; $\quad \dfrac{I}{\upsilon} = \dfrac{k.m}{6}.\dfrac{m^2\pi^2}{4} = \dfrac{k.m^3\pi^2}{24}$

$$\frac{k.m^3\pi^2}{24} \geq \frac{2.25.m}{Rp}.Ft$$

$$m^2 \geq 5.47\frac{Ft}{Rp.k}$$

$$m \geq 2.34\sqrt{\frac{Ft}{Rp.k}}$$

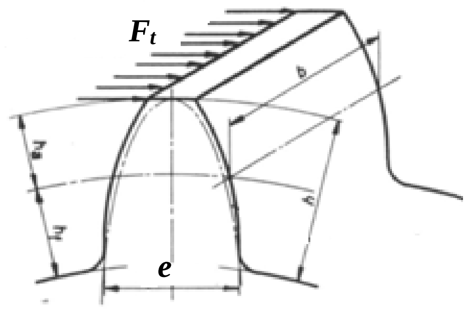


Fig.I. 1.7 : Calculation diagrams.

I. 1. 4. Force Analysis

Wheel 1 is assumed to be the driving wheel (motor) and wheel 2 is assumed to be the driven wheel (receiver), $\mathbf{C_1}$ is the driving torque on wheel 1, $\mathbf{C_2}$ is the driving torque on wheel 2.

$\mathbf{P_1}$ is the power on wheel 1, and $\mathbf{P_2}$ is the power on wheel 2, η is the efficiency of the gear. We have: $\mathbf{P_1 = C_1 . \omega_1}$; $\mathbf{P_2 = P_1 .\eta}$; η =98-99%.

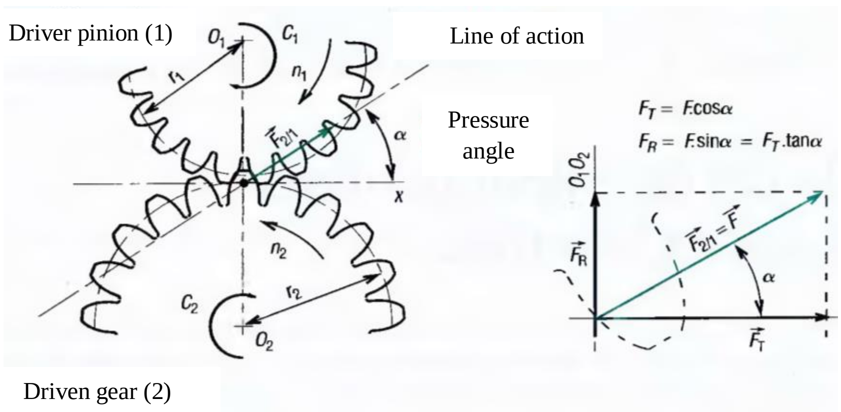


Fig. I. 1. 8: Forces exerted on the tooth

I. 1. 4. 1. Contact force $F_{2/1}$

This diagram shows the action exerted by the gear (2) on the pinion (1). It is always plotted along the pressure line passing through point I, which forms an angle (α = 20°) with the tangent to the pitch circle. $F_t = F_{2/1}\cos(\alpha)$

I. 1. 4. 2. Tangential force F_t

This is the source of the transmitted torque. $F_t = \dfrac{M_t}{r}; r = \dfrac{d}{2}$

I. 1. 4. 3. Radial force F_r Perpendicular to F_t, sometimes called separation force, its action tends to separate the two wheels and results in shaft deflection. $F_r = F_t \tan(\alpha)$

Example

A 30-tooth wheel is coupled with an 18-tooth spur gear with a module of 8 mm and a pressure angle of 20°. If the transmitted power is 40 kW at 500 rpm, find the forces on the teeth.

Solution

We calculate the transmitted torque C

$$P = C \times \omega \Rightarrow C = \frac{30P}{\pi N}$$

$$C = \frac{30 \times 40 \times 10^3}{3.14 \times 500} = 764.33 N.m$$

The tangential force is calculated from the equation:

$$C = F_t \times d_1/2 \Rightarrow F_t = \frac{2C}{d_1}$$

$$F_t = \frac{2C}{Z_1 \times m} = \frac{2 \times 764.33}{18 \times 8 \times 10^{-3}} = 10615.69 N$$

Now, we calculate the radial force

$$F_r = F_t \times \tan(\alpha) = 10615.69 \times 0.36 = 3821.64 N$$

The total force is :

$$F = \frac{F_t}{\cos(\alpha)} = \frac{10615.69}{0.93} = 11414.72 N$$

I. 1. 5. The forming of gear teeth

There are several ways to obtain gears; the choice depends on the quality of the transmission, the materials used, the size of the series, the type of system, etc.

For large series, the toothed wheel blanks can be obtained by casting (sand casting for cast iron or steel wheels, under pressure for light alloy or plastic wheels). The teeth are very often finished on a cutting machine. Toothed wheel blanks or shafted pinions can also be obtained by forging (a method very often used in the automotive industry).

There are many techniques for rough cutting of teeth. They range from artisanal methods to mass production methods, and can be divided into four main categories:

I. 1. 5. 1. Gear milling

Gear milling is an economical and flexible process for cutting a variety of cylindrical and other gear types such as spur, helical and bevel gears, splines, and racks. In gear milling, circular, disc-type cutters and end-mill cutters are used to cut gear teeth. The shape of the milling cutter conforms to the gear-tooth space. Each tooth is cut individually; after completion of a tooth, the cutter is returned to its starting position, the blank is indexed for the second tooth, and the cycle is repeated. This process is recommended for production of small volumes of low-precision gears. Gears having different modules and number of teeth need separate milling cutters. Milling cutters are less costly than hobs and other types of cutters. End-milling cutters are used for cutting the teeth of large gears of high modules. Cylindrical gears made by milling find applications in low-speed machinery and where the microgeometry deviations of the gears are not of major concern.

Gear milling by disc cutter and end-mill cutter are shown in Figures. I.1.11A and 11B. The disc cutters are used for the form cutting of big spur gears of large pitch, while end-mill cutters are for pinions of large pitch and for double helical gears.

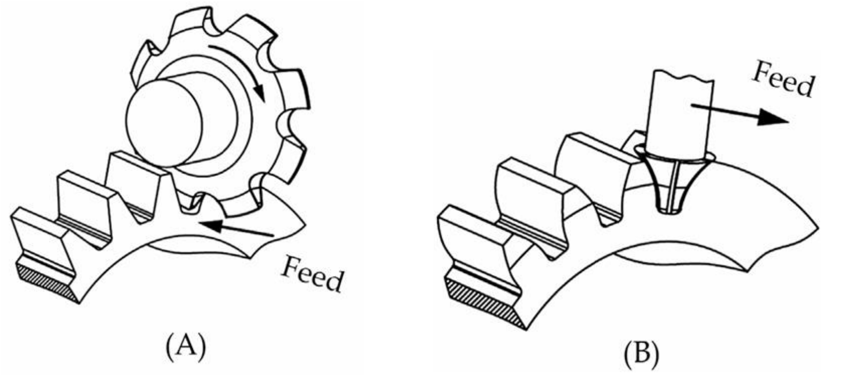
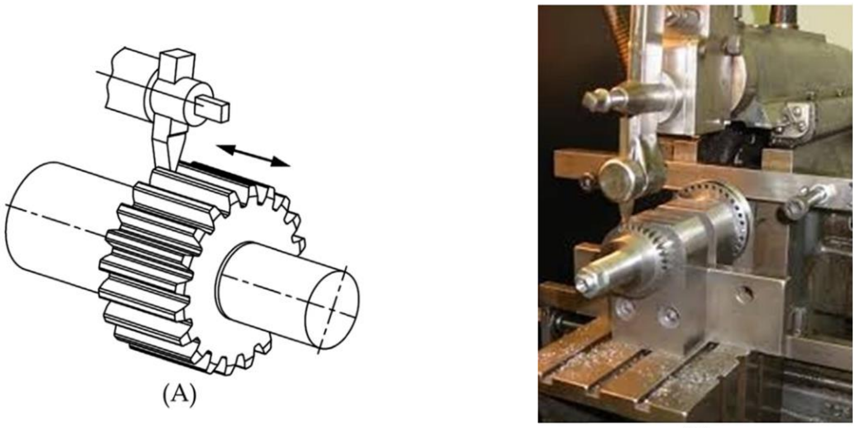


Fig. I. 1. 11: Gear milling; (A) milling by disc cutter; (B) milling by end-mill cutter.

I. 1. 5. 2. Gear cutting on a shaper

A shaper is machine tool utilizing linear motion for cutting. It is used primarily in gear cutting to manufacture lower quality gears of simple profile, such as spur gears, splines, and clutch teeth. Large quantities of gears may be economically cut with a cutting tool with a cutting edge that corresponds to the shape of the tooth space. The tool reciprocates parallel to the center axis of the blank and cuts one tooth space at a time. Successive teeth are cut by rotating the gear blank through an angle corresponding to the pitch of the teeth until all the tooth spaces have been cut. Figure I.1.12 illustrates the principle of cutting gear teeth by means of a shaper.



(A) (B)

Fig. I. 1. 12: Gear teeth cutting on a shaper. (A) Schematic presentation, (B) actual photograph.

I. 1. 5. 3. Hobbing

The hobbing process is illustrated in Figures I.1.13 and 14. The hob is simply a cutting tool that is shaped like a worm. The teeth have straight sides, as in a rack, but the hob axis must be turned through the lead angle in order to cut spur-gear teeth. For this reason, the teeth generated by a hob have a slightly different shape from those generated by a rack cutter. Both the hob and the blank must be rotated at the proper angular-velocity ratio. The hob is then fed slowly across the face of the blank until all the teeth have been cut.

I. 1. 5. 4. Cutting with a pinion tool (useful for space problems, specific machines), the pinion tool is driven by a vertical alternating cutting movement (mortising) Figure I.1.15, and the tool and the part to be machined are driven by a synchronized rotation movement (meshing).

Fig. I. 1.13: Hobbing helical gear

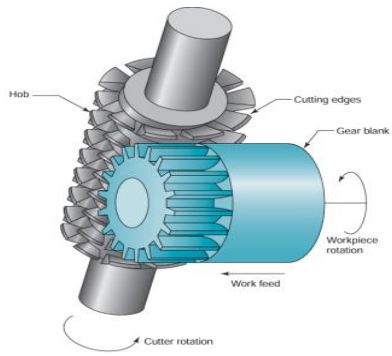


Fig. I. 1.14: Hobbing spur gear

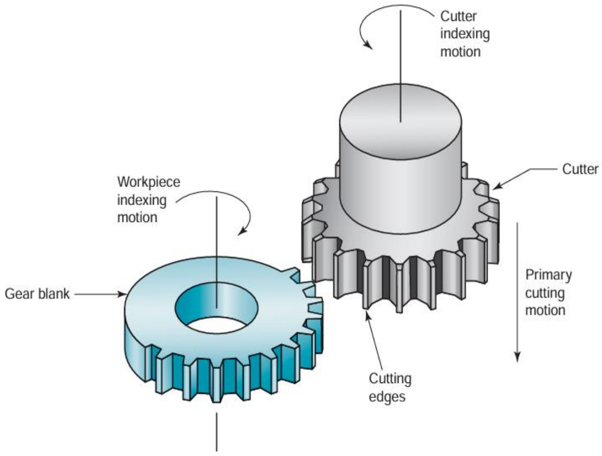


Fig. I. 1.15: Gear cutting: pinion

I. 1. 6. Applications

Exercise 1:

We want to build a gearbox so that the input speed of 1500 rpm is reduced to 500 rpm. If $Z_1 = 18$ teeth, what is the value of Z_2? If m = 3 mm, what is the value of d_2?

Solution

We have $r_{1/2} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{Z_1}{Z_2}$

$$\frac{N_2}{N_1} = \frac{Z_1}{Z_2} \Rightarrow Z_2 = \frac{N_1 \times Z_1}{N_2} = \frac{1500 \times 18}{500} = 54 teeth$$

Calculate of d_2:

$$d_2 = m \times Z_2 = 3 \times 54 = 162 mm$$

Exercise 2:

A gear consisting of three straight-toothed cylindrical gears, $Z_1 = Z_2 = 20$ teeth and $Z_3 = 60$ teeth (see Figure 1). The modulus m = 2 mm. Determine:

1. The transmission ratio of the assembly.
2. The pitch diameters.
3. The center distance between gears 1 and 2, then between gears 2 and

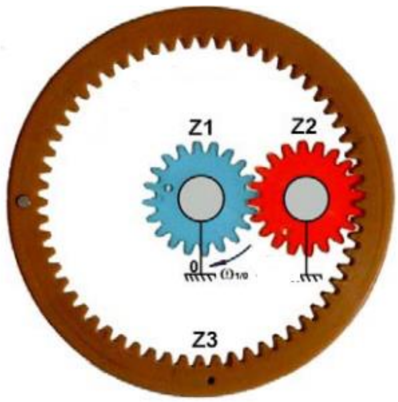


Fig. I. 1.16: Internal and external toothed gears.

Solution

1. Calculating the transmission ratio

$$r_{1/3} = \frac{N_3}{N_1} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$r_{1/3} = \frac{N_3}{N_1} = (-1)\ \frac{Z_1 \times Z_2}{Z_2 \times Z_3} = -\frac{20}{60} = -\frac{1}{3}$$

The sign (-) indicates that gear (2) turns in the opposite direction to gear (1).

2. The pitch diameters of the pinion and gear are, respectively,

$$d_2 = mZ_2 = 2 \times 20 = 40mm$$
$$d_1 = mZ_1 = 2 \times 20 = 40mm$$
$$d_3 = mZ_3 = 2 \times 60 = 120mm$$

3. The center distance between gears (1) and (2)

$$a_{w1/2} = \frac{d_1 + d_2}{2} = \frac{40 + 40}{2} = 40mm$$
$$a_{w2/3} = \frac{d_3 - d_2}{2} = \frac{120 - 40}{2} = 40mm$$

Exercise 3:

We consider a cylindrical gear with straight teeth, pitch 6.28 mm, pressure angle 20°, number of teeth on the gear 80 teeth, transmission ratio 0.25. Determine:

1. The number of teeth on the pinion;

2. The module m;

3. The center distance a_w.

Solution

1. Calculating the number of teeth on the pinion,

$$r_{1/2} = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} = \frac{1}{4} \Rightarrow Z_1 = Z_2 \times \frac{1}{4} = \frac{80}{4} = 20 teeth$$

2. Calculating of the modulus

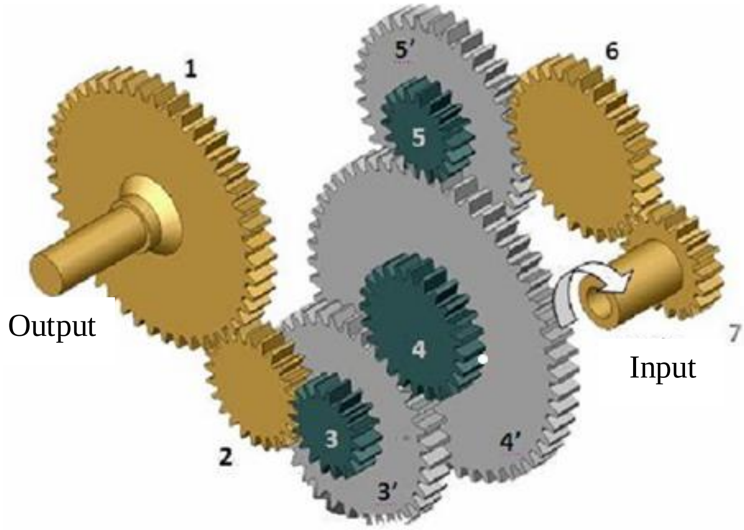
$$p = \pi m = 6.28 \Rightarrow m = \frac{6.28}{\pi} = 2mm$$

3. Calculating of the centre distance

$$a_{w1/2} = \frac{d_1 + d_2}{2} = \frac{m(Z_1 + Z_2)}{2} = \frac{2(20 + 80)}{2} = 100mm$$

Exercise 4:

Calculate the transmission ratio.



Z_1=65 teeth	
Z_2=32 teeth	
Z_3=24 teeth; $Z_{3'}$=48 teeth	
Z_4=38 teeth; $Z_{4'}$=82 teeth	
Z_5=26 teeth; $Z_{5'}$=54 teeth	
Z_6=42 teeth	
Z_7=30 teeth	

Fig. I. 1.17: Simple trains

Solution

$$r_{7/1} = \frac{N_1}{N_7} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$n = 6$$

$$r_{7/1} = \frac{N_1}{N_7} = (-1)^6 \frac{Z_7 \times Z_6 \times Z_5 \times Z_4 \times Z_3 \times Z_2}{Z_6 \times Z_{5'} \times Z_{4'} \times Z_{3'} \times Z_2 \times Z_1} = \frac{Z_7 \times Z_5 \times Z_4 \times Z_3}{Z_{5'} \times Z_{4'} \times Z_{3'} \times Z_1}$$

$$r_{7/1} = \frac{N_1}{N_7} = \frac{30 \times 26 \times 38 \times 24}{54 \times 82 \times 48 \times 65} = \frac{711360}{13815360} = 0.051$$

Exercise 5:

A gear consisting of two spur gears (see Fig. 3), Z_2 = 35 teeth and Z_4 =? The module m = 1.5 mm and the center distance a_w = 116.26 mm. Knowing that motor M_3 rotates at speed N_M = 750 rev/min.

1. Determine the main characteristics of this pinion (2);
2. Determine the rotational speed of nut (6), N6;
3. Calculate the feed rate V_7 of the screw, if the pitch of screw (7) is 3 mm.

Solution

1. The main characteristics of this pinion (2) is given in table below.

Given: $Z_2 = 35 teeth$, $m = 1.5 mm$, $a_w = 116.26 mm$

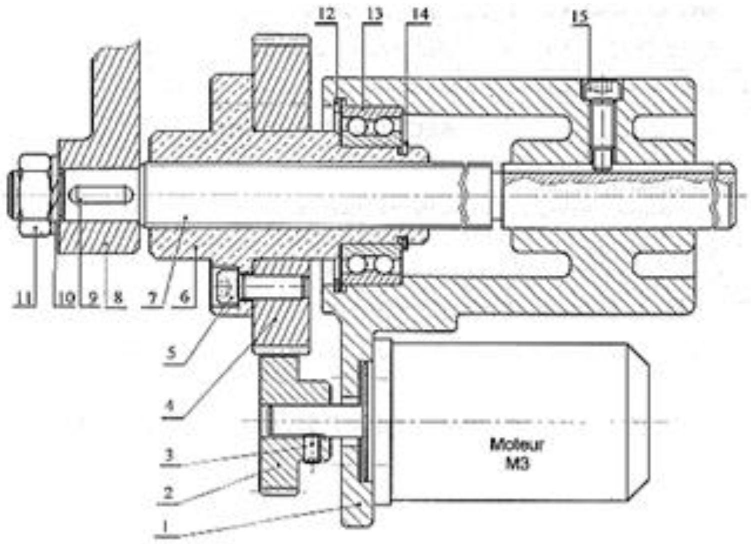


Fig. I. 1.18: Screw and nut control system

Characteristics	Pinion (2)
$d = mZ$	$d_2 = mZ_2 = 1.5 \times 35 = 52.5mm$
$d_a = d + 2m$	$d_{a2} = d_2 + 2m = 52.5 + 3 = 55.5mm$
$d_f = d - 2.5m$	$d_{f2} = d_2 - 2.5m = 52.5 - 2.5 \times 1.5 = 48.75mm$
$h_f = 1.25m$	$h_f = 1.25m = 1.25 \times 1.5 = 1.875mm$
$h_a = m$	$h_a = m = 1.5mm$
$h = h_f + h_a = 2.25m$	$h = 2.25m = 2.25 \times 1.5 = 3.375mm$
$b = km$	$b = 10 \times 1.5 = 15mm$

2. Calculating of the speed of rotation: N_6

Nut (6) makes a complete connection on the gear (4), so:

Calculation of Z_4

we have
$$a_w = \frac{d_2 + d_4}{2} = \frac{m(Z_2 + Z_4)}{2}$$
$$Z_4 = \frac{m2(Z_2 + Z_4)}{2}$$

$$N_6 = N_4$$
$$\frac{N_4}{N_2} = \frac{Z_2}{Z_4}; \qquad N_2 = N_M$$
$$N_6 = N_M \frac{Z_2}{Z_4} = 750 \frac{35}{120} = 218.75 rev / min$$

3. Feed of (7)

$$V_7 = N_6 \times p = 218.75 \times 3 = 656.25 mm / \min$$
$$V_7 = 0.65 m / \min = 0.0109 m / s$$

Exercise 6:

Consider a spur gear transmitting a power of 1.5 kW at N=1500 rev/min. The practical strength of the constituent material is 15 N/mm^2, given: d = 100 mm and k = 10.

1. Calculate the tangential force Ft;

2. Calculate the modulus of this gear;

3. Calculate the torque C and deduce the angular velocity ω.

Solution

1. Calculate the Tangential force F_t

$$P = C \times \omega$$
$$C = F_t \frac{d}{2}; \omega = \frac{\pi N}{30}$$
$$P = F_t \frac{d}{2} \frac{\pi N}{30} \Rightarrow F_t = \frac{60P}{d\pi N}$$

$$F_t = \frac{60P}{d\pi N} = \frac{60 \times 1.5 \times 10^3}{0.1 \times 3.14 \times 1500} = 191.08N$$

2. Calculating of the modulus, we have:

$$m \geq 2.34 \sqrt{\frac{F_t}{kR_p}}$$

So $m \geq 2.34 \sqrt{\frac{191.08}{10 \times 15}}$

$$m \geq 2.62mm$$

We choose from table I.2: m=3mm

3.calculating the torque.

$$C = F_t \frac{d}{2} = \frac{191.08 \times 0.1}{2} = 9.55Nm$$

Then we can deduce the angular velocity as:

$$P = C\omega \Rightarrow \omega = \frac{P}{C}$$
$$\omega = \frac{1.5 \times 10^3}{9.55} = 157.06 rd / s$$

Exercise 7:

An 18-tooth spur gear pinion, modulus 8 mm, and pressure angle 20° meshes with a 30-tooth wheel. Calculate the forces on the teeth if the transmitted power is 40 kW at 500 rev/min.

Solution

From equation below, we find the transmitted load to be

$$F_{t1/2} = \frac{60P}{d\pi N} = \frac{60P}{mZ_1\pi N} = \frac{60\times40\times10^3\times10^3}{8\times18\times3.14\times500} = 10615.71N$$

Thus, the tangential force of gear 2 on gear 1 is $F_{t2/1} = 10615.71N$

Therefore $F_{r2/1} = F_{r1/2} = F_{t1/2}\tan(\alpha) = 10615.71\times0.36 = 3821.65N$

and so $F = \frac{F_{t1/2}}{\cos(\alpha)} = \frac{10615.71}{0.93} = 11414.74N$

Exercise 8:

The relationship: $m \geq 2.34\sqrt{\frac{F_t}{kR_p}}$ allows us to calculate the modulus m as a function of the tangential force F_t, the coefficient k, and the practical resistances Rp.

Show that it can also be written in the following form: $m \geq \sqrt[3]{\frac{10.94C}{KZR_p}}$

Z: number of teeth

C: torque acting on the gear

Solution

We start from the expression:

$$m \geq 2.34\sqrt{\frac{F_t}{kR_p}}$$

Therefore

$$m^2 \geq (2.34)^2\frac{F_t}{kR_p}$$

$$F_t = \frac{2C}{d} = \frac{2C}{mZ}$$

$$m^2 \geq \frac{10.94C}{mZkR_p}$$

$$m^3 \geq \frac{10.94C}{ZkR_p}$$

$$m \geq \sqrt[3]{\frac{10.94C}{ZkR_p}}$$

I. 2. Helical Gears

Similar to spur gears, they are widely used in power transmission; the teeth of the wheels are inclined relative to the axis of rotation of the two shafts.

At equal size, they are more efficient than straight-toothed gears in transmitting power and torque. Due to better progressiveness and continuity of the mesh, they are also quieter.

Most often helical gears are working in paralel shafts. Helical angle on both gears is the same but have different direction left and right (figure. I.2.1). If helical angle is the same and have the same direction shafts are at 90^{o} angle and this type of gear is named crossed helical or screw gears (figure. I.2.3). It is possible to combine two gears with different value of helical angle and the same direction to receive non standard angle between shafts

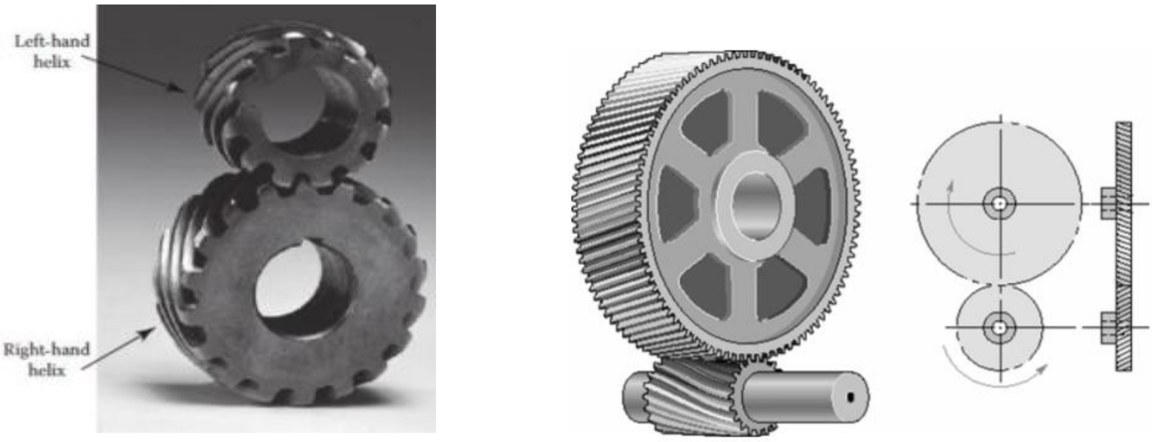


Fig. I. 2.1: Helical cylindrical gears

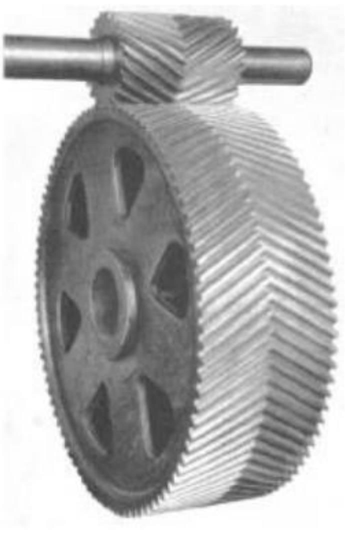


Fig. I. 2.2: Herring Bone gear

Fig. I. 2.3: Helical gear meshed on crossed axes

I. 2. 1. Comparison between straight teeth and helical teeth

I.2.1.a. Advantage of helical gears: More flexible, more progressive and less noisy transmission than straight gears, greater driving force 2, 3, or 4 pairs of teeth always meshing; transmission of significant forces at high speeds.

I.2.1.b. Disadvantages

- Helical gears subject the shaft bearings to both radial and thrust loads. When the thrust loads become high or are objectionable for other reasons, it may be desirable to use double helical gears (see figure I. 2.2).
- Use is impossible in the form of a sliding gear; these gears must always remain in mesh.

I. 2. 2. Characteristics

- **Helix angle** β: it measures the inclination of the teeth, or helix, relative to the axis of the wheel; usual values are between 15° and 30°. The helix angle is the same on each gear, but one gear must have a right-hand helix and the other a left-hand helix.

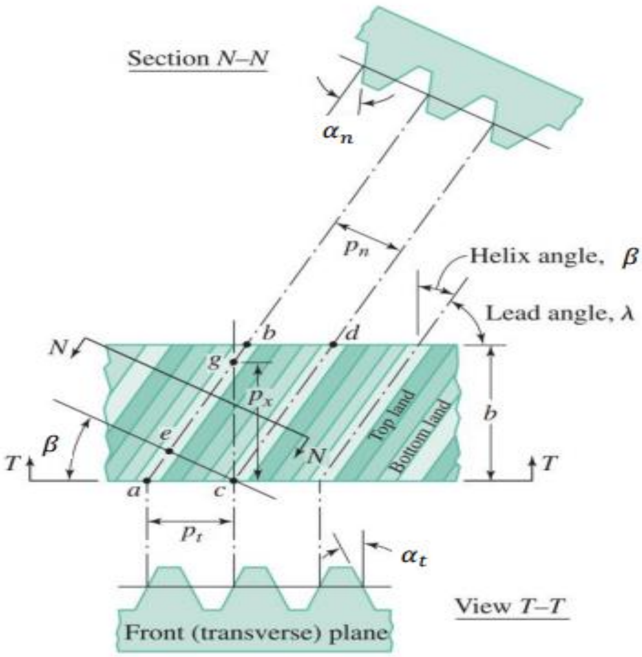


Fig. I. 2.4: Geometric profile in helical gear

- **Normal quantities:** P_n, m_n, and α_n(20°).

They are standardized and measured perpendicular to the helix.

- **Apparent quantities:** P_t, m_t, and α_t (20°)are not standardized and depend on the value ofβ. They are measured in the plane of rotation of the wheel.

- **Center distance a_w:** it depends on the angle β. By varying the angle β any desired center distance can be obtained.
- **Width *b*:** for reasons of continuity and progressiveness, the width ***b*** of the wheel must be greater than the axial pitch P_x. (Usual value $b \geq 2P_x$).

Table I. 2.1: Helical gear's geometrical characteristic

Description	*symbol*	*usual observation and formula*
Helix angle	β	$15° \leq \beta \leq 30°$
Helix direction		One gear must have a ***right-hand*** helix and the other a ***left-hand*** helix
Normal module	m_n	m_n is to be chosen from the series of standardized modules
Transverse module	m_t	$m_t = \frac{m_n}{\cos(\beta)}$
Normal circular pitch	P_n	$p_n = \pi m_n$
Transverse circular pitch	P_t	$P_t = \frac{P_n}{\cos(\beta)} = \pi m_t$
Pitch diameter	d	$d = m_t Z$
Addendum	ha	$ha = m_n$
Dedendum	hf	$hf = 1.25 m_n$
Depth of tooth	h	$h = ha + hf = 2.25 m_n$
Outside diameter	da	$da = d + 2m_n$
Root diameter	df	$df = d - 2.5 m_n$
Center distance	a	$a = \frac{d_1 + d_2}{2} = \frac{m_t(Z_1 + Z_2)}{2} = \frac{m_n(Z_1 + Z_2)}{2\cos(\beta)}$
Primitive helix Pitch	P_z	$P_z = \frac{\pi d}{\tan(\beta)} = Z.P_x$
Axial helix pitch	P_x	$P_x = \frac{P_t}{\tan(\beta)} = \frac{P_n}{\sin(\beta)}$
Face width	b	$b \geq \frac{\pi m_n}{\sin(\beta)} = 2P_x$
Normal pressure angle	α_n	$\alpha_n = 20°$
Transversal pressure angle	α_t	$\tan(\alpha_n) = \tan(\alpha_t).\cos(\beta)$

Example: A stock helical gear has a normal pressure angle of 20°, a helix angle of 25°, and a transverse pitch of 6 mm, and has 18 teeth. Find:

(a) The pitch diameter.

(b) The normal and the axial pitches.

(c) The transverse pressure angle.

Solution

The pitch diameter:

$$d = m_t Z = \frac{P_t}{\pi} Z; \qquad d = \frac{6\times 18}{3.14} = 34.39mm$$

The normal pitch:

$$P_n = P_t \cos(\beta) = 6\times\cos(25°) = 5.4mm$$

The axial pitch:

$$P_x = \frac{P_t}{\tan(\beta)} = \frac{6}{\tan(25°)} = 13.04mm$$

The transverse pressure angle:

$$\tan(\alpha_n) = \tan(\alpha_t).\cos(\beta) \Rightarrow \tan(\alpha_t) = \frac{\tan(\alpha_n)}{\cos(\beta)}$$

$$\alpha_t = \tan^{-1}(\frac{\tan(\alpha_n)}{\cos(\beta)}); \qquad \alpha_t = \tan^{-1}(\frac{\tan(20°)}{\cos(25°)}) = 21.80°$$

I. 2. 3. Forces analyses

The inclination of the teeth generates axial (Thrust forces) forces along the axis of the shaft, which must be supported by the bearings and the additional torques which accentuate the bending of the shafts.

When the gear mesh transmits power, forces act on the gear teeth. As shown in Figure I. 2. 5, if the Z-axis of the orthogonal 3-axes denotes the gear shaft, forces are defined as follows: The force that acts in the X-axis direction is defined as the tangential force ***Ft***.

$$Ft = \frac{C}{r}$$

The force that acts in the Y-axis direction is defined as the radial force ***Fr***.

$$Fr = Ft\frac{\tan(\alpha_n)}{\cos(\beta)}$$

The force that acts in the Z-axis direction is defined as the axial force ***Fa*** or **thrust.**

$$Fa = Ft\tan(\alpha_n)$$

Analyzing these forces is very important when designing gears. In designing a gear, it is important to analyze these forces acting upon the gear teeth, shafts, bearings, etc.

The total force:

$$F = \sqrt{Ft^2 + Fr^2 + Fa^2}$$

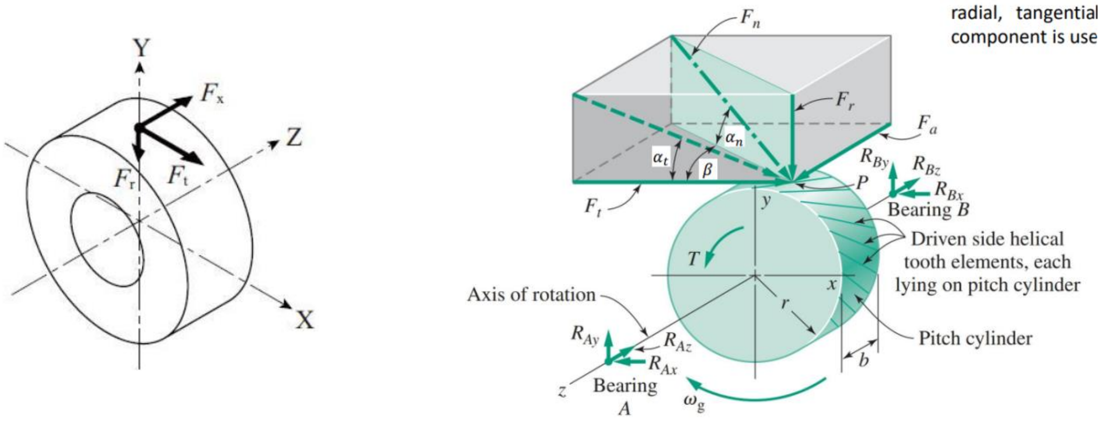


Fig. I. 2. 5: Distribution of forces in helical gear.

I. 2. 4. Applications

Exercise 1:

Consider a helical cylindrical gear such that Z_1 = 33, Z_2 = 44, and m_n = 2.

1. Calculate the helix angle required to obtain a center distance of 80 mm.

2. What are the possible values for the center distance if β varies between 0° and 40°?

3. If β = 35° and α_n = 20°, calculate the values of m_t, P_n, P_t, P_x, P_{z1}, P_{z2}, d_1, d_2, and α_t.

Solution

1. Calculating of the helix angle, we have:

$$a_w = \frac{d_1 + d_2}{2} = \frac{m_t(Z_1 + Z_2)}{2} = \frac{m_n(Z_1 + Z_2)}{2\cos(\beta)}$$

therefor

$$\cos(\beta) = \frac{m_n(Z_1 + Z_2)}{2a_w} = \frac{2(33+44)}{2\cos(\beta)} = \frac{77}{80} = 0.96$$

$$\beta = 16.26° = 16°15'36"$$

2. The possible values of the center distance

$$a_w = \frac{2(33+40)}{2\cos(\beta)} = \frac{77}{\cos(\beta)}$$

β°	0	10	20	30	40
a_w	77	78.57	82.79	89.53	101.31

3. The case of: $\underline{\beta = 35°}$

- Transverse modulus

$$m_t = \frac{m_n}{\cos(\beta)} = \frac{2}{\cos(35°)} = 2.44mm$$

- Normal pitch

$$p_n = m_n\pi = 2\times 3.14 = 6.28mm$$

- Transverse pitch

$$p_t = m_t\pi = 2.44\times 3.14 = 7.66mm$$

- Axial helix pitch

$$P_x = \frac{P_t}{\tan(\beta)} = \frac{P_n}{\sin(\beta)} = \frac{P_Z}{Z}$$

$$P_x = \frac{P_t}{\tan(\beta)} = \frac{7.66}{\tan(35°)} = 10.93mm$$

- Primitive helix Pitch

$$P_{Z1} = Z_1 \times P_x = 33\times 10.93 = 360.69mm$$
$$P_{Z2} = Z_2 \times P_x = 40\times 10.93 = 480.92mm$$

- Pitch diameters

$$d_1 = Z_1 \times m_t = 33\times 2.44 = 80.52mm$$
$$d_2 = Z_2 \times m_t = 44\times 44 = 107.36mm$$

- $$\tan(\alpha_n) = \tan(\alpha_t).\cos(\beta)$$
  $$\tan(\alpha_t) = \frac{\tan(\alpha_n)}{\cos(\beta)} = \frac{\tan(20°)}{\cos(35°)} = \frac{0.36}{0.81} = 0.44$$

So $\alpha_t = 23.74°$

<u>Exercise 2 :</u>

The proposed two-stage helical gear reducer has the particularity of having the input shaft coaxial with the output shaft (see Figure I.2.6). Gears (1, 2): Z_1 = 30, Z_2 = 60, helix angle β_1 = 30°, normal module m_n = 5mm.

Gears (3, 4): Z_3 = 22, Z_4 = 35, Normal module 8mm. If the center distance is the same for both gears, determine the helix angle β_2 of the second gear set.

➢ Calculate the overall transmission ratio, then deduce the value of N_4 if N_1 = 1500 rpm.

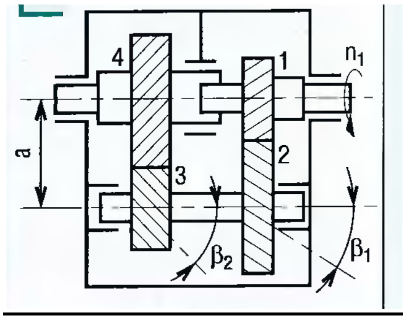


Fig. I. 2.6: Two-stage helical gear reducer

Solution

1. Determination of the helix angle β_2 of the second gear

We have $a_{w(1-2)} = a_{w(3-4)}$

Therefore

$$\frac{m_{n(1-2)}(Z_1+Z_2)}{2\cos(\beta_{(1-2)})} = \frac{m_{n(3-4)}(Z_3+Z_4)}{2\cos(\beta_{(3-4)})}$$

$$\cos(\beta_{(3-4)}) = \frac{m_{n(3-4)}(Z_3+Z_4)}{m_{n(1-2)}(Z_1+Z_2)}\cos(\beta_{(1-2)})$$

$$\cos(\beta_{(3-4)}) = \frac{8(22+35)}{5(30+60)}0.86 = 0.87$$

$$\beta_{(3-4)} = 29.54°$$

2. Transmission ratio

$$r_{(1/4)} = (-1)^n \frac{Z_1 \times Z_3}{Z_2 \times Z_4} = \frac{30\times 22}{60\times 35} = \frac{11}{35} \approx 0.31$$

The rpm of the gear (4)

$$r_{(1/4)} = \frac{N_4}{N_1} \Rightarrow N_4 = r_{(1/4)} \times N_1 = \frac{11}{35}\times 1500 = 471.42 rev/\min$$

Exercise 3:

The sketch of Figure I.2.6 shows a one-stage gear reducer that utilizes helical gears with a normal circular pitch of 4.45*mm*, normal pressure angle of 20°, and helix angle of 30°. The helix of the 18-tooth drive pinion (1) is left-hand. The input shaft is to be driven in the direction shown (CCW: Counter Clock Wise) by a 1725-rpm electric motor operating steadily at full rated power, and the desired output shaft speed is 575 rpm. Determine the following:

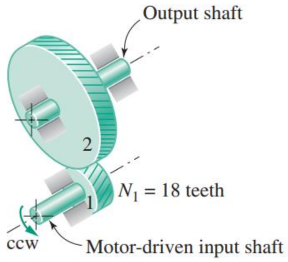


Fig. I. 2. 7: One-stage helical gear reducer

a. Transverse pressure angle

b. Transverse circular pitch

c. Pitch diameter of pinion (1)

d. Pitch diameter of gear (2)

e. Number of teeth on gear (2)

f. Center distance

g. Pitch-line velocity

h. Numerical values and directions of tangential, radial, and axial force components on the pinion while operating at full rated motor of 1 horsepower

i. Minimum recommended face width

Solution

a. Transverse angle pressure

we have $\tan(\alpha_n) = \tan(\alpha_t).\cos(\beta)$

then: $\tan(\alpha_t) = \dfrac{\tan(\alpha_n)}{\cos(\beta)}$

therefor :

$$\alpha_t = \tan^{-1}(\frac{\tan(\alpha_n)}{\cos(\beta)})$$

$$\alpha_t = \tan^{-1}(\frac{\tan(20°)}{\cos(30°)}) = \tan^{-1}(\frac{0.36}{0.86}) = 22.29°$$

b. Transverse diametric pitch

$$p_t = \frac{p_n}{\cos(\beta)}$$

$$p_t = \frac{4.45}{\cos(30°)} = 5.17mm$$

c. Pitch diameter of pinion (1)

$$d_1 = m_t Z_1 = \frac{m_n Z_1}{\cos(\beta)} = \frac{p_n Z_1}{\pi \cos(\beta)}$$

$$d_1 = \frac{4.45 \times 18}{3.14 \times \cos(30°)} = 29.66mm$$

d. Pitch diameter of gear (2):

$$r = \frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow d_2 = \frac{N_1}{N_2} d_1$$

$$d_2 = \frac{1725}{575} \times 29.66 = 88.98mm$$

e. Calculating the number of teeth on the gear Z_2

$$r = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} \Rightarrow Z_2 = \frac{N_1}{N_2} Z_1$$

$$Z_2 = \frac{1725}{575} \times 18 = 54$$

Or

$$d_2 = m_t \times Z_2 \Rightarrow Z_2 = \frac{d_2}{m_t} = \frac{88.98}{1.64} = 54.25 \approx 54$$

f. Center distance

$$a_w = \frac{d_2 + d_1}{2} = \frac{m_t (Z_2 + Z_1)}{2} = \frac{m_n (Z_2 + Z_1)}{2\cos(\beta)}$$

$$a_w = \frac{d_2 + d_1}{2} = \frac{88.98 + 29.66}{2} = 59.32mm$$

g. Pitch-line velocity

$$V_2 = V_1$$

$$V_2 = \omega_2 \times R_2$$

$$V_2 = \frac{\pi N_2}{30} \times R_2 = \frac{3.14 \times 575}{30} \times \frac{88.98}{2} \times 10^{-3} = 2.67 m/s$$

h. Numerical values and directions of tangential, radial, and axial forces

we have : $P = C\omega = \frac{F_t d \omega}{2} = \frac{F_t d \pi N}{60}$

Calculation of the pitch diameter

$$d_1 = m_t Z_1 = \frac{m_n Z_1}{\cos(\beta)} = \frac{p_n Z_1}{\pi \cos(\beta)}$$

$$d_1 = \frac{4.45 \times 18}{3.14 \cos(30°)} = 29.66 \approx 30mm$$

then

$$F_t = \frac{60P}{d_1 \pi N} = \frac{60 \times 736}{29.66 \times 10^{-3} \times 3.14 \times 1725} = 274.87N$$

Calculation of the radial force

$$Fr = Ft \frac{\tan(\alpha_n)}{\cos(\beta)} = 274.87 \frac{\tan(20°)}{\cos(30°)} = 115.06N$$

Calculation of the axial force

$$Fa = Ft \tan(\alpha_n) = 274.87 \times 0.36 = 98.95N$$

The total force

$$F = \sqrt{Ft^2 + Fr^2 + Fa^2} = \sqrt{274.87^2 + 115.06^2 + 98.95^2} = 313.75N$$

i. Minimum recommended face width

$$b \geq \frac{\pi m_n}{\sin(\beta)} = \frac{p_n}{\sin(\beta)}$$

$$b \geq \frac{3.14 \times 4.45}{\sin(30°)} = 27.94mm$$

we chose $b = 28mm$

Exercise 4:

A parallel helical gear set uses a 20-tooth pinion driving a 36-tooth gear. The pinion has a right-hand helix angle of 30°, a normal pressure angle of 20°, and a normal circular pitch of 19.23mm. Find:

(a) The transverse and axial circular pitches

(b) The ratio of transmission

(c) The transverse pressure angle

(d) The addendum, dedendum, and pitch diameter of each gear.

Solution:

- The transverse circular pitch

$$p_t = \frac{p_n}{\cos(\beta)} = \frac{19.23}{\cos(30°)} = \frac{19.23}{0.86} = 22.36mm$$

- The axial circular pitch

$$P_x = \frac{P_t}{\tan(\beta)} = \frac{P_n}{\sin(\beta)}$$

We use

$$P_x = \frac{P_n}{\sin(\beta)} = \frac{19.23}{\sin(30°)} = 38.46mm$$

- The ratio of transmission

$$r = \frac{N_2}{N_1} = \frac{Z_1}{Z_2} = \frac{20}{36} = 0.55$$

- The transverse pressure angle

$$\tan(\alpha_t) = \frac{\tan(\alpha_n)}{\cos(\beta)} = \frac{\tan(20°)}{\cos(30°)} = \frac{0.36}{0.86} = 0.41$$

$$\alpha_t = \tan^{-1}(0.41) = 22.29°$$

- **Pinion:**

Pitch diameter (d):

$$d_1 = \frac{p_t}{\pi} \times Z_1 = \frac{22.36}{3.14} \times 20 = 142.42mm$$

Addendum (h_a):

$$ha = \frac{p_n}{\pi} = \frac{19.23}{3.14} = 6.12mm$$

Dedendum(h_f):

$$hf = 1.25\frac{p_n}{\pi} = 7.65mm$$

- **Gear:**

Pitch diameter (d):

$$d_2 = \frac{p_t}{\pi} \times Z_2 = \frac{22.36}{3.14} \times 36 = 256.35mm$$

I. 3. Bevel gears

I. 3.1. Operating principle of a bevel gear

Bevel gear is a mechanism consisting of two bevel gears, this mechanism is used to transmit motion between intersecting shafts. The shafts usually make 90° angle with each other but also other angles are possible.

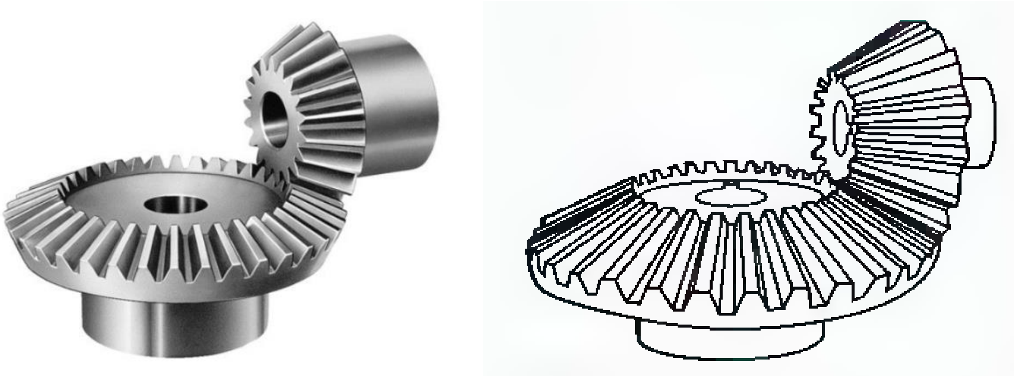


Fig. I. 3. 1: Bevel gears

Like helical gears, bevel gears generate radial and thrust loads on the shafts. Because bevel gears are cut on conical surfaces, they have a large end and a small end.

Because bevel gears are used on intersected shafts, it is extremely difficult to have both gears mounted between two bearings (straddle mounted). In most cases, one of the two gears have to be mounted outboard of the bearings (outboard mounted).

I. 3. 2. Geometric characteristics of bevel gears

In order for two bevel gears intermesh correctly, they must have both a common generator and their summits confused.

A sectional view of two bevel gears in mesh is shown in figure I.3.2. The following terms in connection with bevel gears are important from the subject point view:

- **Pitch angle:** It is the angle made by the pitch line with the axis of the shaft. It is denoted by δ
- **Pitch cone:** It is a cone containing the pitch elements of the teeth.
- **Cone centre:** It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

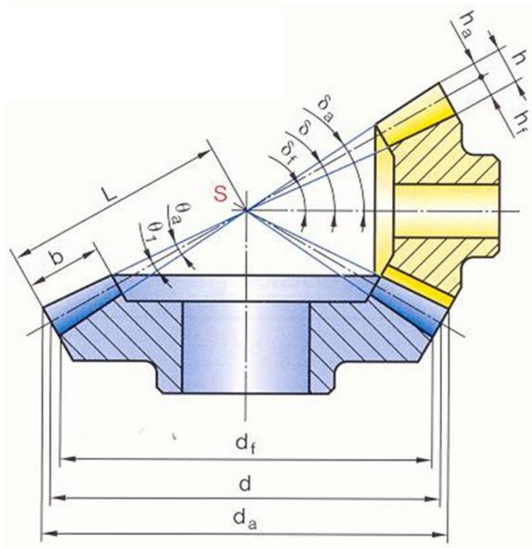


Fig. I. 3. 2: Terms used in bevel gears

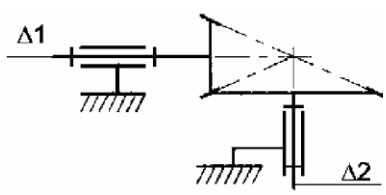
- **Addendum angle:** it is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by δ_a
- **Dedendum angle:** It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by δ_f .
- **Face angle:** It is the angle subtended by the face of the tooth at the cone centre. It is denoted by θ_a .

- **Root angle:** It is the angle subtended by the Root of the tooth at the cone centre. It is denoted by θ_f .

Table I.3.1: Geometrical characteristic of bevel gears

Description	*symbol*	*usual observation and formula*
Module	***m***	*m* is to be chosen from the series of standardized modules
pitch	***P***	$p = \pi m$
pitch diameter	***d***	$d = mZ$
Pitch angle	δ	$\tan(\delta_1) = \frac{Z_1}{Z_2}$; $\tan(\delta_2) = \frac{Z_2}{Z_1}$
Addendum angle	θ_a	$\tan(\theta_a) = 2m.\sin(\delta)/d$
Dedendum angle	θ_f	$\tan(\theta_f) = 2,5m.\sin(\delta)/d$
Face angle	δ_a	$\delta_a = \delta + \theta_a$
Root angle	δ_f	$\delta_f = \delta - \theta_f$
Height angle	θ	$\theta = \theta_a + \theta_f$
Addendum	***ha***	$ha = m$
Dedendum	***hf***	$hf = 1.25m$
Depth of tooth	***h***	$h = ha + hf = 2.25m$
Outside diameter	***da***	$da = d + 2m\cos(\delta)$
Root diameter	***df***	$df = d - 2.5m\cos(\delta)$
Cone distance (generator)	***L***	$L = \frac{d_1}{2.\sin(\delta_1)} = \frac{d_2}{2.\sin(\delta_2)}$
Face width	***b***	$\frac{1}{4}L \le b \le \frac{1}{3}L$
Pressure angle	α	$\alpha = 20°$

I. 3. 3. Transmission ratio



$r_{2/1}=d_1/d_2$, on a $d=2L\sin\delta$.
$r_{2/1}=2L\sin\delta_1/2L\sin\delta_2=\sin\delta_1/\sin\delta_2$.
On a $\delta_2+\delta_1=90°$ d'où $\delta_1=90°-\delta_2$ donc
$\sin\delta_2=\cos\delta_1$
$r_{2/1}=\sin\delta_1/\cos\delta_1=tg\delta_1$.
$r_{2/1}=N_2/N_1=\omega_2/\omega_1=d_1/d_2=Z_1/Z_2=1/(Z_2/Z_1)=1/tg\delta_2$.
$tg\delta_2=Z_2/Z_1$
$tg\delta_1=Z_1/Z_2$

I. 3. 4. Forces acting on a bevel gear

Consider a bevel gear and pinion in mesh. The total force (***F***) on the tooth is perpendicular to the tooth profile and thus makes an angle equal to the pressure angle (α) to the pitch circle.

The conical shape of the teeth generates axial forces (thrust) (F_a) the point of application ***I*** of the force (***F***) carried by ***IK*** on the tooth is assumed to be located in the middle of the teeth (b/2 on each side and on the primitive cone).

The tangential force (F_t) depends on the mean radius (r_m) and not on the pitch radius. The radial force (F_r) like the axial force (F_a) depends on the pitch angle δ of the pitch cone.

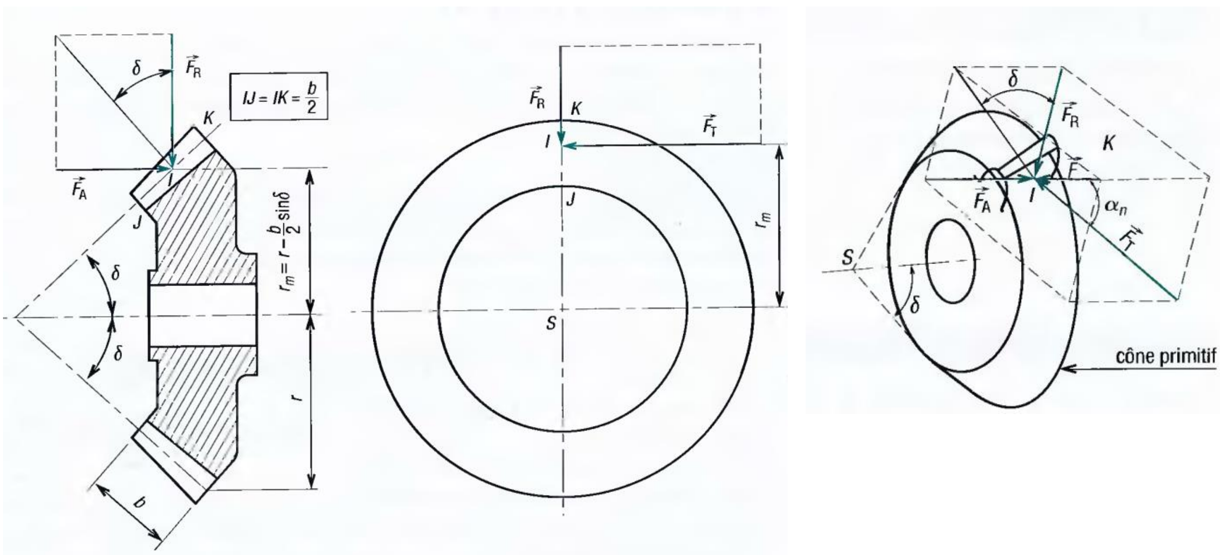


Fig. I. 3. 3: Forces acting in teeth

The magnitude of the tangential, axial and radial components is as follows:

$F_t = \frac{2C}{d_m}$; Where **C** is the torque and $\mathbf{d_m}$ is the mean pitch diameter.

$F_a = F_t.\tan(\alpha).\sin(\delta)$

$F_r = F_t.\tan(\alpha).\cos(\delta)$

The force on teeth:

$$F = \frac{F_t}{\cos(\alpha)}$$

$$F = \sqrt{F_t^2 + F_r^2 + F_a^2}$$

(F_r) and (F_a) do not participate in the transmission of torque ***C***.

If the axes of the two wheels are perpendicular, the axial force on wheel (1) becomes the radial force on wheel (2), and vice versa ($F_{r1}=F_{a2}$ and $F_{a1}=F_{r2}$).

I. 3. 5. Determination of the tooth module

The calculation of the modulus (m) is carried out from the modulus calculated on the mean radius of the primitive cones. This approach is justified by the following considerations:

1. If we calculated the module on the large end of the tooth, the tangential force inversely proportional to the radius, would be less than on the average radius:

$\boldsymbol{F_t=(C/r_m) > (C/r_a)}$.

- The obtained module would be too low than the module calculated on the average radius.
- As the tooth has decreasing thickness from the large end to the tip, the tooth would be too weak.

2. For reasons contrary to the previous ones, the module calculated on the small end of the tooth would give a tooth that is too strong.

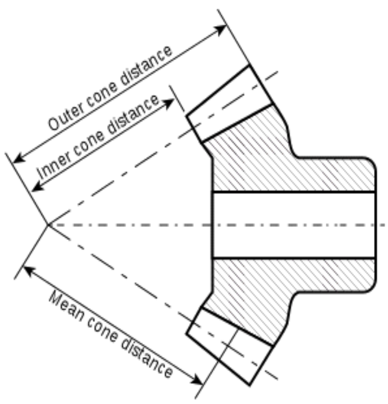


Fig. I. 3. 4: Cone distance

It is therefore logical to calculate the module on the large end of the tooth and correct the value found if necessary, in order to obtain one of the normal modules of cylindrical wheels.

$$m_m \geq 2,34\sqrt{\frac{F_t}{K \times Rp}}$$

- ✓ Standardized module:

If ***d*** is the primitive diameter at the base of the primitive cone we have *d=2r*, *r* is the primitive radius.

$r = r_m + \frac{b}{2}\sin\delta$, from which:

$$m = m_m . \frac{r}{r_m} = m_m . \frac{r_m + \frac{b}{2}\sin\delta}{r_m}$$

Note: It is possible to calculate the average modulus m_m using the relation:

When the number of teeth is known for one of the two gears:

$$m_m \geq \sqrt[3]{\frac{10.94 \times C_m}{K \times Z \times Rp}} .$$

Example

The transmission shaft (2) is connected to the electric motor (1) with a power of ***P*** = 2 kW and rotates at a speed of 1400 rpm. The bevel gears (2) and (3) are straight-toothed. The gears (4) and (5) are spure gears. We give: Z_2 = 32 teeth, Z_3 = 64 teeth, Z_4 = 25 teeth, Z_5 = 80 teeth and the module $m_{(4\text{-}5)}$ = 2 mm. The material of the elements (2-3) has a practical resistance: Rp = 150 N/mm^2. Take (k = 10).

- ✓ Calculate the overall transmission ratio $r_{2/5}$, and then deduce the output speed N_5.
- ✓ Calculate the pitch angles of the gears (2 and 3).
- ✓ Show that the modulus of the gears (2-3) can be obtained using the following formula:

$$m \geq \sqrt[3]{\frac{10.94C}{K \times r_{2/3} \times Z_3 \times Rp}}$$

Where $r_{2/3}$ is the transmission ration of gears (2) and (3).

- ✓ Calculate the geometrical characteristics of the bevel gear (2).

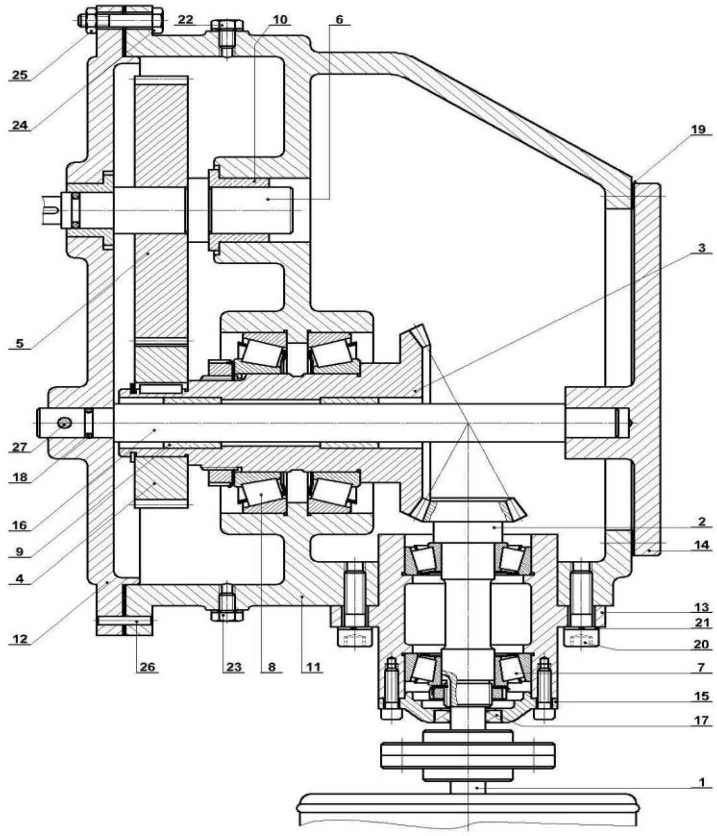


Fig. I. 3. 5: Reducer with bevel gears

Solution

- ✓ Calculate the overall transmission ratio, and then deduce the output speed N_5.

$$r_{tot} = (-1)^2 \times (\frac{Z_2 \times Z_4}{Z_3 \times Z_5}) = \frac{32 \times 25}{64 \times 80} / = \frac{5}{32} = 0.15$$

$$r_{tot} = \frac{N_5}{N_2};\ N_5 = N_2 \times r_{tot} = 1400 \times 0.15 = 210 tr/min$$

- ✓ Calculate of the pitch angles of the gears (2 and 3).

$$\tan(\delta_2) = Z_2 / Z_3 = 32/64\ ; \delta_2 = 26.56°$$

$$\tan(\delta_3) = Z_3 / Z_2 = 64/32\ ; \delta_3 = 63.43°$$

- ✓ Show that the modulus of the gears (2-3) can be obtained by the following formula:

we have:

$$m \geq 2,34\sqrt{\frac{F_t}{K \times Rp}}$$

$$F_t = \frac{C}{d_2/2} = \frac{2C}{d_2} = \frac{2C}{m \times Z_2}$$

$$m \geq 2,34\sqrt{\frac{2C}{m \times Z_2 \times K \times Rp}}$$

$$m^2 \geq 5,47\frac{2C}{m \times Z_2 \times K \times Rp} = \frac{10.94C}{m \times Z_2 \times K \times Rp}$$

$$m^3 \geq \frac{10.94C}{Z_2 \times K \times Rp}$$

$$m \geq \sqrt[3]{\frac{10.94 \times C}{Z_2 \times K \times Rp}}$$

$$m \geq \sqrt[3]{\frac{10.94 \times C}{K \times Rp \times Z_3 \times r_{2/3}}}$$

✓ Calculation of the module $m_{(2\text{-}3)}$

$$m \geq \sqrt[3]{\frac{10.94 \times C}{Z_2 \times K \times Rp}}$$

$$C = \frac{P}{\omega} = \frac{30P}{\pi \times N} = \frac{30 \times 2 \times 10^3}{3,14 \times 1400} = 13,64 N.m$$

$$m \geq \sqrt[3]{\frac{10.94 \times C}{Z_2 \times K \times Rp}} = \sqrt[3]{\frac{10.94 \times 13,64 \times 10^3}{32 \times 10 \times 150}} = \sqrt[3]{3,108} = 1.45$$

$$m \geq 1,45mm$$

We choose *m*=2mm.

✓ Calculate of the geometrical characteristics of the gear (2)

Pitch diameter: $d_2 = m \times Z_2 = 2 \times 32 = 64mm$

Outside diameter: $d_{a2} = d_2 + 2mcos(\delta_2) = 64 + 2 \times 2 \times cos(26.52°) = 67.56mm$

Root diameter: $d_{f2} = d_2 - 2,5mcos(\delta_2) = 64 - 2.5 \times 2 \times cos(26.52°) = 59.55mm$

Addendum, dedendum and height of teeth:

$$h_a = m = 2mm$$
$$h_f = 1.25m = 1.25 \times 2 = 2.5mm$$
$$h = h_a + h_f = 4.5mm$$

Cone distance: $L_2 = \frac{d_2}{2\sin(\delta_2)} = \frac{64}{2\sin(26.52°)} = 72.72mm$

Face width:

$$\frac{1}{4}L \leq b \leq \frac{1}{3}L$$
$$18.18 \leq b \leq 24.24mm$$

Addendum angle:

$$\tan(\theta_{a2}) = \frac{2m\sin(\delta_2)}{d_2} = \frac{2\times 2\times \sin(26.52°)}{64} = 0.027$$

$$\theta_{a2} = 1.57°$$

Dedendum angle:

$$\tan(\theta_{f2}) = \frac{2.5m\sin(\delta_2)}{d_2} = \frac{2.5\times 2\times \sin(26.52°)}{64} = 0.034$$

$$\theta_{f2} = 1.96°$$

Face angle: $\delta_{a2} = \delta_2 + \theta_{a2} = 26.52° + 1.57° = 28.09°$

Root angle: $\delta_{f2} = \delta_2 - \theta_{f2} = 26.52° - 1.96° = 24.56°$

I. 3.6. Spiral bevel gears

They have curved teeth at an angle allowing tooth contact to be gradual and smooth. The transmission efficiency of spiral bevel gear is the highest in all kinds of mechanical transmission. For all kinds of transmission, especially high power drive, is provided with a huge economic benefit. When passing the same torque, the space required of spiral bevel gear is smaller than belt drive and chain drive .The transmission ratio of spiral bevel gear is changeless forever.

Fig. I. 3. 6: Spiral bevel gears

I. 3. 6. 1. Characteristics of Spiral Bevel Gears

1. **Smooth and Quiet Operation:** The helical teeth of spiral bevel gears result in gradual tooth engagement, leading to smoother and quieter operation compared to straight bevel gears. This makes them suitable for applications where noise reduction is important.

2. **High Load-Carrying Capacity:** Spiral bevel gears distribute the load across multiple teeth, enhancing their load-carrying capacity and allowing them to handle higher loads and torque.

3. **Efficiency:** The continuous contact between teeth in spiral bevel gears results in higher gear efficiency compared to straight bevel gears, especially at high speeds.

4. **Versatility:** Spiral bevel gears can be used in a wide range of applications, including automotive, aerospace, industrial machinery, and marine systems.

5. **Precise Tooth Contact:** Spiral bevel gears offer accurate tooth contact, ensuring reliable and precise motion transmission.

6. **Backlash Reduction:** Spiral bevel gears can be designed to minimize backlash, which is the play between gear teeth, contributing to precise motion control.

I. 3. 6. 2. Applications of Spiral Bevel Gears

- **Automotive Transmissions:** Spiral bevel gears are commonly used in automotive rear differentials to transfer power from the driveshaft to the rear wheels. They offer reliable and efficient torque transmission in various road conditions.

- **Aerospace:** Spiral bevel gears are used in aircraft propeller systems and helicopter rotor assemblies. Their smooth operation and high load-carrying capacity are essential in aviation applications.

- **Industrial Machinery:** Gearboxes, machine tools, and printing presses are examples of industrial machinery where spiral bevel gears are found. They provide efficient power transmission and precise motion control.

- **Marine Applications:** Spiral bevel gears are used in marine propulsion systems to transfer power from the engine to the propeller. They are valued for their load-carrying capacity and reliability in harsh marine environments.

- **Robotics:** Spiral bevel gears are used in robotic joints and actuators, where smooth operation and precision are crucial for accurate motion control.

- **Heavy Machinery:** Spiral bevel gears are utilized in heavy-duty machinery like construction equipment and mining machinery due to their ability to handle high loads and provide efficient power transmission.

Overall, the characteristics of spiral bevel gears make them well-suited for applications that require smooth and reliable power transmission, especially in situations involving high loads, high speeds, and the need for low noise and vibration.

Spiral bevel gear is widely used in many industry sector at home and abroad like oilfield petrochemical machinery , Full-size machine tools , various types of machining equipment , engineering machinery , metallurgical equipment , steel rolling machinery , mining machinery , textile machinery , aerospace , electric lift , aircraft industry and so on.

I. 3.7. Applications

Exercise 1

A straight-toothed bevel gear has an 18-tooth pinion meshing with a 54-tooth wheel.
The module is 4 mm. The pressure angle is 20°, and the two shafts are perpendicular.
Determine:

- The pitch;
- The angles of the two pitch cones;
- The pitch diameters;
- The lengths of the pitch cones.

Solution

- Calculating the pitch: $p = \pi \times m = 3{,}14 \times 4 = 12.56mm$
- Calculating of the pitch angles

We have :

$$\tan(\delta_1) = Z_1 / Z_2 = 18/54\ ;\delta_1 = 18{,}43°$$

$$\tan(\delta_2) = Z_2 / Z_1 = 54/18\ ;\delta_2 = 71{,}56°$$

- Calculating of the lengths of pitch cones

$$L = \frac{d_1}{2.\sin(\delta_1)}$$

$$= \frac{mZ_1}{2.\sin(\delta_1)} = \frac{4 \times 18}{2.\sin(18{,}43°)} = 113.87mm$$

Or using:

$$L = \frac{d_2}{2.\sin(\delta_2)}$$

$$= \frac{mZ_2}{2.\sin(\delta_2)} = \frac{4\times 54}{2.\sin(71,56°)} = 113.84mm$$

<u>Exercise 2:</u>

Rotational motion and power are transferred from the drive shaft (1) to the output shaft (24) by a set of a bevel gears with a straight tooth (19-15), and cylindrical gears with straight teeth (1-8) and (8'-14).

Technical data:

Motor rotational speed: $N_m = N_1 = 1500$ rev/mn.

Motor power: $P_m = 2.2$ kW

Total efficiency: $\eta = 0.9$

Transmission ratios: r(8'-14) = 3/5; r(1-8) =2/5

Z_{15}=24; Z_{19}=120 teeth, m=2, $\eta_{Bg} = 0.96$; $a_{(1\text{-}8)}$=24mm

Required work

1. Calculate the ratio of total power transmission.
2. Calculate the rotational speed of the exit shaft (24)
3. Calculate the output power.
4. Calculate the geometrical characteristics of the bevel gear (19 and 15).
5. Calculate the forces acting on the pinion (1).
6. Calculate the forces acting on the bevel gear (15) and present them on the sketch.

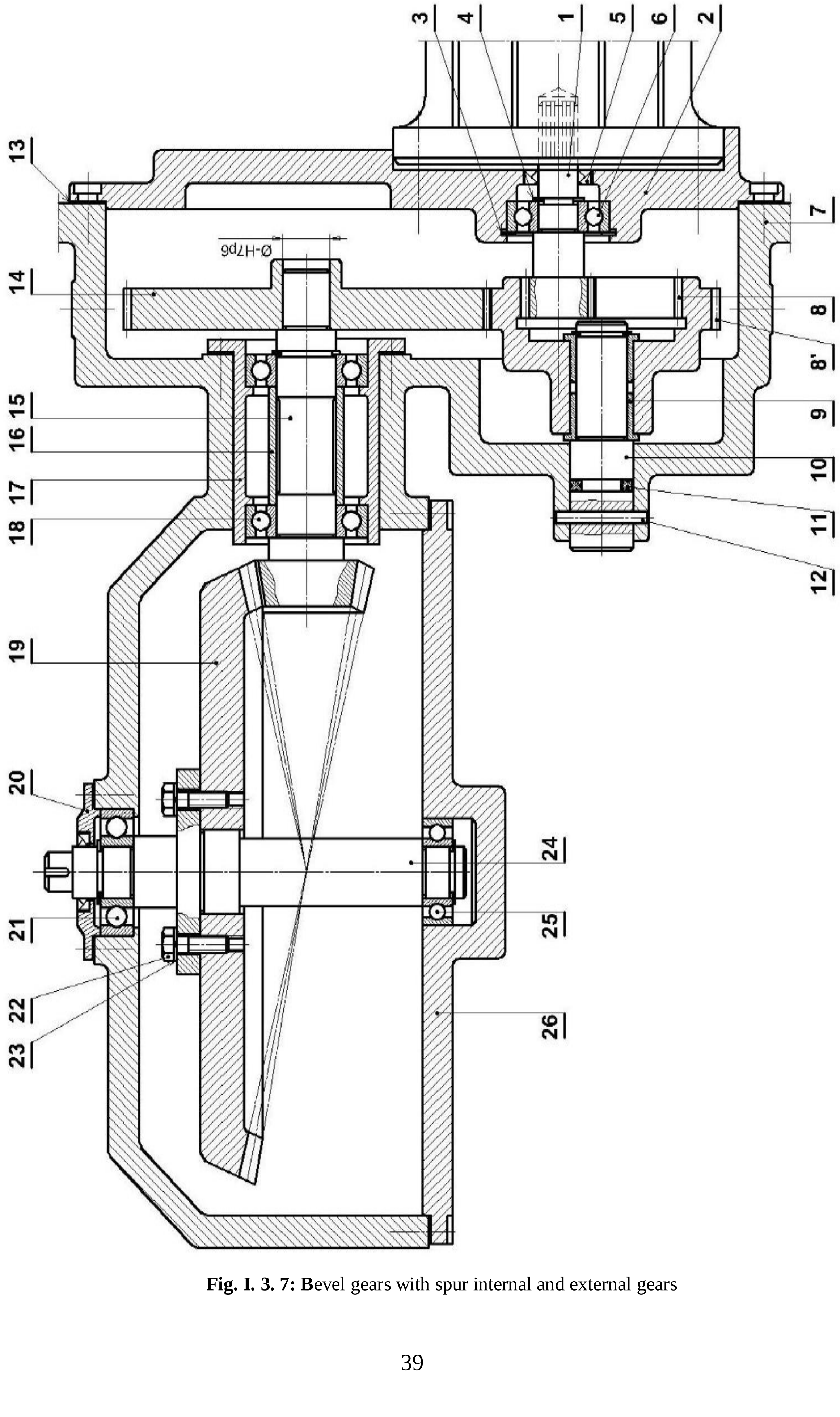


Fig. I. 3. 7: Bevel gears with spur internal and external gears

Solution

1. Calculation of the ratio of the power transmission

$$r_{1/19} = \frac{N_{19}}{N_1} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$r_{1/19} = \frac{N_{19}}{N_1} = (-1)^2 \frac{Z_1 \times Z_{8'} \times Z_{15}}{Z_8 \times Z_{14} \times Z_{19}}$$

$$r_{1/19} = \frac{Z_1}{Z_8} \times \frac{Z_{8'}}{Z_{14}} \times \frac{Z_{15}}{Z_{19}}$$

$$r_{1/19} = r_{1/8} \times r_{8'/14} \times \frac{Z_{15}}{Z_{19}}$$

$$r_{1/19} = \frac{2}{5} \times \frac{3}{5} \times \frac{24}{120} = \frac{6}{125} = 0.048$$

2. Calculation of the rotational speed of the exit shaft (24)

$$r_{1/19} = \frac{N_{24} = N_{19}}{N_1} = \frac{6}{125} \Rightarrow N_{24} = N_1 \times 0.048$$

$$N_{24} = 1500 \times 0.048 = 72\, rev/\min$$

3. Calculation of the output power

$$\eta = \frac{P_{out}}{P_{input}} = \frac{P_{24}}{P_m} \Rightarrow P_{24} = \eta P_m$$

$$P_{24} = 0.9 \times 2.2 \times 10^3 = 1.98 kW$$

4. Calculation of the geometrical characteristics of the bevel gear (19 and 15).

4.1 .Calculation of the pitch angles

$$\tan(\delta_{15}) = \frac{Z_{15}}{Z_{19}} = \frac{24}{120} = 0.2$$

$$\delta_{15} = 11.31°$$

$$\tan(\delta_{19}) = \frac{Z_{19}}{Z_{15}} = \frac{120}{24} = 5$$

$$\delta_{19} = 78.69°$$

$$\delta_{19} + \delta_{15} = 78.69° + 11.31° = 90°$$

4.2 .Calculation of the pitch diameters

$$d_{15} = mz_{15} = 2 \times 24 = 48mm$$

$$d_{19} = mz_{19} = 2 \times 120 = 240mm$$

Cone distance: $L = \frac{d_{15}}{2\sin(\delta_{15})} = \frac{48}{2\sin(11.31°)} = 126.24mm$

Face width: $\frac{1}{4}L \le b \le \frac{1}{3}L$

$$31.56 \le b \le 42.08mm$$

5. Calculation of the forces acting on the pinion (1)

In this case we have a spur gears.

$$P_m = C_1 \times \omega_1 = F_{t1} \times \frac{d_1}{2} \times \frac{\pi N_1}{30} \Rightarrow F_{t1} = \frac{60P_m}{d_1 \times \pi N_1}$$

$$F_{t1} = \frac{60P_m}{d_1 \times \pi N_1}$$

$$r_{1/8} = \frac{2}{5} = \frac{Z_1}{Z_8}$$

$$a_{(1-8)} = \frac{d_8 - d_1}{2} = \frac{m(Z_8 - Z_1)}{2} = 24$$

$$Z_8 - Z_1 = 24; m = 2$$

$$Z_1 = Z_8 \frac{2}{5}$$

$$Z_8 - Z_8 \frac{2}{5} = 24 \Rightarrow Z_8 = 40\,tooth$$

$$Z_1 = Z_8 - 24 = 16\,tooth$$

$$F_{t1} = \frac{60P_m}{d_1 \times \pi N_1} = \frac{60 \times 2.2 \times 10^3}{32 \times 10^{-3} \times 3.14 \times 1500} = 875.79N$$

$$F_{r1} = F_{t1}\tan(\alpha) = 875.79 \times 0.36 = 315.28N$$

$$F = \frac{F_{t1}}{\cos(20)} = \frac{875.79}{0.93} = 941.70N$$

6. Calculation of the forces acting on the bevel gear (15)

the tangential force Ft:

$$P_{15} = C_{15} \times \omega_{15} = F_{t15} \times \frac{d_{m15}}{2} \times \frac{\pi N_{15}}{30} \Rightarrow F_{t15} = \frac{60P_{15}}{d_{m15} \times \pi N_{15}}$$

hence,

$$P_{15} = \eta_{sg} \times P_m$$

and, in the other hand we have:

$$\eta = \eta_{sg} \times \eta_{bg} \Rightarrow \eta_{sg} = \frac{\eta}{\eta_{bg}} = \frac{0.9}{0.96} = 0.93$$

then $P_{15} = 0.93 \times 2.2 = 2.04kW$

now we calculate the mean pitch diameter d_{m15}

$$d_{m15} = d_{15} - b\sin(\delta_{15})$$
$$b = 36.82mm$$
$$d_{m15} = 48 - 36.82\sin(11.31°) = 41mm$$

Calculation of the rotational spssd of pinion (15)

$$\frac{N_{15} = N_{14}}{N_1} = \frac{Z_1 \times Z_{8'}}{Z_8 \times Z_{14}} = r_{1/8} \times r_{8'/14}$$

$$N_{15} = N_1 \times r_{1/8} \times r_{8'/14}$$

$$N_{15} = 1500 \times \frac{3}{5} \times \frac{2}{5} = 360 rev / \min$$

$$F_{t15} = \frac{60 P_{15}}{d_{m15} \times \pi N_{15}} = \frac{60 \times 2.04 \times 10^3}{41 \times 10^{-3} \times 3.14 \times 360} = 2640.98N$$

Calculate of both Fr and Fa

$$Fa = Ft\tan(\alpha)\sin(\delta_{15}) = 2640.98 \times 0.36 \times 0.19 = 180.57N$$

$$Fr = Ft\tan(\alpha)\cos(\delta_{15}) = 2640.98 \times 0.36 \times 0.98 = 931.73N$$

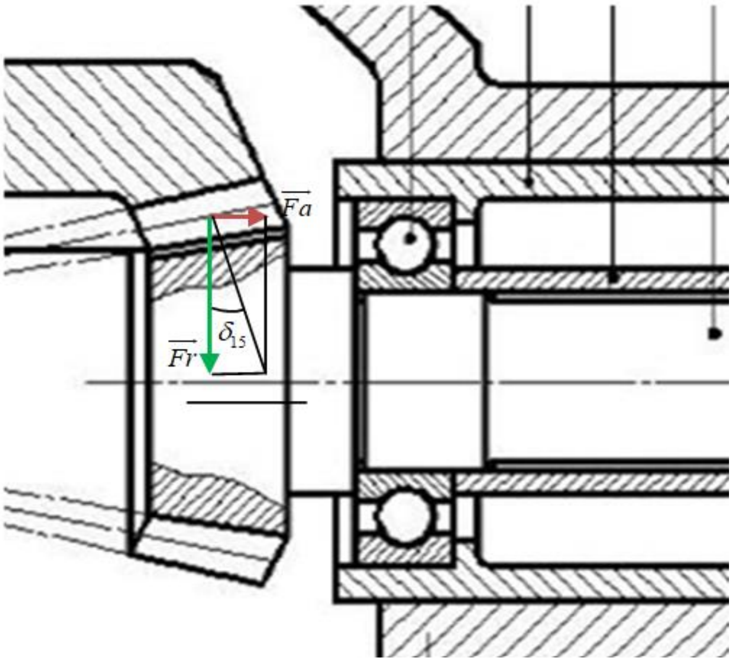


Fig. I. 3. 8: Forces sketch on pinion bevel gears.

II. 4. Worm gearset

A worm gearset is used for transmitting power between two non-parallel and non-intersecting shafts. It is useful when a large speed reduction ratio is required between crossed-axis shafts. A worm is similar to a screw, and a wormgear is similar to a helical gear. So, rotating the worm can cause the worm gear to rotate due to the screw's worm-like action.

II. 4. 1. Operating of a worm gear

A worm gear is a mechanism composed of two elements: the worm (screw 1) with one or more threads, which mesh with the teeth of the wormgear (wheel 2). These gears allow large reduction ratios of around 1/500 and offer possibilities of irreversibility.

They provide the smoothest engagement of all gears, quiet and shock-free.

High sliding and friction cause poor performance. Therefore, good lubrication is essential, as are low-friction material pairs (steel for the worm and bronze for the wormgear).

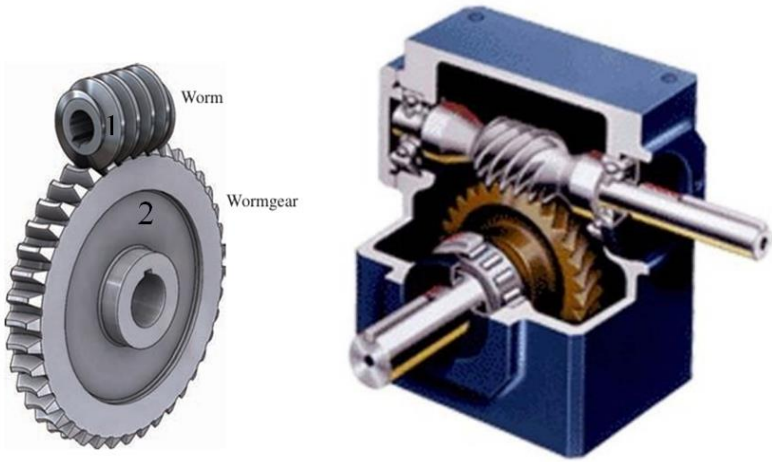


Fig. I. 4. 1: Worm and wormgear

II. 4. 1. 1. Advantages

- ✓ Silent transmission and smooth (noiseless operation);
- ✓ Self-locking action;
- ✓ A large speed ratio;
- ✓ Small drive size
- ✓ Better load distribution;

II. 4. 1. 2. Disadvantages

- ✓ Large power loss: the transmitted power cannot exceed 150 kW because of the very low efficiency (η = 0.8 to 0.85);

- ✓ Expensive antifriction materials, use of high -quality bronzes (Figure I.4.2);
- ✓ Manufacturing by very expensive tools;
- ✓ Considerable sliding speed;
- ✓ Considerable heat generated.

Fig. I. 4. 2: Worm and toothed wheel (wormgear)

II. 4. 2. Classification of worm gears

II. 4. 2. 1. According to the shape of the worm body, we distinguish between cylindrical worm transmissions (Figure I.4.2a) and globular worm transmissions (Figure I.4.2b).

II. 4. 2. 2. Depending on the position of the worm in relation to the wormgear

- ✓ Lower transmissions (Figure I.4.3a)
- ✓ Upper transmissions (Figure I.4.3b)
- ✓ Lateral transmissions (Figure I.4.3c)

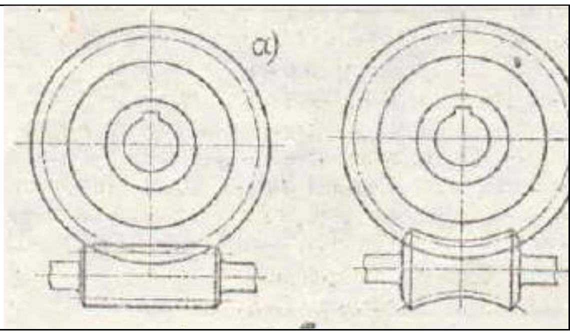


Fig. I. 4. 3: Shape of the worm.

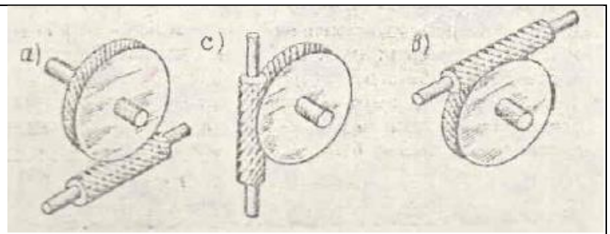


Fig. I. 4. 4: Relative position of worm and wormgear.

II. 4. 2. 3. According to the shape of the worm teeth profile

There are two common types of worm gear sets, which are cylindrical and globoid. Figure II.4.3a shows a diagram of a cylindrical wormset, while Figure II.4.3b shows that of a globoid wormset.

Archimedes' worm is a single-enveloping or single-throated wormset that consists of a cylindrical worm with straight edges engaging a throated gear that partly wraps around the worm. A globoidal worm (Figure I.4.3b) is a double enveloping or double-throated wormset that consists of a throated worm with curved sides engaging a throated gear.

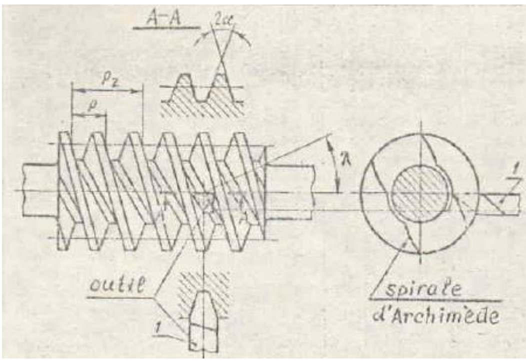


Fig. I. 4. 5: Archimedes spiral worm

Archimedean (Figure I.4.5) worms are more popular than globoid worm sets due to manufacturing difficulties associated with globoid drives, but they have higher power capacity for the same size as cylindrical drives.

II. 4. 3. Geometric characteristics

The ratio of the number of teeth is different from the ratio of the pitch diameters. The characteristics of the gear are those of the cylindrical gear with helical teeth.

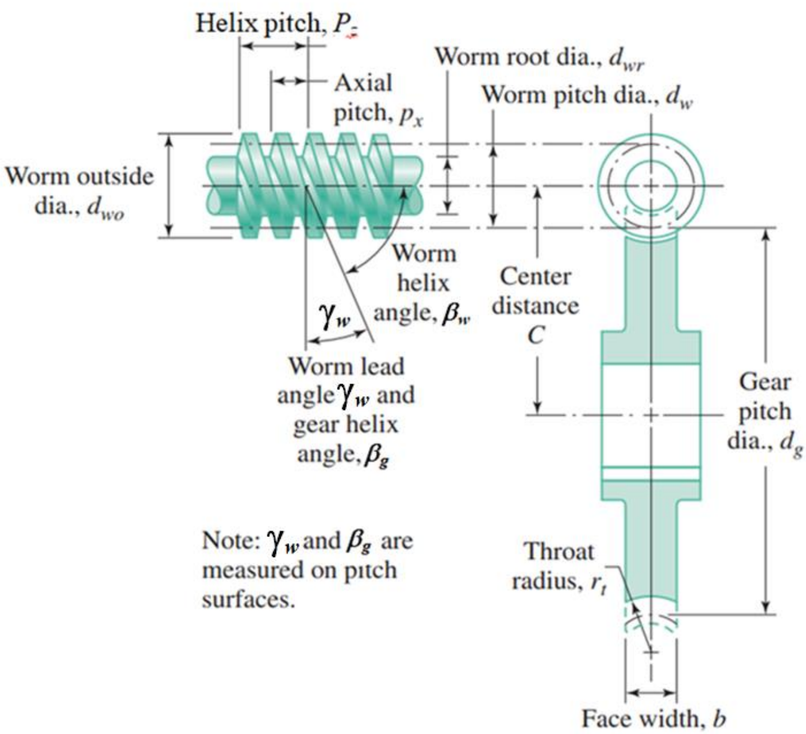


Fig. I. 4. 5: Geometric characteristics of worm and wormgear

Table I. 4. 1: Geometric characteristics of worm and wormgear

Worm characteristics	*Symbols*	*formula*	
Number of screw threads	Z_w	Function of the ratio of angular velocities: $\frac{\omega_w}{\omega_g}=\frac{N_w}{N_g}=\frac{Z_g}{Z_w}$	
Worm helix angle	β_w	irreversibility if $\gamma_w \leq 5°$ $\beta_w+\gamma_w=90°$	
Gear helix angle	β_g	$\beta_w+\beta_g=90°$	$\beta_g=\gamma_w$
Helix direction		The wormgear and the worm have the same helix direction	
Normal worm modulus	m_n	Determine on the wormgear.	
Axial module of the worm	m_x	$m_x=\frac{P_x}{\pi}=\frac{m_n}{\cos\beta_g}=\frac{m_n}{\sin\beta_w}$	
Normal pitch	P_n	$P_n=\pi.m_n$	
Transverse pitch	P_t	$P_t=\frac{P_n}{\cos\beta_g}=\pi.m_t$	
Axial pitch (worm)	P_x	$P_x=P_t$	
Helix pitch	P_z	$P_z=Z_w.P_x$	
Primitive wormgear diameter	d_g	$d_g=m_t.Z_g$	
Primitive Worm diameter	d_w	$d_w=\frac{P_z}{\pi\tan\beta_g}=\frac{P_z}{\pi\tan\gamma_w}$	
Center distance between worm and wormgear	a	$a=\frac{d_g+d_w}{2}$	
Addendum	h_a	$h_a=m_n$	
Dedendum	h_f	$h_f=1.25.m_n$	
Tooth height	h	$h=h_a+h_f$	
Outside worm diameter	da_w	$da_w=d_w+2m_n$	
Root worm diameter	d_{fw}	$d_{fw}=d_w-2.5m_n$	
Normal pressure angle	α_n	$\alpha_n\approx 20°$	
Axial pressure angle of the worm	α_x	$\alpha_x=\alpha_t(gear)$	
Worm length (Lead)	L	$L\approx 5P_x$	

II. 4. 4. Direction of rotation of worm drive

The direction of rotation in a worm and wormgear system of drive will depend upon the direction of the helix of the worm and the position of the wormgear. This can be easily observed in figure I.4.6.

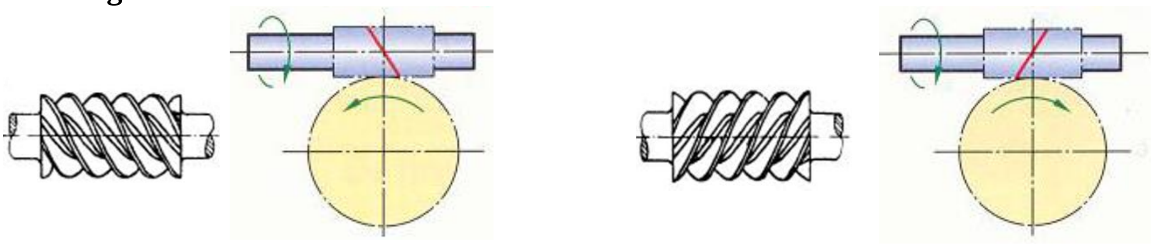


Fig. I. 4. 6: Helix direction

II. 4. 5. Worm gear threads

The number of threads (or starts) on a worm determines how many gear teeth are advanced per revolution. This factor directly impacts the gear reduction ratio, speed control, and overall drive efficiency.

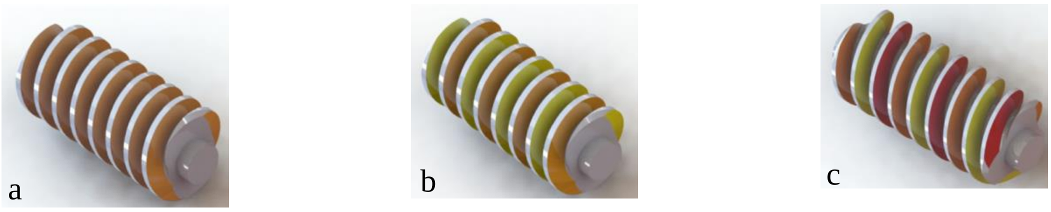


Fig. I. 4. 7: Worm gear threads, (a) single start thread; (b) double start thread; (c) triple start thread

II. 4. 6. Sliding speed

During the operation of the worm transmission, the worm threads slide along the teeth of the gear wheel. The sliding speed has the tangential direction to the screw profile and is determined from the parallelogram of speeds (Figure I.4.8).

$$\vec{V}_{sl} = \vec{V}_w - \vec{V}_g$$

$\vec{V}_w$ is the peripheral speed of the worm and $\vec{V}_g$ the peripheral speed of the gear.

$$V_{sl} = V_w / \cos(\gamma_w) = \omega_w d_w / 2\cos(\gamma_w)$$

We can see that the sliding speed $\vec{V}_{sl}$ is greater than the speed $\vec{V}_w$.

High sliding speed in these transmissions increases the wear of the active surfaces of the wheel teeth. To reduce wear, the worm and the gear must be anti-friction and have sufficient strength and wear resistance.

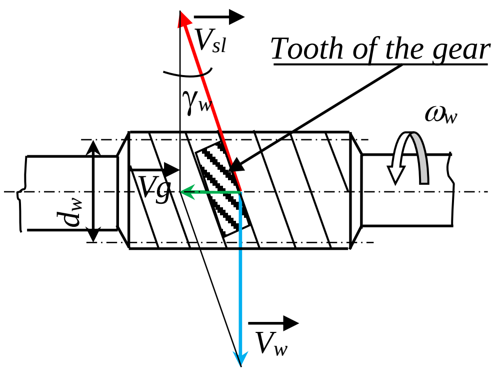


Fig. I. 4. 8: Sliding speed.

II. 4. 7. Forces supported by the worm gear

To reduce the volume of calculation, it is assumed that the force acts in the meshing zone of the worm and the wormgear, which is concentrated and applied at the meshing pole (***p***) in the direction of the common normal to the contact surfaces. Neglecting friction for the time being, the diagram of the distribution of forces in this type of transmission is shown in figure I. 4. 9. The only force $\overrightarrow{F_n}$ is the one acting normal to the tooth profile.

Force $\overrightarrow{F_n}$ can be resolved into three mutually perpendicular components: the tangential component $\overrightarrow{F_t}$, radial component $\overrightarrow{F_r}$ and axial component $\overrightarrow{F_a}$.

- Axial force : $F_a = F_n \cos(\alpha)\cos(\gamma)$
- Tangential force: $F_t = F_n \cos(\alpha)\sin(\gamma)$
- Radial force: $F_r = F_n \sin(\alpha)$

Furthermore, we have:

- Tangential force on the gear ($\overrightarrow{F_{tg}}$) is equal to the axial force on the worm ($\overrightarrow{F_{aw}}$)

$$F_{tg} = F_{aw} = \frac{2.C_g}{d_g} ;$$

- Tangential force on the worm ($\overrightarrow{F_{tw}}$) is equal to the axial force on the gear ($\overrightarrow{F_{ag}}$)

$$F_{tw} = F_{ag} = \frac{2.C_w}{d_w}$$

- Radial forces on the worm ($\overrightarrow{F_{rw}}$) and the worm-wheel ($\overrightarrow{F_{rg}}$)

$$F_{r_w} = F_{rg} = F_{tg} tg\alpha$$

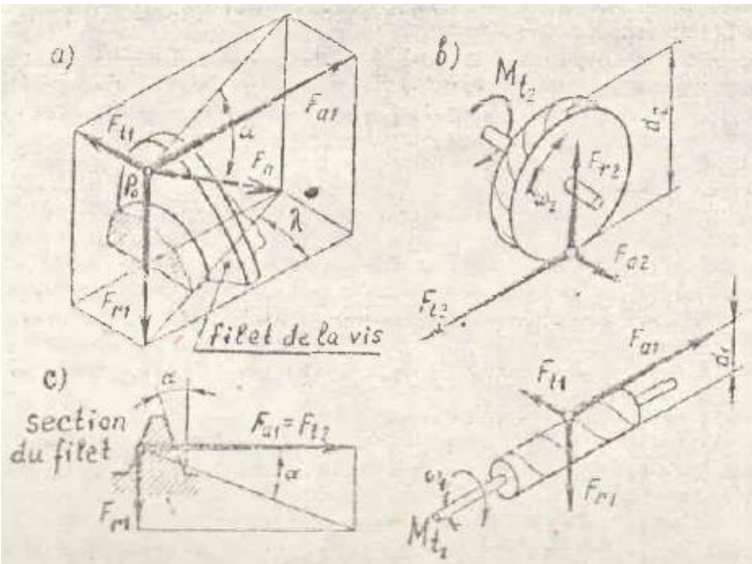


Fig. I. 4. 9: The forces exerted on the gear and the worm.

II. 4. 8. Applications

Exercise 1.

A double-start worm has a helix pitch of 60 mm. The meshing worm gear has 30 teeth, and has been cut using a hob having a module of 8.5 mm in the normal plane. Do the following:

a. Calculate the pitch diameter of the worm.

b. Calculate the pitch diameter of the wormgear.

c. Calculate the center distance.

d. Calculate the reduction ratio of the worm gearset.

e. Calculate the outside diameter of the worm (mm).

f. Calculate the outside diameter of the worm gear (mm).

Solution

a. Calculate of the pitch diameter of the worm

To calculate the pitch diameter of the worm, we use Eq.(1):

$$d_w = \frac{P_z}{\pi \tan(\beta_g)} = \frac{P_z}{\pi \tan(\gamma_w)} \quad (1)$$

Since:

$$P_z = Z_w \times P_x \quad (2)$$

and

$$P_x = P_t(gear) = \pi m_t = \frac{\pi m_n}{\cos(\beta_g)} \quad (3)$$

From Eq.(2):

$$P_x = \frac{P_z}{Z_w} = \frac{60}{2} = 30mm$$

From Eq.(3):

$$\cos(\beta_g) = \frac{\pi m_n}{P_x} = \frac{3.14 \times 8.5}{30} = 0.88 \quad (4)$$

$$\beta_g = 28.35°$$

Therefore:

$$d_w = \frac{60}{3.14 \times \tan(28.35°)} = 36.05mm \quad (5)$$

b. Calculate the pitch diameter of the worm gear.

$$d_g = m_t \times Z_g = \frac{m_n}{\cos(\beta_g)} Z_g$$
$$d_g = \frac{8.5}{0.88} 30 = 289.5mm \quad (6)$$

c. Calculate of the center distance.

$$a = \frac{d_g + d_w}{2} = \frac{36.05 + 289.5}{2} = 162.77mm \quad (7)$$

d. Calculate of the reduction ratio of the worm gearset.

$$r_{w/g} = \frac{N_g}{N_w} = \frac{Z_w}{Z_g} = \frac{2}{30} = \frac{1}{15} \quad (8)$$

e. Calculate of the outside diameter of the worm (mm).

$$d_{aw} = d_w + 2m_n = 36.05 + 2 \times 8.5 = 53.05mm \quad (9)$$

f. Calculate of the outside diameter of the worm gear (mm).

$$d_{ag} = d_g + 2m_n = 289.5 + 2 \times 8.5 = 306.5mm \quad (10)$$

<u>Exercise 2</u>

It is proposed to drive an industrial crushing machine, designed to crush out-of-tolerance scrap ceramic bearing liners, with an in-stock 2-hp, 1200-rpm electric motor coupled to an appropriate speed reducer. The crushing machine input shaft is to rotate at 60 rpm. A worm gear speed reducer is being considered to couple the motor to the crushing machine. A preliminary sketch of the wormset to be used in the speed reducer proposes a double-start right-hand worm with axial pitch of 15.875 mm, a normal pressure angle of 20°, and a center distance of 127mm. The proposed material for the worm is steel with a minimum surface hardness of Rockwell C58. The proposed gear material is forged bronze.

Calculate the following:

a. Number of teeth on the gear.

b. Lead angle of the worm.

c. Sliding velocity between worm and gear.

d. Tangential force on the worm.

e. Axial force on the worm.

f. Radial force on the worm.

g. Tangential force on the gear.

h. Axial force on the gear.

i. Radial force on the gear.

j. Power delivered to the crushing machine input shaft.

k. Whether the wormset is self-locking.

Solution

Given: P =2hp; N_m=1200rev/min; Ng=60rev/min; P_x=15.875mm; a=127mm; α_n=20°

a. Calculate of the number of teeth on the gear (Z_g)
Ration of transmission:

$$r_{w/g} = \frac{N_g}{N_w} = \frac{Z_w}{Z_g} \Rightarrow Z_g = \frac{N_w}{N_g} Z_w = \frac{1200}{60} \times 2 = 40 tooth \tag{1}$$

b. Calculate of the lead angle of the worm.

We have:

$$d_w = \frac{P_z}{\pi \tan(\gamma_w)} \Rightarrow \tan(\gamma_w) = \frac{P_z}{\pi d_w} \tag{2}$$

Calculate of helix pitch P_z

$$P_z = P_x \times Z_w = 15.875 \times 2 = 31.75 mm \tag{3}$$

Calculate of the pitch diameter of the worm d_w

$$a = \frac{d_w + d_g}{2} \Rightarrow d_w = 2a - d_g \tag{4}$$

$$P_x = P_t = \pi m_t \Rightarrow m_t = \frac{P_x}{\pi} = \frac{15.875}{3.14} = 5.05 mm \tag{5}$$

$$d_g = m_t \times Z_g = 5.05 \times 40 = 202 mm \tag{6}$$

Then:

$$d_w = 2 \times 127 - 202 = 52 mm \tag{7}$$

By replacing in Eq.(2) we can obtaining

$$\tan(\gamma_w) = \frac{31.75}{3.14 \times 52} = 0.194$$
$$\gamma_w = 10.97° \tag{8}$$

c. Calculating of the sliding velocity between worm and gear

$$V_s = \frac{\omega_w d_w}{2\cos(\gamma_w)} = \frac{\pi N_w d_w}{60\cos(\gamma_w)}$$
$$V_s = \frac{3.14 \times 1200 \times 52 \times 10^{-3}}{60\cos(10.97°)} = 3.33 m/s \tag{9}$$

d. Calculate of the tangential force on the worm.

We know that the tangential force in the worm is given by the relation:

$$F_{tw} = \frac{2C_w}{d_w} \tag{10}$$

And in other hand, we have:

$$P_M = P_w = C_w \times \omega_w \Rightarrow C_w = \frac{P_w}{\omega_w} = \frac{30P_w}{\pi N_w} \tag{11}$$

Then: $F_{tw} = \frac{60P_w}{\pi d_w N_w} = \frac{60 \times 2 \times 736 \times 10^3}{3.14 \times 52 \times 1200} = 450.75N$ (12)

e. Calculate of the axial force on the worm.

The axial force can be calculated using the relation between them and the tangential force:

$$F_{aw} = \frac{F_{tw}}{\tan(\gamma_w)} = \frac{450.75}{\tan(10.97°)} = 2320.69N \tag{13}$$

f. Calculate of the radial force on the worm.

$$F_{rw} = F_n \sin(\alpha_n)$$

$$F_{tw} = F_n \cos(\alpha_n)\sin(\gamma_w)$$

$$F_{rw} = F_{tw} \times \frac{\tan(\alpha_n)}{\sin(\gamma_w)} \tag{14}$$

$$F_{rw} = 450.75 \times \frac{\tan(20°)}{\sin(10.97°)}$$

$$F_{rw} = 854.05N$$

g. Tangential force on the gear.

$$F_{tg} = F_{aw} = 2320.69N$$

h. Axial force on the gear.

$$F_{ag} = F_{tw} = 450.75N$$

i. Radial force on the gear.

$$F_{rg} = F_{rw} = 854.05N$$

II. Introduction to dynamic study

II. 1. Introduction

The theoretical contact between curved surfaces is generally a point or a line (as a ball or cylinder and plane, a pair of mating gear teeth, etc.). When curved elastic bodies are pressed together, finite contact areas are developed because of deflections. These contact areas are so small, however, that corresponding compressive stresses tend to be extremely high.

In the case of machine components like ball bearings, roller bearings, gears, and cam and followers (figure II.1) these contact stresses at any specific point on the surface are cyclically applied (as with each revolution of a bearing or gear), hence fatigue failures tend to be produced. These failures are caused by minute cracks that propagate to permit small bits of material to separate from the surface. This surface damage, sometimes referred to as “wear,” is preferably called surface fatigue.

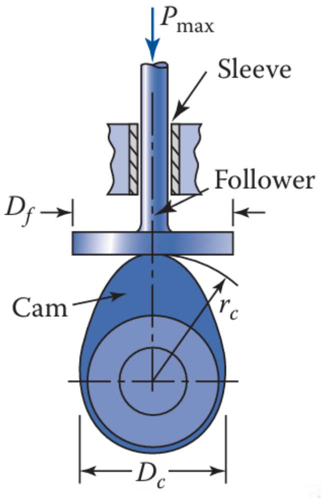


Fig. II. 1: Contact surfaces

II. 2. Surface pressure

Even though a gear tooth may not break due to bending stresses during its life, it could develop pits on the tooth face due to high contact stresses fatiguing the surface by compression (Figure II.2). The contact pressure is intensified near the pitch circle, where the contact is pure rolling with zero sliding velocity. There the elastohydrodynamic oil film is minimal and the load is less distributed.

This condition is modeled as a pair of cylinders in line contact, and a Hertzian contact stress analysis is used.

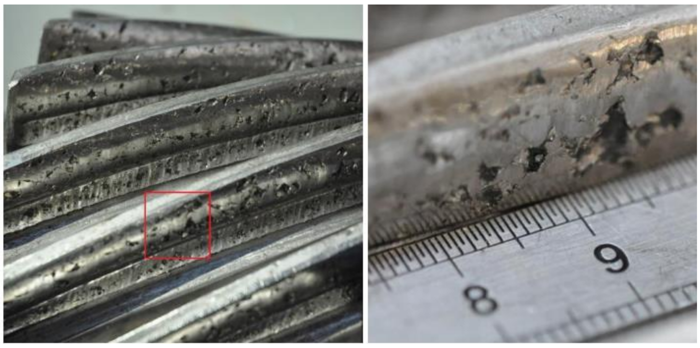


Fig. II. 2: Observation of pitting that spreads and causes flaking.

II. 2. 1 Hertz Theory

We briefly recall the contact theory of Heinrich Rudolf Hertz (1881), in particular the case of linear contact. Let us consider two cylinders of diameters d_1 and d_2 and length L pressed against each other by a force $\vec{F}$.

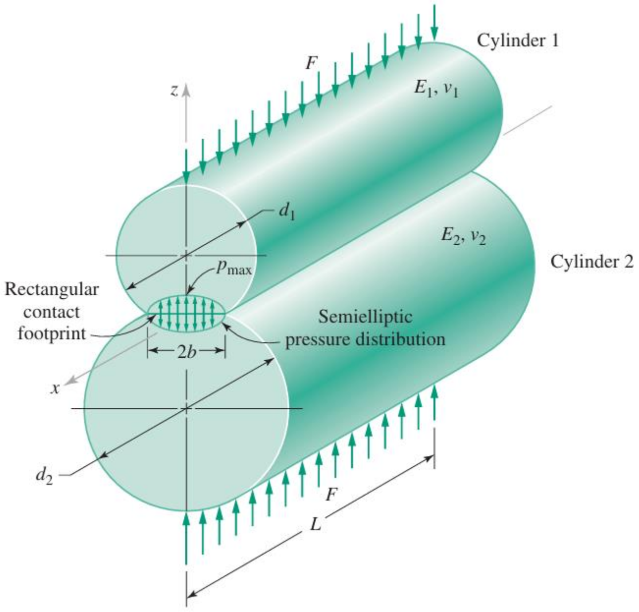


Fig. II. 3: Two parallel cylinders in contact, loaded by force F

At the point of contact, we observe a flattening of the 2 curvatures to form a small flat contact surface whose length is L and whose width is 2b (Figure II.3). The half of width (b) can be calculated by:

$$b = \sqrt{\frac{2F\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}{\pi L\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)}} = \sqrt{\frac{2F}{\pi L}\frac{\Delta}{\rho}} \tag{1}$$

$$\Delta = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right); \qquad (2)$$
$$\rho = \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)$$

with

ν: Poisson's ratio and E: Young's modulus of materials 1 and 2.

In the contact surface, the pressure distribution is semi-elliptical. Its maximum value is:

$$\sigma_{\lim} = \frac{2F}{\pi L b} = \sqrt{\frac{F}{\pi L} \frac{\left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right)}{\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}} = \sqrt{\frac{2F}{\pi L} \frac{\rho}{\Delta}} \qquad (3)$$

These equations can be applied to the case of spur or helical cylindrical gears.

The application of equations 1 and 2 assumes that:

- The contact is frictionless;
- The contacting bodies are elastic, isotropic, homogeneous, and smooth;
- The radii of curvature ρ_1 and ρ_2 are very large in comparison with the dimensions of the boundary of the surface of contact.

The table II.1 presents à values of Young's and Poisson's for some materials.

Table II. 1: Young and Poisson's modulus for different materials.

Materials	Young's Modulus : E (MPa)	Poisson's ratio : ν
Construction steel	2.1 x 10^5	0.27 - 0.30
Stainless steel	2.2 x 10^5	0.3 - 0.31
Malleable iron	1.7 x 10^5	0.21-0.26
Nodular iron	1.7 x 10^5	0.21-0.26
Cast iron	0.83 to 1.70 x 10^5	0.21-0.26
Bronze (copper + 9 to 12% tin)	1.25 x 10^5	0.33
Aluminum	(0.682 – 0.785) x 10^5	0.32 – 0.34

II. 3. Contact pressure in the case of spur gears

F: contact action on the teeth,

$$F_t = F \cos \alpha \Rightarrow F = F_t / \cos \alpha \ \ (in\ N)$$

F_t: tangential force, L: tooth width L=b= K. m (mm)

ρ_1 and ρ_2 : being the radii of curvature (in mm) of the profile sides

$$\sigma_{\lim} = \frac{2F}{\pi L b} = \sqrt{\frac{F}{\pi L} \frac{\left(\frac{1}{\rho_1} \pm \frac{1}{\rho_2}\right)}{\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}} \qquad (3)$$

$\begin{cases} + \text{ for external gears} \\ - \text{ for internal gears} \end{cases}$

We have already noted that the first evidence of wear occurs near the pitch line. The radii of curvature of the tooth profiles at the pitch point are:

$$\rho_1 = \frac{d_p \sin(\alpha)}{2} \qquad \rho_2 = \frac{d_g \sin(\alpha)}{2} \qquad (4)$$

where α is the pressure angle and d_p and d_g are the pitch diameters of the pinion and gear, respectively.

II. 4. Contact pressure in the case of helical gears

To calculate the contact pressure, we need to estimate the length of this contact on the various meshing teeth. This calculation is not simple. If the number of meshing teeth is sufficiently large, the ISO standard estimates the contact length to be equal to *b*. Furthermore, the actual radius of curvature of the teeth (in the normal plane) ρ_n, is expressed relative to the apparent radius of curvature ρ_t, as follows:

$$\rho_n = \frac{\rho_t}{\cos(\beta)} \qquad (5)$$

$$\sigma_{\lim} = \frac{2F}{\pi L b} = \sqrt{\frac{F\cos(\beta)}{\pi L} \frac{\left(\frac{1}{\rho_{t1}} + \frac{1}{\rho_{t2}}\right)}{\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)}} \qquad (6)$$

$$\rho_{t1} = \frac{d_p \sin(\alpha)}{2}; \rho_{t2} = \frac{d_g \sin(\alpha)}{2}$$

With β is the helical angle.

II. 5. Applications

<u>Exercise 1:</u>

A 25mm-diameter cylindrical roller is preloaded against a 75mm-diameter cylindrical roller in a traction drive. The steel rollers are 25mm wide and the preloaded force is 200N. The axes of the cylinders are parallel (see Figure II.4).

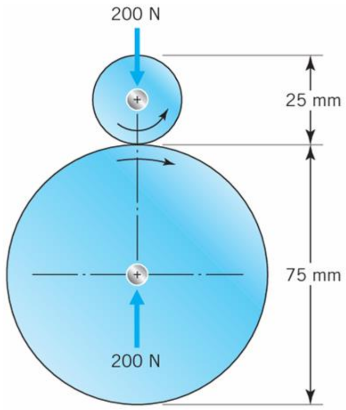


Fig. II. 4: Cylindrical roller

Calculate the maximum contact pressure, the width, and the area of contact.

Given:

$$E_1 = 2.1 \times 10^5 MPa; E_2 = 2 \times 10^5 MPa$$
$$\nu_1 = 0.28; \nu_2 = 0.30$$

Solution

- This case corresponds to the external contact so, from equation (1) we have the maximum contact pressure:

$$\sigma_{\lim} = \sqrt{\frac{F}{\pi L}\frac{\rho_1+\rho_2}{\rho_1\rho_2}\frac{1}{\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}}$$

Let us consider:

$$\Delta = \frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}$$
$$\Delta = \frac{1-0.28^2}{2.1\times10^5}+\frac{1-0.3^2}{2\times10^5} = (0,438+0.455)\times10^{-5} = 8.93\times10^{-5} mm^2/N$$
$$\rho = \frac{\rho_1+\rho_2}{\rho_1\rho_2} = \frac{37.5+12.5}{37.5\times12.5} = 0.106 mm^{-1}$$

then

$$\sigma_{\lim} = \sqrt{\frac{F}{\pi L}\frac{\rho}{\Delta}}$$

$$\sigma_{\lim} = \sqrt{\frac{200}{3.14\times25}\times\frac{0.106\times10^5}{8.93}}$$

$\sigma_{\lim} = 54.99 \quad N/mm^2$
$\sigma_{\lim} = 55 \quad MPa$

- Calculation of the width of the contact area.

To calculate b, we use:

$$b = \sqrt{\frac{2F}{\pi L}\frac{\Delta}{\rho}} = \sqrt{\frac{2\times 200\times 8.93\times 10^{-5}}{3.14\times 25\times 0.106}} = 0.065mm$$

- The area is :

$$A = b\times L = 0.065\times 25 = 1.62 \quad mm^2$$

Exercise 2

A gear pair is to be designed to transmit 15.0 kilowatts of power to a large meat grinder in a commercial meat processing plant. The pinion (Z_p = 18; m=5) is attached to the shaft of an electric motor rotating at 575 rpm. The gear must operate at 272 rpm. The gear unit will be enclosed and of commercial quality. Pressure angle is equal to 20°.

1. Specify the design of the gears. k=10

2. Calculate the pressure surface between the pair gear.

Given: E_1=2*10^{5}MPa and E_2=2.1*10^{5}MPa; ν_1= ν_2=0.3

Solution

1. Specify design:

Calculation of the number of the teeth gear Z_g

The ratio of transmission power is:

$$\frac{N_g}{N_p} = \frac{Z_p}{Z_g} \Rightarrow Z_g = Z_p\frac{N_p}{N_g} = 18\frac{575}{272} = 38 \quad teeth$$

Calculation of the pitch diameters of the pinion and the gear

$$d_p = mZ_p = 18\times 5 = 90mm$$

$$d_g = mZ_g = 38\times 5 = 190mm$$

Calculation of the centre distance

$$a_w = \frac{d_p + d_g}{2} = \frac{90+190}{2} = 140mm$$

Calculation of the width of the gear

$b = km = 10 \times 5 = 50mm$

2. Calculation of the pressure surface between the pair gears

In this case we have an external gear contact, so we use equation (3) in the form:

$$\sigma_{\lim} = \sqrt{\frac{F}{\pi L} \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \frac{1}{\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}} = \sqrt{\frac{F}{\pi L} \frac{\rho}{\Delta}}$$

Hence

$$\rho = \frac{\rho_1 + \rho_2}{\rho_1 \rho_2}$$

$$\rho_1 = \frac{d_p}{2} \sin(20°) = \frac{90}{2} \sin(20°) = 45 \times 0.34 = 15.3mm$$

$$\rho_2 = \frac{d_g}{2} \sin(20°) = \frac{190}{2} \sin(20°) = 95 \times 0.34 = 32.3mm$$

$$\rho = \frac{15.3 + 32.3}{15.3 \times 32.3} = 0.096mm^{-1}$$

$$\Delta = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

$$\Delta = \frac{1-0.3^2}{2.1 \times 10^5} + \frac{1-0.3^2}{2 \times 10^5} = (0,433 + 0.455) \times 10^{-5} = 8.88 \times 10^{-5} mm^2 / N$$

Calculate of the normal force

$$F = \frac{F_t}{\cos(\alpha)}$$

$$F_t = \frac{60P}{\pi d_p N_p} = \frac{60 \times 15 \times 10^3}{3.14 \times 90 \times 10^{-3} \times 575} = 5538.63N$$

$$F = \frac{5538.63}{\cos(20°)} = \frac{5538.63}{0.93} = 5955.51N$$

$$\sigma_{\lim} = \sqrt{\frac{F}{\pi L} \frac{\rho}{\Delta}} = \sqrt{\frac{5955.51}{3.14 \times 50} \times \frac{0.096}{8.88 \times 10^{-5}}} = 202.50N / mm^2$$

$$\sigma_{\lim} = 202.50MPa$$

Exercise 3

An electric motor rotating at 750 rpm transmits a power of 4 kW to a driven shaft which must rotate at 250 rpm. The gears used are cylindrical with helical teeth and parallel axes with β = 25°; the approximate center distance a_w = 240 mm and Rpe = 30 MPa, K = 10

1) Determine the cutting elements for the two gears.

2) Determine the pressure superficial between the teeth on contact.

3) Deduce the width of the surface area and the surface corresponding.

$E_1 = 2 \times 10^5 MPa; E_2 = 2 \times 10^5 MPa$

$\nu_1 = 0.3; \nu_2 = 0.30$

Solution

1. Determination of the cutting elements

First, we start with the calculation of the pitch diameters

$$\frac{N_g}{N_p} = \frac{Z_p}{Z_g} = \frac{d_p}{d_g} = \frac{250}{750} = \frac{1}{3}$$
$$d_g = 3d_p$$

Hence,

$$a_w = \frac{d_p + d_g}{2} = 240 \Rightarrow d_p + d_g = 240 \times 2 = 480$$
$$d_g = 3d_p$$
$$d_p + d_g = 480$$
$$4d_p = 480 \Rightarrow d_p = \frac{480}{4} = 120mm$$
$$d_g = 3 \times 120 = 360mm$$

Calculation of the real module

$$m_n \geq 2.34\sqrt{\frac{Ft}{Rpe \times k}}$$

The tangential force can be calculated from:

$$Ft = \frac{60P}{\pi d_p \times N_p} = \frac{60 \times 4 \times 10^3}{3.14 \times 120 \times 10^{-3} \times 750} = 849.25N$$

$$m_n \geq 2.34\sqrt{\frac{849.25}{30 \times 10}} = 3.93$$
$$m_n \geq 3.93$$
$$m_n = 4$$

Calculation of the transverse module

$$m_t = \frac{m_n}{\cos(\beta)} = \frac{4}{\cos(25)} = 4.44$$

Then, the number of teeth is:

$$d_p = m_t \times Z_p \Rightarrow Z_p = \frac{d_p}{m_t} = \frac{120}{4.44} = 27 tooth$$

$$d_g = m_t \times Z_g \Rightarrow Z_g = \frac{d_g}{m_t} = \frac{360}{4.44} = 81 tooth$$

- Calculation of the teeth width

$$b \geq \frac{\pi m_n}{\sin(\beta)} = \frac{\pi \times 4}{\cos(25°)} = 29.90 mm$$

$$b \approx 30 mm$$

2. Calculation of surface stress

$$\sigma_{\lim} = \sqrt{\frac{F\cos(\beta)}{\pi L}\frac{\left(\frac{1}{\rho_{t1}}+\frac{1}{\rho_{t2}}\right)}{\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} = \sqrt{\frac{F_t\cos(\beta)}{\pi L\sin(\alpha)}\frac{\rho_t}{\Delta}}$$

with:

$$\rho_t = \left(\frac{1}{\rho_{t1}}+\frac{1}{\rho_{t2}}\right)$$

$$\rho_{t1} = \frac{d_p \sin(20°)}{2} = \frac{120}{2}\sin(20°) = 20.4 mm$$

$$\rho_{t2} = \frac{d_g \sin(20°)}{2} = \frac{360}{2}\sin(20°) = 61.2 mm$$

$$\rho_t = \frac{1}{\rho_{t1}}+\frac{1}{\rho_{t2}} = \frac{1}{20.4}+\frac{1}{61.2} = 0.065 \quad mm^{-1}$$

$$\Delta = \frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2} = 2\frac{1-0.3^2}{2\times 10^5} = 0.91\times 10^{-5} mm^2/N$$

$$\sigma_{\lim} = \sqrt{\frac{F_t\cos(\beta)}{\pi b s\cos(\alpha)}\frac{\rho_t}{\Delta}} = \sqrt{\frac{849.25\times\cos(25°)}{3.14\times 30\times\cos(20°)}\frac{0.065}{0.91\times 10^{-5}}}$$

then, we obtain

$$\sigma_{\lim} = 249.63 N/mm^2 = 249.63 MPa$$

3) The width of the contact surface area

$$\sigma_{\lim} = \frac{2F}{\pi L b} \Rightarrow b = \frac{2F}{\pi L \sigma_{\lim}}$$

$$b = \frac{2 \times 846.25}{3.14 \times 50 \times 249.63} = 0.043mm$$

the surface area is

$$A = Lb = 50 \times 0.043 = 2.16mm^2$$

Chapter III: Shafts and Axles

III. 1. Introduction

Virtually all machines involve the transmission of power and/or motion from an input source to an output work site. The input source, usually an electric motor or internal combustion engine, typically supplies power as a rotary driving torque to the input shaft of the machine under consideration, through some type of a coupling.

A shaft is typically a relatively long cylindrical element supported by bearings, and loaded torsionally, transversely, and/or axially as the machine operates.

An axle, a non rotating member that carries no torque, is used to support rotating parts.

III. 2. Types of shafts and axles

According to their geometries, the axles are straight (it's a ruler).

For the shafts, they are classified according to:

III. 2. 1. The geometric shape of the axis

- ✓ The straight shaft (constant diameter)

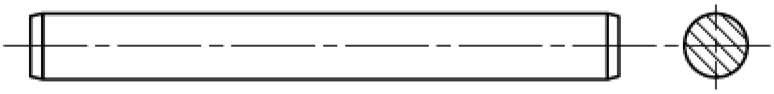


Fig. III. 1: Straight shaft.

- ✓ The coded shaft (crankshaft)

Fig. III. 2: Coded shaft.

- ✓ The flexible shaft.

Fig. III. 3: Flexible shaft

- ✓ The smooth (none stepped) shafts (Fig. III. 1).
- ✓ The stepped shafts.

The formation of the steps on the shafts is necessary due to the requirements of manufacturing, assembly and disassembly.

III. 2. 2. According to the types of transverse sections

- ✓ The solid shafts.
- ✓ The hollow shafts.

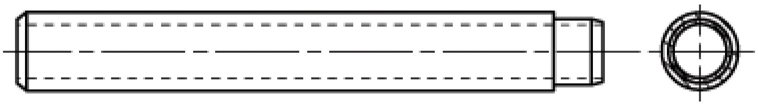


Fig. III. 4: Hollow shafts

- ✓ Fluted shafts.

Fig. III. 5: Fluted shafts

In special applications, shafts may be square, rectangular, or some other cross sectional shape.

We only examine straight, stepped shafts.

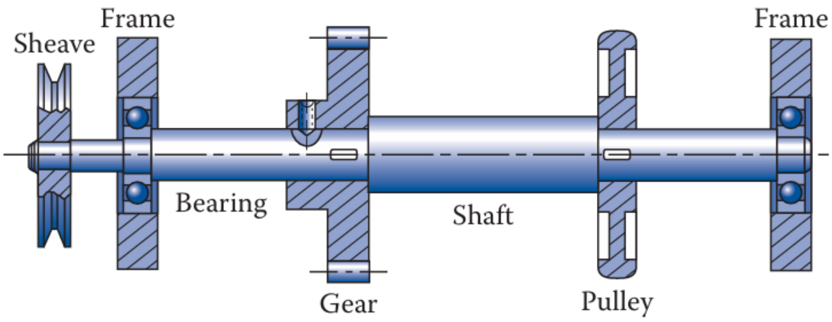


Fig. III. 6: A stepped shaft with various elements attached

III. 3. Shaft calculation criteria

The calculation of the forces can generally be carried out on a schematic diagram of the kinematic chain. The preliminary project consists of drawing the actual shaft, in such a way that it is capable of supporting these forces:

- by resisting long enough,
- Without deforming too much,
- Without exaggerated vibrations.

And, as an absolute condition, it must be achievable as simply as possible. Let us add that in certain applications, weight conditions are significantly involved (automobile and especially aeronautics).

III. 3. 1. Mechanical stresses

III. 3. 1. a. Tensile and compression

When a material is subjected to an external force (***F***), a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point.

The normal stress is given by:

$$\sigma = \frac{N}{A} \quad (1)$$

The condition of resistance is given by:

$$\sigma \leq \sigma_{adm} \quad (2)$$

Fig. III. 7: Traction of solid shaft.

III. 3. 1. b. Torsion

Torque is a moment that twists a structure. Unlike axial loads which produce a uniform, or average, stress over the cross section of the object, a torque creates a distribution of stress over the cross section. To keep things simple, we're going to focus on structures with a circular cross section, often called shafts. When a torque is applied to the structure, it will twist along the long axis of the rod, and its cross section remains circular.

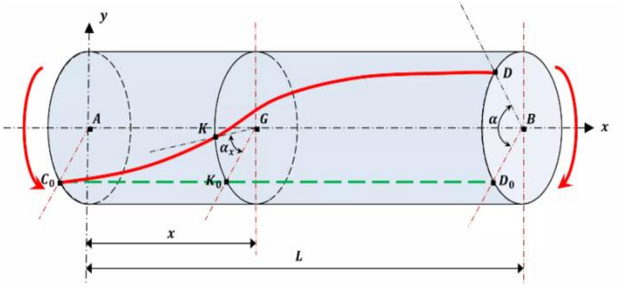


Fig. III. 8: Shaft torsion angle.

The tangential stress is given by: $\tau = \dfrac{Mt}{(I_0/\rho)}$ (2)

such as

I_0: is the polar moment relative to the center of gravity of the cross section, for a full circular section: $I_0 = \dfrac{\pi d^4}{32}$; $\rho_{\max} = \dfrac{d}{2}$.

$\left(\dfrac{I_0}{\rho}\right)$: is the torsion modulus; *Mt:* is the torsion moment.

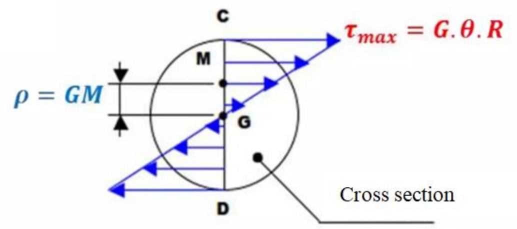


Fig. III. 9: Distribution of the tangential stress over the cross section.

- The torsion resistance condition is:

$$\tau_{\max} = \frac{Mt}{(I_0/\rho_{\max})} \leq Rpg \qquad (3)$$

Rpg : practical shear resistance (N/mm^2). $R_{pg} = \frac{R_{eg}}{s}$, s: is the safety coefficient.

- Deformation condition:

$$\theta = \frac{Mt}{GI_0} \leq \theta_{adm} \qquad (4)$$

With: $\theta = \frac{\alpha}{L}$; α is the rotation angle in radian and L the length in meters.

θ : is the unit strain angle [rad/m]

G : is the transverse elasticity coefficient [N/m^2].The following table provides a reminder of the mechanical characteristics of some materials :

Table III. 1: Mechanical characteristics of some materials

Materials	**Re**(MPa)	**Rr** (MPa)	**E** (MPa)	**G** (MPa)
General purpose steel	235	340	200 000	80 000
Special steel	700	930	200 000	80 000
Font	200	320	80 000 to 170 000	40 000
Aluminum	200	330	72 000	32 000

III. 3. 1. c. Flexion

Bending is the deformation of an object (beam) under the action of a load. It results in a curvature. The pure bending normal stress is given by:

$$\sigma = \frac{M_f}{(I_{GZ}/\rho)} \qquad (5)$$

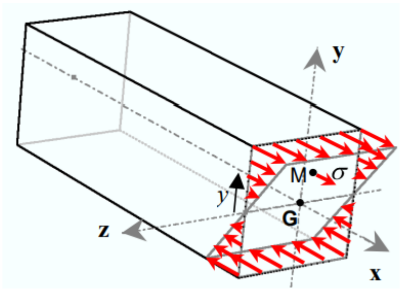


Fig. III. 10: Distribution of the normal stress over the cross section.

Such that: I_{GZ} is the quadratic moment of cross-section with respect to the axis (Gz). [m ^{4}].

For a round and solid section $I_{GZ} = I_{Gy} = \frac{\pi d^4}{64}$

It follows from the general case that the maximum principal stress is equal according to Rankine's formula:

$$\sigma_1 = \frac{1}{2}\sigma + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} \tag{6}$$

$$\sigma_1 = \frac{1}{2}\left[\frac{M_f}{0.1d^3} + \sqrt{\left(\frac{M_f}{0.1d^3}\right)^2 + \left(\frac{Mt}{0.1d^3}\right)^2}\right]$$

$$\sigma_1 = \frac{1}{2}\frac{1}{0.1d^3}\left[M_f + \sqrt{M_f{}^2 + Mt^2}\right]$$

we pose:

$$M_{if} = \frac{1}{2}\left[M_f + \sqrt{M_f{}^2 + Mt^2}\right] \tag{7}$$

which presents the perfect moment.

$\sigma_1 = \sigma_{\max} = \frac{M_{if}}{0.1d^3}$

- **The resistance condition** : $\sigma_{\max} \le \sigma_{adm}$

This is the maximum stress of fictitious bending in a shaft undergoing a compound stress (torsion + bending).

III. 3. 2. Calculation of shaft

The shaft is guided by bearings, we have the transmitted power:

$P = C.\omega$

1. Indication of the loads applied to the shaft (calculation of reactions).

2. The determination of the applied loads, knowing that the transmitted power is:

$P_a = P_m.\eta$

3. The preliminary design calculation.

- Construction of bending distribution, horizontal plane, vertical plane,
- $Mf_t = \sqrt{Mf^2{}_H + Mf^2{}_V}$
- Construction of torsion distribution.

We are looking for the dangerous section, where $Mf = Mf_{\max}$ and $Mt = Mt_{\max}$

$$\sigma_{max} = \frac{M_{if}}{0.1d^3} \le \sigma_{adm} \Rightarrow d \ge \sqrt[3]{\frac{M_{if}}{0.1.\sigma_{adm}}}$$

III. 4. Applications

Exercise 1

Consider a shaft with a solid circular cross-section. Its diameter is d = 30 mm, and the torsionally moment applied at its ends is Mt = 50 Nm. This shaft is made of steel with G= 80 000 MPa.

1. Calculate the unit angle of twist in °/m.
2. Calculate the maximum shear stress.

Solution

1. Calculation of the unit angle:

we have from equation (4)

$$\theta = \frac{Mt}{GI_0} \text{ and } I_0 = \frac{\pi d^4}{32} = \frac{3.14 \times 30^4}{32} = 79481.25 mm^4 = 79481.25 \times 10^{-12} m^4$$

and from it

$$\theta = \frac{50 \times 10^3}{80 \times 10^3 \times 10^6 \times 79481.25 \times 10^{-12}} = 0.007 rd / m$$

Conversion to °/m:

$$\left.\begin{matrix} \pi \to 180° \\ 0.007 \to \theta° \end{matrix}\right\} \theta° = \frac{180° \times 0.007}{3.14} = 0.40°$$

$$\theta = 0.40° / m$$

2. Calculation of the maximum shear stress.

$$\tau_{max} = \frac{Mt}{(I_0/\rho_{max})}; \rho_{max} = \frac{d}{2}$$

$$\tau_{max} = \frac{50 \times 10^3}{(79481.25/15)} = 9.43 N / mm^2$$

Exercise 2

Consider a cylindrical copper specimen with a diameter of 25 mm subjected to a torque of 210 Nm during a torsion test. The measured angle of torsion is 4.9° for a length of 1 m.

1. Calculate the transverse modulus of elasticity G of the tested copper.
2. Determine the angle of twist of a beam made of the same material, with the same diameter and a length of 1.8 m, if it is subjected to a maximum shear stress of 140 N/mm².

Solution

1. Calculation of the transverse modulus of elasticity **G** of the tested copper.

We start from:

$$\theta = \frac{Mt}{GI_0} \text{ then}: G = \frac{Mt}{\theta I_0}; I_0 = \frac{\pi d^4}{32} \Rightarrow G = \frac{32Mt}{\theta \pi d^4}$$

Conversion of °/m to the rd/m

$$\left.\begin{array}{l} \pi \to 180° \\ \theta \to 4.9°/m \end{array}\right\} \theta(rd) = \frac{3.14 \times 4.9}{180°} = 0.085 rd/m$$

$$\theta = 0.085 rd/m$$

so

$$G = \frac{32 \times 210}{0.085 \times 3.14 \times 25^4} = 64455.60 MPa$$

2. Determination of the angle of twist, from equation:

$$\theta = \frac{Mt_{\max}}{GI_0}$$

Calculation of the *Mt*

$$\tau_{\max} = \frac{Mt_{\max}}{\left(I_0/\rho_{\max}\right)}; \rho_{\max} = \frac{d}{2}; \left(I_0/\rho_{\max}\right) = \frac{\pi d^3}{16}$$

$$Mt_{\max} = \tau_{\max} \frac{\pi d^3}{16}$$

$$Mt_{\max} = \frac{140 \times 10^6 \times 3.14 \times \left(25 \times 10^{-3}\right)^3}{16} = 429.29 Nm$$

$$\theta = \frac{Mt_{\max}}{GI_0} = \frac{429.29}{64455.60 \times 10^6 \times 38330.07 \times 10^{-12}} = 0.173 rd/m$$

From other side

$$\theta = \frac{\alpha}{l} \Rightarrow \alpha = \theta l = 1.8 \times 0.173 = 0.311 rd$$

Conversion to the degree

$$\left.\begin{array}{l} \pi \to 180° \\ 0.3114 \to \alpha \end{array}\right\} \alpha = \frac{180° \times 0.3114}{180°} = 17.57°$$

<u>Exercise 3</u>

Figure (III. 11) shows a paint mixer. The shaft rotates at a speed of 630 rpm to transmit a power of 1.4 kW. It can be approximated as a cylindrical beam. Its length is 500 mm. The yield strength of the shaft material is 200 MPa. We want to verify the sizing of the propeller shaft. We take the safety factor s = 2.

First case 1: the shaft is considered a solid shaft.

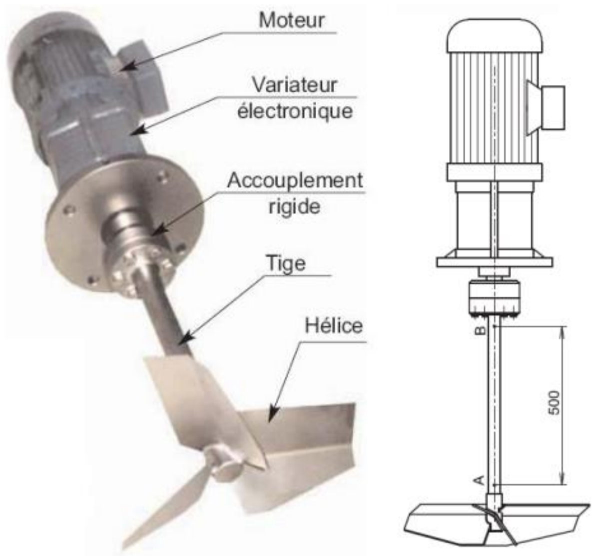


Fig. III. 11: Paint mixer

1. Calculate the transmitted torsionally moment.
2. Calculate the diameter of the rod.
3. Calculate the angle of twist between the two ends of the rod if G = 80 GPa.
4. Calculate the new diameter of the rod in the case where the unit angle of twist must not exceed 0.1 degrees per meter.
5. Deduce the diameter of the rod that meets both conditions (strength and stiffness).

Second case: the rod is considered a hollow shaft whose outside diameter is twice it's inside diameter.

1. Calculate the outside diameter of the hollow rod.
2. Calculate the angle of twist between the two ends of the hollow rod if G = 80 GPa.
3. Calculate the new outside diameter of the hollow rod if the unit angle of twist must not exceed 0.1 degrees per meter.
4. Deduce the outside diameter of the hollow rod that meets both conditions (strength and stiffness).
5. Calculate the ratio (mass of the solid rod / mass of the hollow rod).

6. What can be concluded?

Solution

First case solid shaft

1. Calculation of the transmitted torsionally moment.

$$P = Mt\omega = \frac{\pi N Mt}{30}$$

$$Mt = \frac{30}{\pi N} = \frac{30 \times 1.4 \times 10^3}{3.14 \times 630} = 21.23 Nm$$

2. Calculation of the diameter of the rod

Condition of resistance gives:

$$\tau_{\max} \leq Rpg$$

$$\frac{Mt}{\left(I_0 / \rho_{\max}\right)} \leq Rpg; \qquad I_0 / \rho_{\max} = \frac{\pi d_1^3}{16}$$

$$\frac{16Mt}{\pi d_1^3} \leq Rpg$$

$$16Mt \leq \pi d_1^3 Rpg$$

$$d_1^3 \geq \frac{16Mt}{\pi Rpg}$$

$$d_1 \geq \sqrt{\frac{16Mt}{\pi Rpg}}$$

so

$$d_1 \geq \sqrt{\frac{16 \times 21.23 \times 10^3 \times 2}{3.14 \times 200}} = 10.26mm$$

$$d_1 \geq 10.26mm$$

3. Calculation of the twist angle between the two ends of the rod. Given: G = 80 GPa.

From: $\theta = \frac{Mt}{GI_0}$ with $\theta_{\lim} = 0.1° / m$

$$\theta = \frac{\alpha}{L} = \frac{Mt}{GI_0} \Rightarrow \alpha = \frac{LMt}{GI_0}$$

$$\alpha = \frac{32LMt}{\pi d_1^{\ 4} G}$$

and

$$\alpha = \frac{32 \times 500 \times 21.23 \times 10^3}{3.14 \times 10.26^4 \times 8 \times 10^4} = 0.122rd$$

$$\left.\begin{matrix} \pi \to 180° \\ 0.047 \to \alpha \end{matrix}\right\} \Rightarrow \alpha = \frac{0.12 \times 180°}{\pi} = 6.87°$$

4. Calculation of the new diameter of the rod in the case where the unit angle of twist must not exceed 0.1 degrees per meter.

From condition of deformation we can see:

$$\theta = \frac{Mt}{GI_0} \leq [\theta] = \theta_{\lim}$$

$$Mt \leq GI_0 \theta_{\lim}$$

$$I_0 \geq \frac{Mt}{G\theta_{\lim}}$$

$$\frac{\pi d_2^{\ 4}}{32} \geq \frac{Mt}{G\theta_{\lim}}$$

$$d_2^{\ 4} \geq \frac{32Mt}{\pi G \theta_{\lim}}$$

$$d_2 \geq \sqrt[4]{\frac{32Mt}{\pi G \theta_{\lim}}}$$

After conversion of $\theta_{\lim}$ from degree per meter to a radian per meter, we have:

$$\theta_{\lim} = 1.74 \times 10^{-6} rd / mm$$

$$d_1 \geq \sqrt[4]{\frac{32 \times 21.23 \times 10^3}{3.14 \times 8 \times 10^4 \times 1.74 \times 10^{-6}}} = 35.30mm$$

$$d_2 \geq 35.30mm$$

5. The minimum diameter d of the shaft that meets both conditions (of strength and rigidity) is :

$$d = d_2 = 35.30mm$$

<u>Second case: hollow rod</u>

1. Calculation the outside diameter of the hollow rod (D=2d).

we have

$$I_0 = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \frac{\pi (D^4 - d^4)}{32} = \frac{15\pi D^4}{32\times 16}$$

from condition of resistance

$$\tau_{\max} \leq Rpg$$

$$\frac{Mt}{\left(I_0/\rho_{\max}\right)} \leq Rpg$$

$$I_0/\rho_{\max} = \frac{15\pi D^4}{32\times 16}\times\frac{2}{D} = \frac{15\pi D^3}{16^2}$$

$$\rho_{\max} = \frac{D}{2}$$

so

$$\frac{16^2 Mt}{15\pi D^3} \leq Rpg$$

$$D^3 \geq \sqrt{\frac{16^2 Mt}{15\pi Rpg}}$$

$$D \geq \sqrt[3]{\frac{16^2 Mt}{15\pi Rpg}}$$

$$D \geq \sqrt[3]{\frac{16^2 \times 21.23\times 10^3}{15\times 3.14\times 100}}$$

$$D \geq 10.48mm$$

2. Calculation of the angle of twist between the two ends of the hollow rod if G = 80 GPa

$$\theta = \frac{Mt}{GI_0} = \frac{\alpha}{L}$$

$$\alpha = \frac{LMt}{GI_0} = \frac{32LMt}{G\pi (D^4 - d^4)}$$

$$\alpha = \frac{16\times 32LMt}{15\pi G D^4}$$

$$\alpha = \frac{16\times 32\times 500\times 21.23\times 10^3}{15\times 3.14\times 8\times 10^4\times 10.48^4} = 0.119rd$$

$$\alpha = 6.82°$$

3. Calculation of the new outside diameter of the hollow rod if the unit angle of twist must not exceed 0.1 degrees per meter.

$$\theta = \frac{Mt}{GI_0} \le [\theta]$$

$$D_2^{\ 4} \ge \frac{16 \times 32 Mt}{15 \pi G [\theta]}$$

$$D_2 \ge \sqrt[4]{\frac{16 \times 32 Mt}{15 \pi G [\theta]}}$$

$$D_2 \ge \sqrt[4]{\frac{16 \times 32 \times 21.23 \times 10^3}{15 \times 3.14 \times 8 \times 10^4 \times 1.74 \times 10^{-6}}}$$

$$D_2 \ge 35.88 mm$$

4. The outside diameter D of the shaft that meets both conditions (of strength and rigidity) is:

$$D = D_2 = 35.88 mm$$

5. Calculation of the ratio (mass of the solid rod / mass of the hollow rod).

$$\eta = \frac{M_s}{M_h} = \frac{\rho V_s}{\rho V_h} = \frac{LS_s}{LS_h}$$

with

M_s: is the masse of the solid shaft and M_h: is the hollow shaft.

ρV_s: is the density and volume of solid shaft, Vh: is the volume of the hollow shaft.

S_s and S_h are the solid and hollow surfaces respectively.

$$\eta = \frac{M_s}{M_h} = \frac{\rho V_s}{\rho V_h} = \frac{LS_s}{LS_h}$$

so

$$\eta = \frac{\frac{\pi D_s^{\ 2}}{4}}{\frac{\pi (D_h^{\ 2} - d^2)}{4}} = \frac{D_s^{\ 2}}{(D_h^{\ 2} - d_h^{\ 2})}$$

$$D_h = 2d_h \Rightarrow d_h = \frac{D_h}{2}$$

$$\eta = \frac{D_s^{\ 2}}{\frac{3}{4} D_h^{\ 2}} = \frac{4 D_s^{\ 2}}{3 D_h^{\ 2}}$$

$$\eta = \frac{4 \times 35.30^2}{3 \times 35.85^2} = 1.29$$

6. Conclusion

It is preferable to use a hollow stem for the following reasons

- Lightness
- Material savings
- Low inertia.

Exercise 4

Figure (III. 12) shows a belt drive. The shaft, with a diameter d = 45 mm, rests on two cylindrical supports at A and B. It carries two pulleys. The drive pulley (1) has a radius r1 = 100 mm, and the driven pulley (2) has a radius R2 = 200 mm. The other dimensions are as follows: a = 300 mm, b = 400 mm, and c = 500 mm. The tension values are: t1 = 3.5 kN, T1 = 7.5 kN, t2 = 2 kN, and T2 = 4 kN.

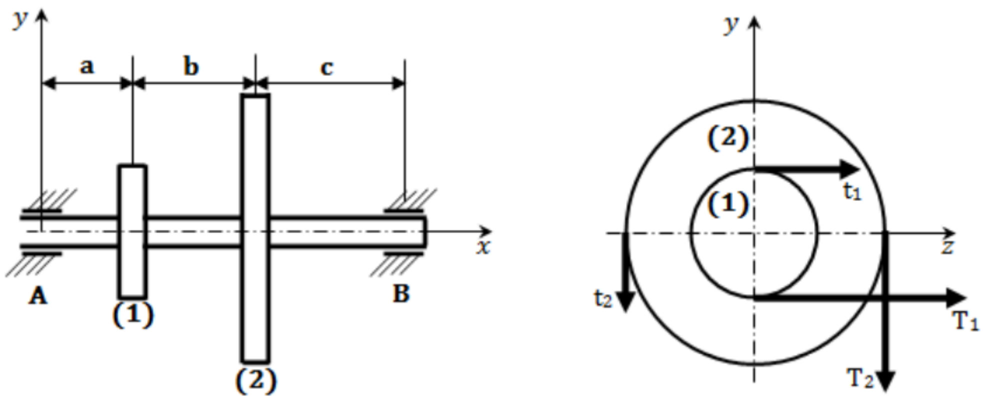


Fig. III. 12: Belt drive.

1. Calculate the torsionally moment
2. Calculate the reactions at points A and B
3. Calculate the maximum bending moment
4. Identify the danger zone
5. Calculate the ideal (resultant) moment
6. Verify the shaft's strength, knowing that it is made of steel with an allowable normal stress of 300 MPa.

Solution

1. Calculation of the torsionally moment

$$Mt = (T_1 - t_1) \times r_1 = (T_2 - t_2) \times r_2$$

$$Mt = (7500 - 3500) \times 0.1 = 400 Nm$$

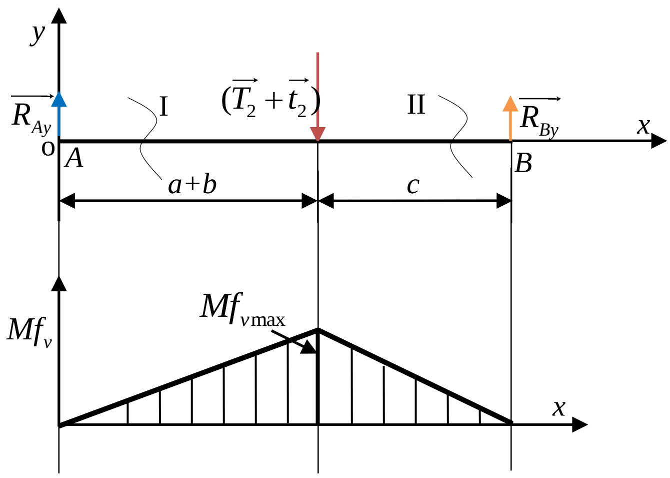
or

$Mt = (4000 - 2000) \times 0.2 = 400 Nm$

2. Calculation of the reactions at points A and B

to calculate the reaction and drawing the distribution of the bending moment in both the vertical and horizontal plans, we divide the study into two plans.

a. In the vertical plane (oxy)



$$\sum M_A \vec{F} = 0$$

$$\sum M_A \vec{F} = (T_2 + t_2)(a+b) - R_{By}(a+b+c) = 0$$

$$R_{By} = \frac{(T_2 + t_2)(a+b)}{(a+b+c)} = \frac{6000 \times 0.7}{1.2} = 3500N$$

$$\sum M_B \vec{F} = 0$$

$$\sum M_B \vec{F} = -(T_2 + t_2)c + R_{Ay}(a+b+c) = 0$$

$$R_{Ay} = \frac{(T_2 + t_2)c}{(a+b+c)} = \frac{6000 \times 0.5}{1.2} = 2500N$$

- Calculation of the vertical bending

Zone one: $0 \le x \ge 700mm$

$$Mf_v = R_{Ay}x = 2500x$$

$$\begin{cases} x = 0 \rightarrow Mf_v = 0 \\ x = 700 \rightarrow Mf_v = 1750Nm \end{cases}$$

Zone two: $700 \le x \ge 1200mm$

$$Mf_v = R_{Ay}x - (T_2 + t_2)(x - 700)$$

$$Mf_v = 2500x - 6000(x - 700)$$

$$\begin{cases} x = 700 \rightarrow Mf_v = 1750Nm \\ x = 1200 \rightarrow Mf_v = 0Nm \end{cases}$$

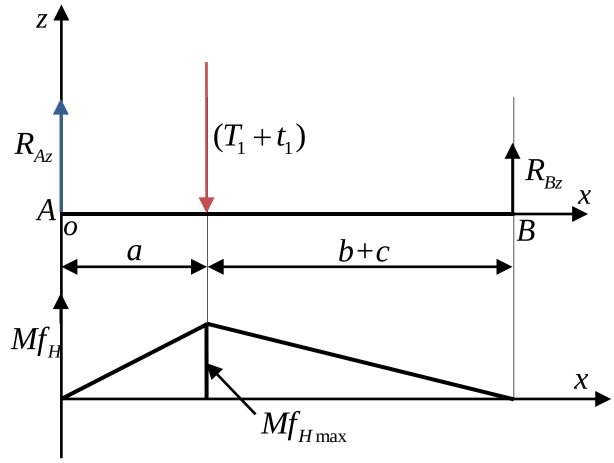
b. In the horizontal plane (ozx)

$$\sum M_A \vec{F} = 0$$

$$\sum M_A \vec{F} = -(T_1 + t_1)a + R_{Bz}(a+b+c) = 0$$

$$R_{Bz} = \frac{(T_1 + t_1)a}{(a+b+c)} = \frac{11000 \times 0.3}{1.2} = 2750N$$

$$\sum M_B \vec{F} = 0$$



$$\sum M_B \vec{F} = (T_1 + t_1)(b+c) - R_{Az}(a+b+c) = 0$$

$$R_{Az} = \frac{(T_2 + t_2)(b+c)}{(a+b+c)} = \frac{6000 \times 0.9}{1.2} = 8250N$$

- Calculation of the horizontal bending

Zone one: $0 \le x \ge 300mm$

$$Mf_H = R_{Az}x = 8250x$$

$$\begin{cases} x = 0 \rightarrow Mf_H = 0 \\ x = 700 \rightarrow Mf_H = 2475Nm \end{cases}$$

Zone two: $300 \le x \ge 1200mm$

$$Mf_H = R_{Az}x - (T_1 + t_1)(x - 300)$$

$$Mf_v = 2500x - 11000(x - 300)$$

$$\begin{cases} x = 300 \rightarrow Mf_H = 2475Nm \\ x = 1200 \rightarrow Mf_H = 0Nm \end{cases}$$

3. Calculation the maximum bending moment

1. The maximum bending in vertical plane is localized at the cross section distance (a+b) from A.

$$Mf(a+b) = \sqrt{Mf_V^2(a+b) + Mf_H^2(a+b)}$$

$$Mf_V(a+b) = R_{Ay} \times 0.7 = 2500 \times 0.7 = 1750Nm$$

$$Mf_H(a+b) = R_{Az} \times 0.7 - 11000 \times 0.4$$

$$Mf_H(a+b) = 8250 \times 0.7 - 11000 \times 0.4 = 1375Nm$$

$$Mf(a+b) = \sqrt{1375^2 + 1750^2} = 2225.56Nm$$

2. The maximum bending in horizontal plane is localized at the cross section distance (*a*) from A.

$$Mf(a) = \sqrt{Mf_V^2(a) + Mf_H^2(a)}$$

$$Mf_H(a) = 2475Nm$$

$$Mf_V(a) = R_{Ay} \times 0.3 = 2500 \times 0.3 = 750Nm$$

$$Mf(a) = \sqrt{2475^2 + 750^2} = 2586.14Nm$$

so the maximum bending is : $Mf_{\max} = Mf(a) = 2586.14Nm$

4. The danger zone is located at a distance (*a*) from support A.

5. Calculation of the ideal (resultant) moment.

$$M_{if} = \frac{1}{2}\left[Mf_{\max} + \sqrt{Mf_{\max}{}^2 + Mt^2} \right]$$

$$M_{if} = \frac{1}{2}\left[2586.14 + \sqrt{2586.14^2 + 400^2} \right]$$

$$M_{if} = 2601.51Nm$$

6. Shaft resistance check.

Resistance condition is:

$$\sigma_{\max} \leq [\sigma]$$

$$\sigma_{\max} = \frac{Mfi}{\left(\dfrac{I_{Gz}}{\rho_{\max}}\right)} = \frac{32 \times 2601.51 \times 10^3}{3.14 \times 45^3} = 292.7N/mm^2$$

$$\sigma_{\max} = 292.7 \leq 300N/mm^2$$

The shaft resists

Exercise 5

The transmission shaft is shown in figure III. 13 below has an outer diameter D_{ou} = 60 mm and an inner diameter D_{in} = 20 mm. It is guided in rotation at points A and C by two roller bearings and carries at points B and D two spur gears (1) and (2) supporting loads F_1 = 3000 N and F_2 = 6000 N respectively. The other dimensions are as follows: a = 300 mm, b = 400 mm, c = 300 mm, d_1 = 400 mm and d_2 = 200 mm and α=20°.

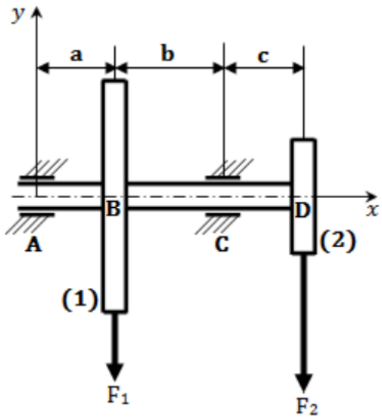


Fig. III. 13: Shaft transmission

1. Calculate the reactions at points A and C.
2. Calculate the bending moments and deduce the maximum bending moment.
3. Calculate the torsional moment.
4. Draw the force and moment diagrams along this shaft.
5. Indicate the danger zone.
6. Calculate the ideal moment.
7. Calculate the equivalent stress in the shaft.
8. Verify the shaft's strength if its allowable normal stress is 250 MPa.

Solution

Given: D_{out}=60mm; Din=20mm for the shaft.

First we calculate the components of the loading forces F_1 and F_2.

$$F_{t1} = F_1 \cos(\alpha) = F_1 \cos(20°) = 3000 \times 0.93 = 2790N$$

$$F_{r1} = F_1 \sin(\alpha) = F_1 \sin(20°) = 3000 \times 0.34 = 1020N$$

$$F_{t2} = F_2 \cos(\alpha) = F_2 \cos(20°) = 6000 \times 0.93 = 5580N$$

$$F_{r2} = F_2 \sin(\alpha) = F_2 \sin(20°) = 6000 \times 0.34 = 2040N$$

1- Calculation of reactions en A et C

$$\sum \vec{F} = \vec{R_A} + \vec{F_{r1}} + \vec{F_{r2}} + \vec{R_C} = \vec{0}$$

$$R_C - R_A = F_{r1} + F_{r2} = 3060N$$

$$\sum M_A \vec{F} = M_A \overrightarrow{R_A} + M_A \overrightarrow{F_{r1}} + M_A \overrightarrow{F_{r2}} + M_A \overrightarrow{R_C} = 0$$

$$F_{r1}.a + F_{r2}(a+b+c) - R_C(a+b) = 0$$

$$R_C = \frac{F_{r1}.a + F_{r2}(a+b+c)}{(a+b)}$$

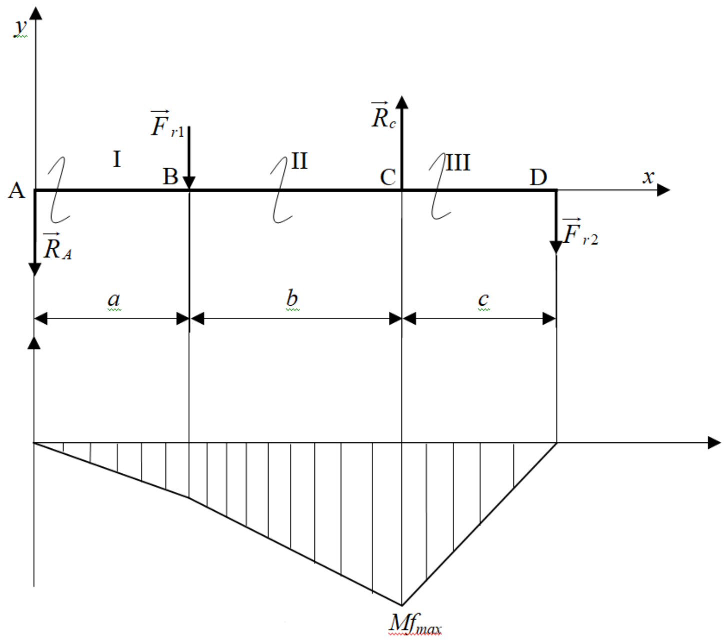
$$R_C = \frac{1020\times 0.3 + 2040\times 1}{0.7} = 3351.42N$$

$$\sum M_C \vec{F} = M_C \overrightarrow{R_A} + M_C \overrightarrow{F_{r1}} + M_C \overrightarrow{F_{r2}} + M_C \overrightarrow{R_C} = 0$$

$$-F_{r1}.b + F_{r2}.c - R_A(a+b) = 0$$

$$R_A = \frac{-F_{r1}.b + F_{r2}.c}{(a+b)}$$

$$R_A = \frac{-1020\times 0.4 + 2040\times 0.3}{0.7} = 291.42N$$



2. Calculation of the bending moment.

In this case, three zones are distinguished.

Zone one: $0 \le x \le 300mm$

$Mf(x) = -R_A.x$

$x = 0 \rightarrow Mf(0) = 0$

$x = 300 \rightarrow Mf(300) = -291.42 \times 300 = -87.42Nm$

Zone two: $300 \leq x \leq 700mm$

$Mf(x) = -R_A.x - F_{r1}(x - 300)$

$x = 300 \rightarrow Mf(300) = -87.42Nm$

$x = 700 \rightarrow Mf(700) = -291.42 \times 700 - 1020 \times 400 = -611.99Nm$

Zone three: $700 \leq x \leq 1000mm$

$Mf(x) = -R_A.x - F_{r1}(x - 300) + R_C(x - 700)$

$x = 700 \rightarrow Mf(700) = -611.99Nm$

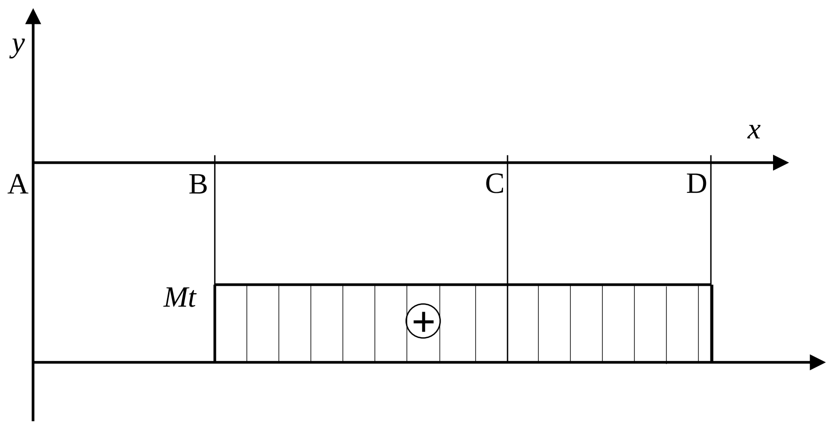
$x = 1000 \rightarrow Mf(1000) = -291.42 \times 1000 - 1020 \times 700 + 3351.42 \times 300 = 0Nm$

3. Calculation the torsional moment.

$$Mt_1 = F_{t1}\frac{d_1}{2} = 2790 \times 0.2 = 558Nm$$

or

$$Mt_2 = F_{t2}\frac{d_2}{2} = \times 0.1 = 558Nm$$



5. The danger zone is situated at the point c, because in this point we have Mf_{max}=611.99Nm and Mt =558Nm.

6. Calculation of the ideal moment.

we have the ideal bending moment expressed by :

$$M_{if} = \frac{1}{2}\left[M_f + \sqrt{M_f^{\ 2} + Mt^2}\right]$$

with M_f is the maximum bending moment and Mt is the torsional moment.

$$M_{if} = \frac{1}{2}\left[Mf_{\max} + \sqrt{Mf_{\max}^{\ 2} + Mt^2}\right]$$

$$M_{if} = \frac{1}{2}\left[611.99 + \sqrt{611.99^2 + 558^2}\right] = 720.08Nm$$

7. Verification of the shaft's strength if its allowable normal stress is 250 MPa.

$$\sigma_{\max} = \frac{Mfi}{\left(\dfrac{IGz}{\rho_{\max}}\right)}; \qquad IGz = \frac{\pi(D_{out}^4 - D_{in}^4)}{64}$$

$$\rho_{\max} = \frac{D_{out}}{2}.$$

$$\frac{IGz}{\rho_{\max}} = 20933.33mm^3$$

then

$$\sigma_{\max} = \frac{720.08\times10^3}{20933.33} = 34.39N/mm^2 = 34.39MPa$$

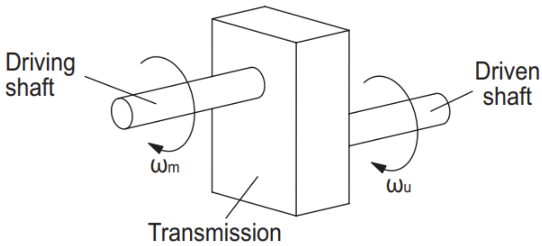
$$\sigma_{\max} = 34.39 \le [\sigma] = 250MPa.$$

so, the shaft resists the applied forces.

IV. Introduction to Motion Transmission

A mechanical transmission is constituted by the complex of the elements required to transmit power in a mechanical system, thus transferring energy from an engine to a user for a certain period of time:

The transfer of this power from the engine to the transmission takes place generally via the driving shaft. A driven shaft allows instead the transfer of this power from the transmission to the user.



The driving and driven shafts are guided in rotation by various components such as plain bearings, thrust bearings, or different types of roller bearings. In the remainder of this course, we will focus on thrust bearings and roller bearings.

IV. 1. Rolling element bearings and thrust bearing

IV. 1.1. Introduction

Bearings are mass-produced. They are available in various sizes and load capacities. To compare products from one manufacturer to another, load capacities are evaluated for standardized speed and duration conditions. Essentially, a bearing is selected by comparing the specific application conditions to the standardized conditions. This section covers rolling bearings, calculating their service life, and mounting them on shafts.

IV. 1. 2. Description of a bearing

A bearing is composed of four elements (Figure IV.1.1 and 2):

- the outer ring (race) mounted in the housing (frame), which can be fixed or rotating;
- the inner ring (race) mounted on the shaft, which can also be fixed or rotating;

- rolling elements (balls, cylindrical rollers or not) which allow the rotation of one ring relative to the other;
- the cage that holds the rolling elements in place.

There are exceptions:

- bearings without cages;
- bearings without outer ring, inner ring or both (the rolling elements are in direct contact with the housing or the shaft).

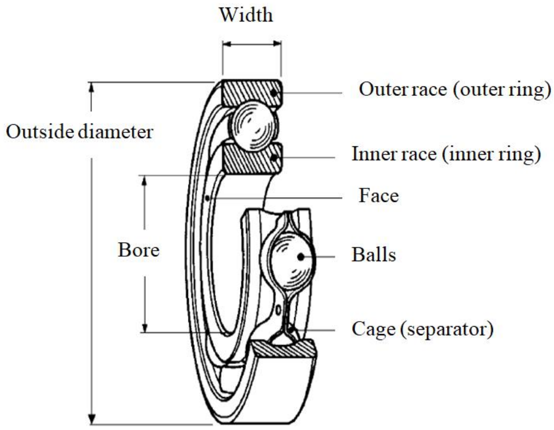


Fig. IV. 1. 1: Elements making up a ball bearing.

Fig. IV. 1. 2: Exploded view of ball bearing

The rings and rolling elements are made of low alloy (1% carbon C) carbon steel (mainly Si (Silicon), Mn (Manganese), Cr (Chromium) in proportions of 0.2 to 1%) for applications at temperatures of 120 to 180 °C. Similar percentages of W (Tungsten) and Mo (Molybdenum) are added for applications up to 450 °C. The rings and rolling elements are heat treated to increase their hardness to about 600 HB to give them greater fatigue resistance. The cage can be made of steel, bronze or plastic. The symbol used to represent the bearing bore is ***d***; the outside diameter is ***D*** and the width ***B***.

Standard bearings are manufactured to standardized dimensions and tolerances, making them interchangeable. Several international organizations govern standardization. The main ones are ANSA */AFBMA* (American National Standards Association/Anti-Friction Bearing Manufacturing

Association), *JIS* (Japanese Industrial Standard), *ISO* (International Organization for Standardizations).

IV. 1. 3. Different types of bearings

Rolling bearings can carry radial, thrust, or combinations of the two loads, depending on their design. Accordingly, most rolling bearings are categorized in one of the three groups: radial for carrying loads that are primarily radial, thrust or axial contact for supporting loads that are primarily axial, and angular contact for carrying combined axial and radial loads.

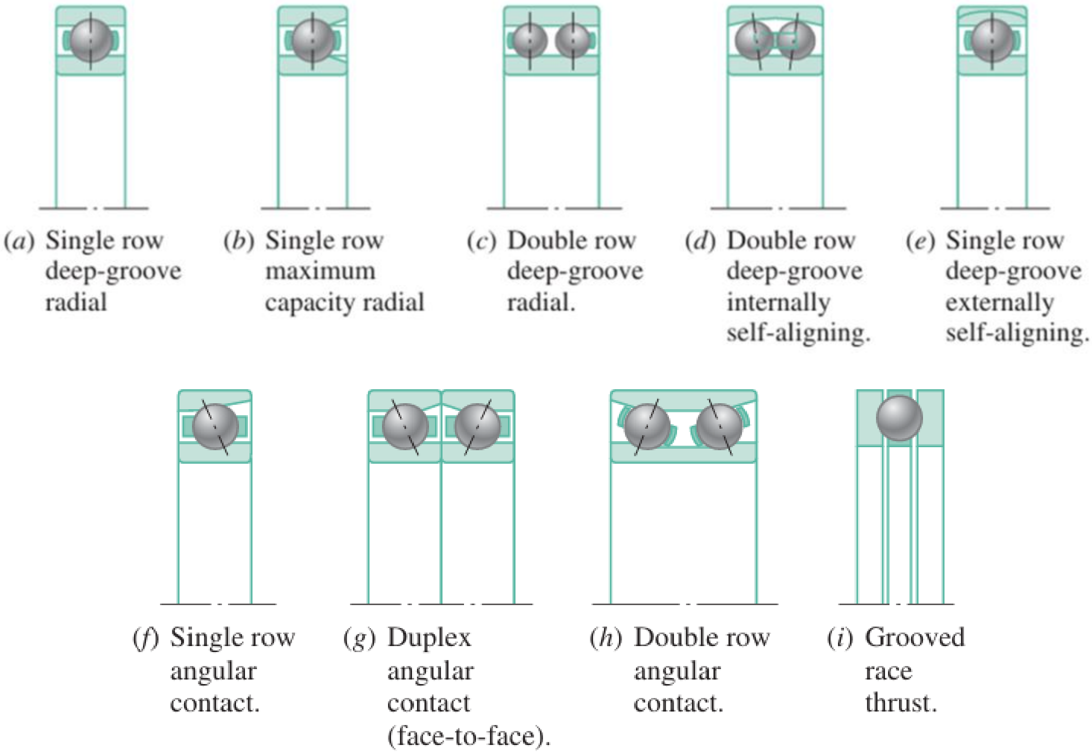


Fig. IV. 1. 3: Different types of ball bearings commonly used.

In contrast with ball bearings, straight roller bearings are unable to support thrust loads, and are usually made so that the races can be axially separated, as shown in Figure III. 1. 4(a). They can, therefore, readily accommodate small axial displacements of a shaft relative to the housing. Such a displacement might occur, for example, because of differential thermal expansion between the shaft and the housing. A small axial thrust capacity can be incorporated into straight roller bearings for locational purposes [by incorporating shoulders as shown in Figure III. 1. 4 (b)], and heavier radial load support can be achieved with double-row roller bearings, as illustrated in Figure III. 1. 4 (c).

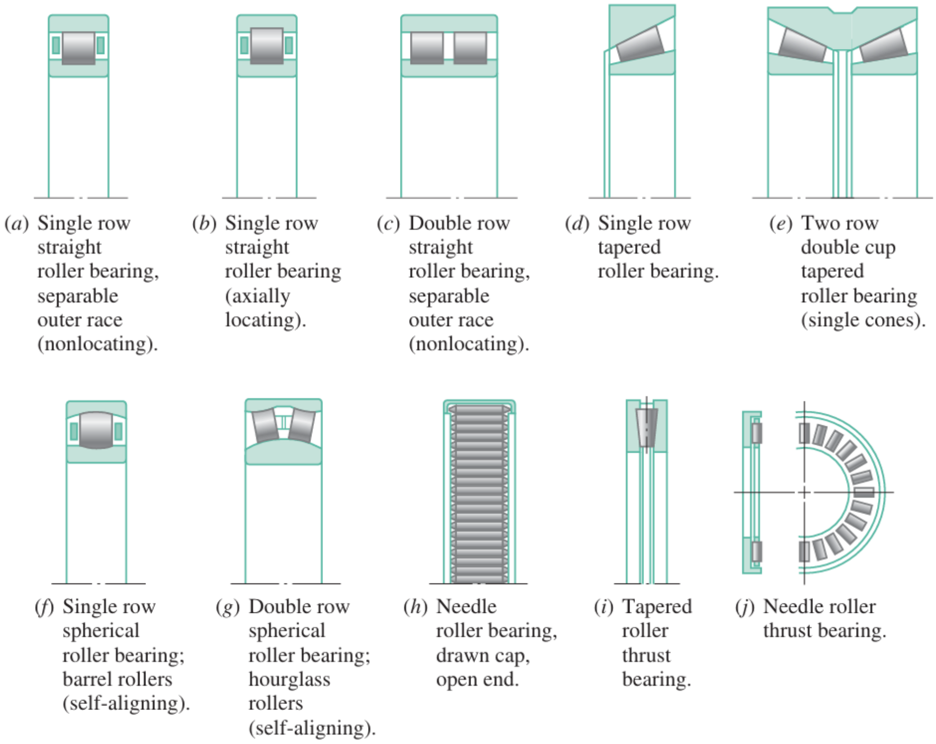


Fig. IV. 1. 4: Different types of roller bearings commonly used.

Two parameters govern the choice of bearings to ensure proper operation of the bearing:

- ***the direction of the applied load which can be*** (Figure IV. 1. 5) :
  - Radial,
  - Axial (thrust),
  - Both radial and axial (combined).
- ***the importance of the deflection*** *of the shaft which tends to modify the initial alignment of the inner and outer rings.*

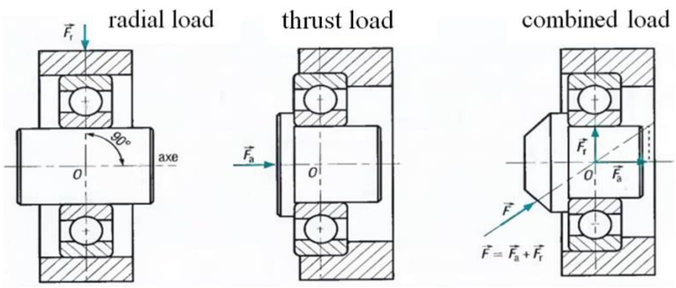


Fig. IV. 1. 5: Loads supported by a bearing

In addition to these parameters, the maximum permissible speed for each type of bearing depends on its average diameter and the lubrication method used. Generally speaking, ball bearings are best suited to high-speed applications requiring high precision, low friction, and low vibration.

Roller bearings have a high load capacity, making them more suitable for low-speed applications with significant shocks.

IV. 1. 4. Bearings life

There are three recognized modes of bearing failure: *fatigue*, *wear* and *permanent deformation.*

Fatigue and wear are associated with dynamic operation, whereas permanent deformation occurs at standstill. Since fatigue and wear times are not necessarily the same, bearing *life* is the shorter of the two. Permanent deformation will put the bearing out of service instantly.

IV. 1. 4. 1. Standardized bearing life (L_{10})

The service life L_{10} of a series of identical bearings, subjected to the same load at some uniform speed, is equal to the number of turns, or revolutions, made by 90% of the bearings in the series before the first signs of fatigue appear.

Units: Standardized life is calculated in **millions of revolutions,** sometimes for convenience in operating **hours.**

IV. 1. 5. Basic dynamic load rating C

C is the constant radial load (axial for a thrust bearing) that a group of apparently identical bearings can take for a rating life of 1 million (ie, 10^6) revolutions of the inner ring in a stationary load (outer ring does not rotate), before the first signs of fatigue appear.

In other words, if a group of 100 identical bearings is subjected during a test to its basic load ***C*** [$\boldsymbol{F_r} = \boldsymbol{C}$], 90 bearings of the group (90%) will have a service life which will reach or exceed 1 million revolutions ($L_{10} = 1$).

Remarks

-Capacity ***C*** is one of the basic characteristics of bearings; it is indicated in manufacturer's catalogs.

-For the same standardized bearing reference, the value of ***C*** can vary significantly from one manufacturer to another.

IV. 1. 5. 1. Relationship between bearing life L_{10} and dynamic load C

Dynamic capacity is related to the bearing life by the following relationship:

$$L_{10} = \left(\frac{C}{P}\right)^n$$

where:

L_{10} : bearing life in millions of revolutions

C: basic dynamic load (Newton)

P: equivalent load exerted on the bearing (Newton)

n = 3 for ball bearings;

n = 10/3 for roller bearings

Bearing Life L_{10H} in operating hours

$$L_{10h} = \frac{L_{10}.10^6}{N.60}$$

N: rotation speed in rpm.

Checking the chosen bearings by calculating their L_{10h} (in hours) service life. The service life must be greater than an admissible service life which is noted by: L_{ha}.

IV. 1. 6. Basic static load rating C_s

C_s refers to the maximum allowable static load that does not impair the running characteristics of the bearing. The basic load ratings for different types of bearings are listed in Tables 1. The value of C_s depends on the bearing material, number of rolling elements per row, the bearing contact angle, and the ball or roller diameter. Except for an additional parameter relating the load pattern, the value of C is based on the same factors that determine C_s.

Table IV.1.1: Dimension and basic load rating for 02-series ball bearings

				Load Ratings (kN)			
				Deep Groove		Angular Contact	
Bore, D (mm)	OD, D_o (mm)	Width, w (mm)	Fillet Radius, r (mm)	C	C_s	C	C_s
10	30	9	0.6	5.07	2.24	4.94	2.12
12	32	10	0.6	6.89	3.10	7.02	3.05
15	35	11	0.6	7.80	3.55	8.06	3.65
17	40	12	0.6	9.56	4.50	9.95	4.75
20	47	14	1.0	12.7	6.20	13.3	6.55
25	52	15	1.0	14.0	6.95	14.8	7.65
30	62	16	1.0	19.5	10.0	20.3	11.0
35	72	17	1.0	25.5	13.7	27.0	15.0
40	80	18	1.0	30.7	16.6	31.9	18.6
45	85	19	1.0	33.2	18.6	35.8	21.2
50	90	20	1.0	35.1	19.6	37.7	22.8
55	100	21	1.5	43.6	25.0	46.2	28.5
60	110	22	1.5	47.5	28.0	55.9	35.5
65	120	23	1.5	55.5	34.0	63.7	41.5
70	125	24	1.5	61.8	37.5	68.9	45.5
75	130	25	1.5	66.3	40.5	71.5	49.0
80	140	26	2.0	70.2	45.0	80.6	55.0
85	150	28	2.0	83.2	53.0	90.4	63.0
90	160	30	2.0	95.6	62.0	106	73.5
95	170	32	2.0	108	69.5	121	85.0

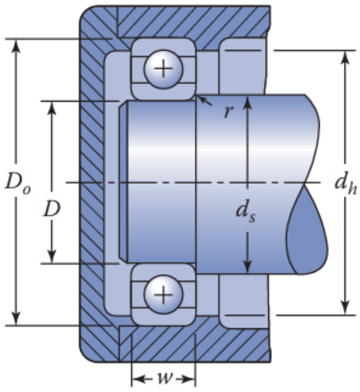


Fig. IV. 1. 6: Dimensions of ball bearing, shaft, and housing

IV. 1. 7. Equivalent Radial Load (P)

The equivalent radial load ***P*** is a pure radial load that presented in the figure IV.1.7, giving exactly the same life as the combination of axial load $\boldsymbol{F_a}$ and radial load $\boldsymbol{F_r}$ actually exerted on the bearing. ***P*** is different from the combined load ***F***.

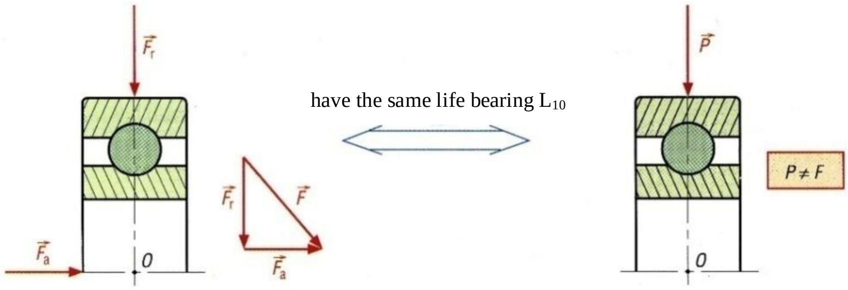


Fig. IV. 1. 7: Equivalent radial load ***P*** and exerted loads $\boldsymbol{F_a}$ and $\boldsymbol{F_r}$.

IV. 1. 7. 1. Special cases

- In the case of needle roller bearings and cylindrical roller bearings with separable rings (Figure IV. 1. 8): $\boldsymbol{F_a} = 0$ and $\boldsymbol{P} = \boldsymbol{F_r}$.
- With thrust supporting only axial loads (Figure IV. 1. 9): $\boldsymbol{F_r} = 0$ and $\boldsymbol{P} = \boldsymbol{F_a}$.

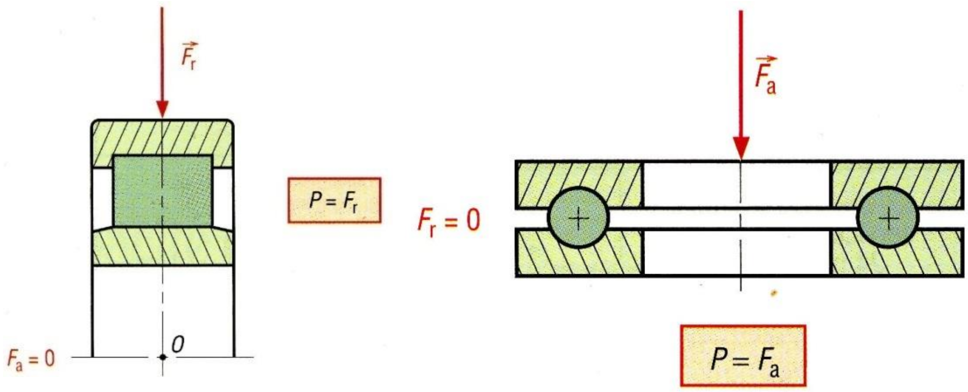


Fig. IV. 1. 8: Value of P in the case of cylindrical roller .bearings

Fig. IV. 1. 9: Value of P in the case of needle roller

IV. 1. 7. 2. General case of a combined charge

F_a and F_r being known, the radial equivalent load P is calculated using the relation:

$$P = X.V.F_r + Y.F_a$$

where

X: a radial factor

Y: a thrust factor

V: a rotational factor

$$V = \begin{cases} 1.0 & for \quad inner-ring \quad rotation \\ 1.2 & for \quad outer-ring \quad rotation \end{cases}$$

- The previous relationships are obtained from the experimental equiduration curves.

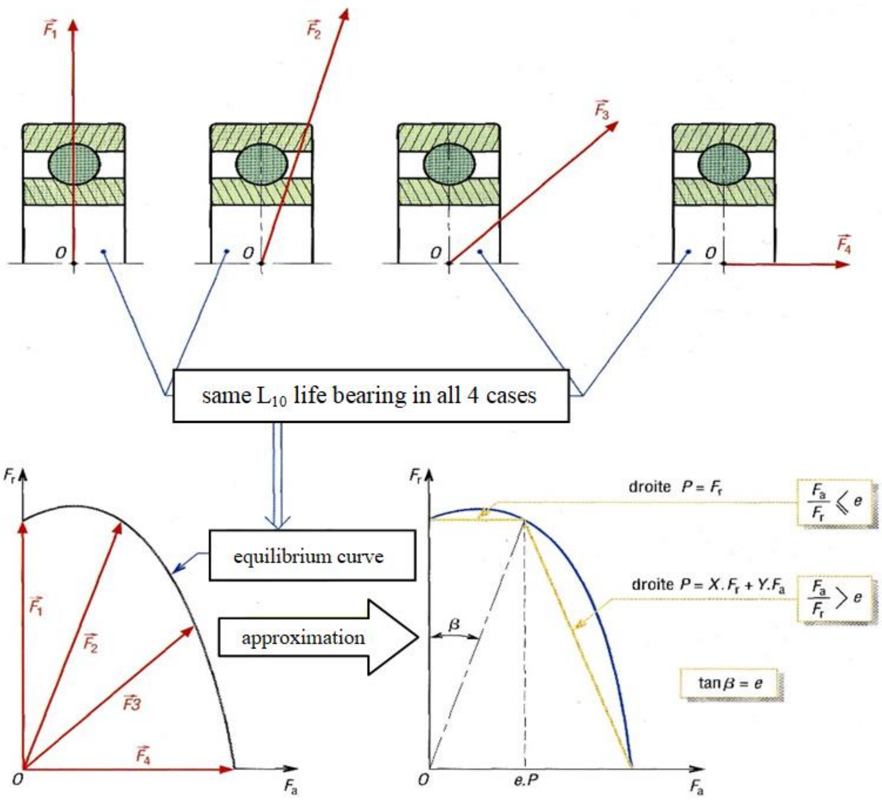


Fig. IV. 1. 10: Under the action of loads **F1**, **F2**, **F3** or **F4** the bearing has the same service life.

The equiduration curve (Figure IV.1.10) is obtained by plotting the curve passing through the ends of the previous loads, all plotted from the same point of application O. This curve is then approximated by straight lines to simplify its use (allows the coefficients X, Y, $e = tan\ \beta$ to be defined...).

Table IV. 1. 2: Factors for deep groove ball bearings

		$F_a/VF_r \leq e$		$F_a/VF_r > e$	
F_a/C_s	e	X	Y	X	Y
0.014[a]	0.19				2.30
0.21	0.21				2.15
0.028	0.22				1.99
0.042	0.24				1.85
0.056	0.26				1.71
0.070	0.27	1.0	0	0.56	1.63
0.084	0.28				1.55
0.110	0.30				1.45
0.17	0.34				1.31
0.28	0.38				1.15
0.42	0.42				1.04
0.56	0.44				1.00

Use 0.014 if (*Fa/Cs*) ≤0.014.

Table IV. 1. 3: Factors for commonly used angular –contact ball bearings

			Single-Row Bearing		Double-Row Bearing			
			$F_a/VF_r > e$		$F_a/VF_r \leq e$		$F_a/VF_r > e$	
Contact Angle (α)	e	$\frac{iF_a}{C_s}$ [a]	X	Y	X	Y	X	Y
	0.38	0.015		1.47		1.65		2.39
	0.40	0.029		1.40		1.57		2.28
	0.43	0.058		1.30		1.46		2.11
	0.46	0.087		1.23		1.38		2.00
15°	0.47	0.12	0.44	1.19	1.0	1.34	0.72	1.93
	0.50	0.17		1.12		1.26		1.82
	0.55	0.29		1.02		1.14		1.66
	0.56	0.44		1.00		1.12		1.63
	0.56	0.58		1.00		1.12		1.63
25°	0.68		0.41	0.87	1.0	0.92	0.67	1.41
35°	0.95		0.37	0.66	1.0	0.66	0.60	1.07

IV. 1. 8. Mounting the bearings

There are two fundamental concepts involved in the assembly of bearings. These are:

1- in the axial direction: the fixed bearing and the floating bearing.

2- in the radial direction: the tight ring and the sliding ring.

Proper mounting requires that one bearing be fixed and the other floating. The choice of fixed bearing is determined by convenience, displacement, and other considerations.

The axial freedom of the floating bearing can come from it

- of its construction;
- of the fit of its inner ring on the shaft or of its outer ring in the housing.

IV. 1. 8.1. Floating bearing

The floating bearing has one of its rings fixed in the housing or on the shaft and the other floating

- Axial freedom by sliding fit of the outer ring in the housing.

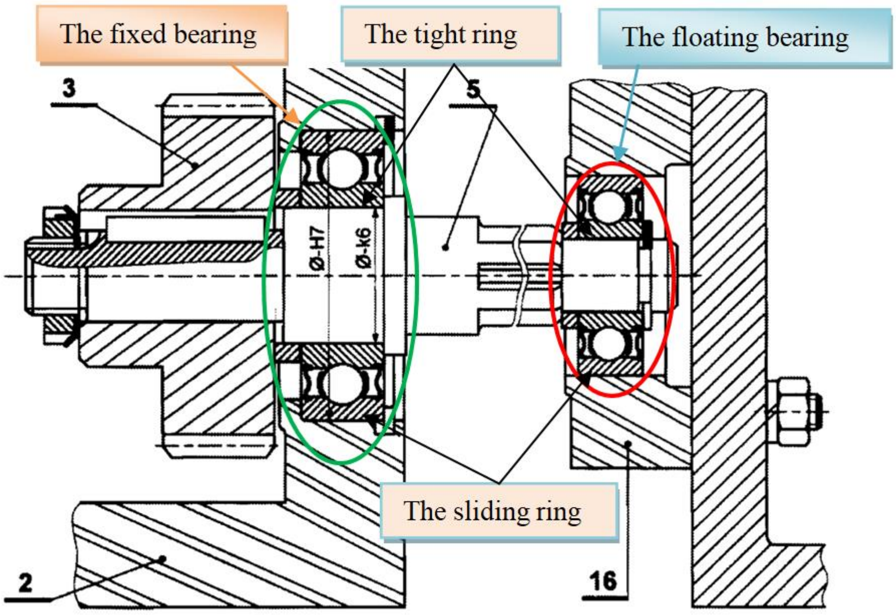


Fig. IV. 1. 11: Fixed and floating bearings.

IV. 1. 8. 2. Fixed bearing

The fixed bearings have its two rings fixed on the shaft and in the housing (Figure IV.1.11).

IV. 1. 8. 3. In the radial direction: the clamped ring and the sliding ring

The spinning ring is:

- tightened on the shaft or in the housing;
- prevented from moving in the axial direction on the shaft or in the housing.

The another ring is:

- slippery mounting on the shaft or in the housing;
- has one axial degree of freedom to move.

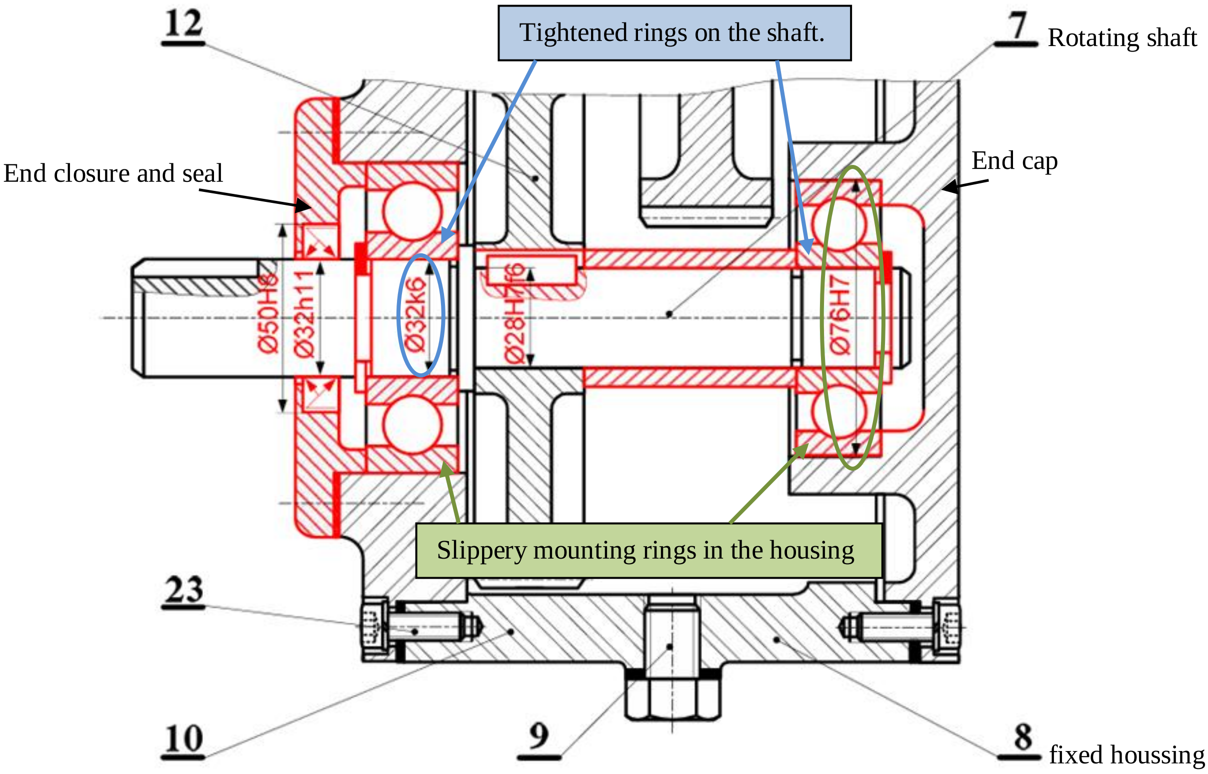


Fig. IV. 1. 12: Tightened and slippery mounting on the shaft or in the housing.

IV. 1. 9. Examples of ball bearing assembly

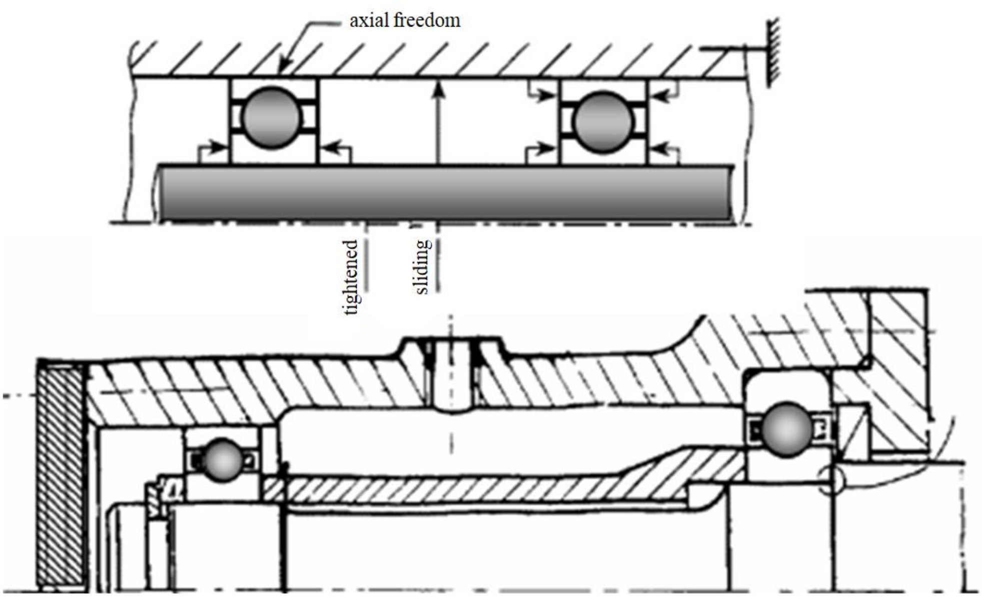


Fig. IV.1. 13: Mounting of a rotating shaft and fixed housing.

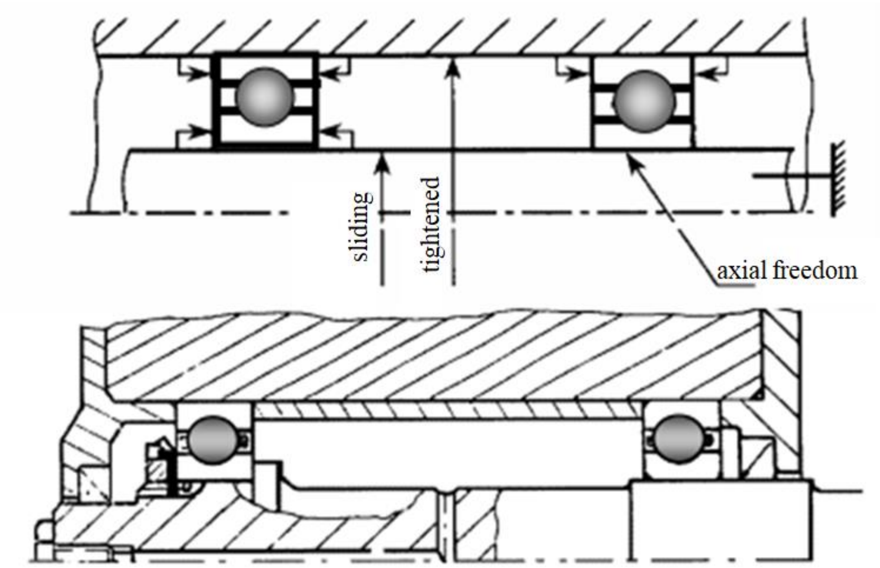
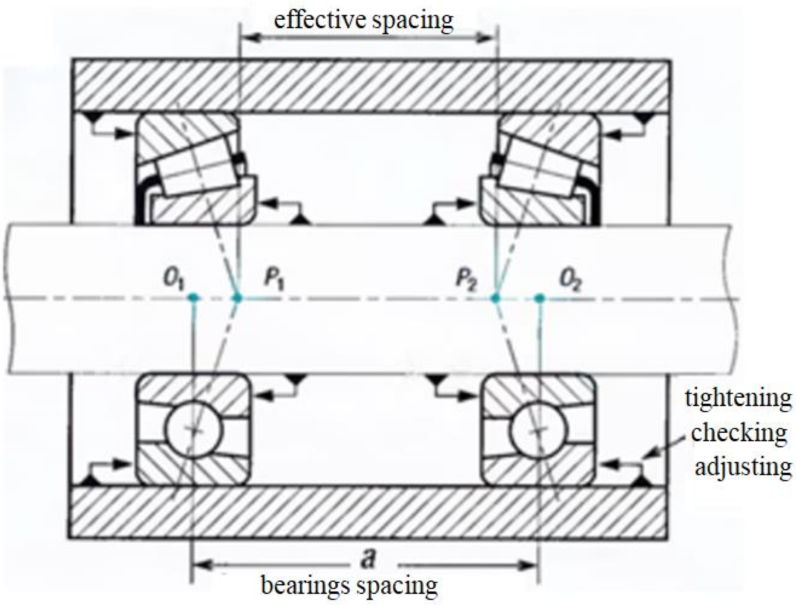


Fig. IV.1. 14: Assembly of a fixed shaft and rotating housing.

IV. 1.10. Mounting of angular contact and tapered roller bearings

IV. 1.10. a. *X-mount or direct mount*

- Simple and economical solution: fewer adjacent parts and less machining.
- It is preferred in the case of rotating shafts with transmission components located between the bearings (gears, etc.).
- The inner rings rotating relative to the load are mounted tightly and the outer rings are mounted sliding.
- The internal clearance adjustment of the connection is carried out on the outer rings.
- Shaft expansions tend to load the bearings a little more and reduce internal clearance.



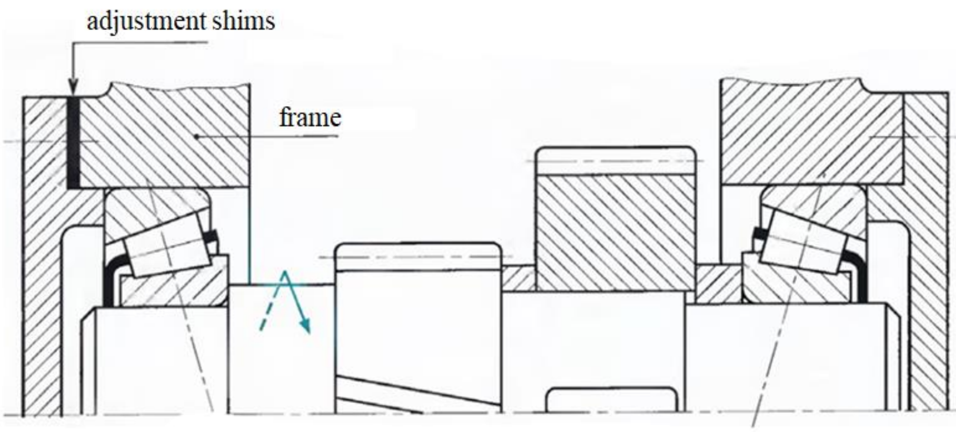
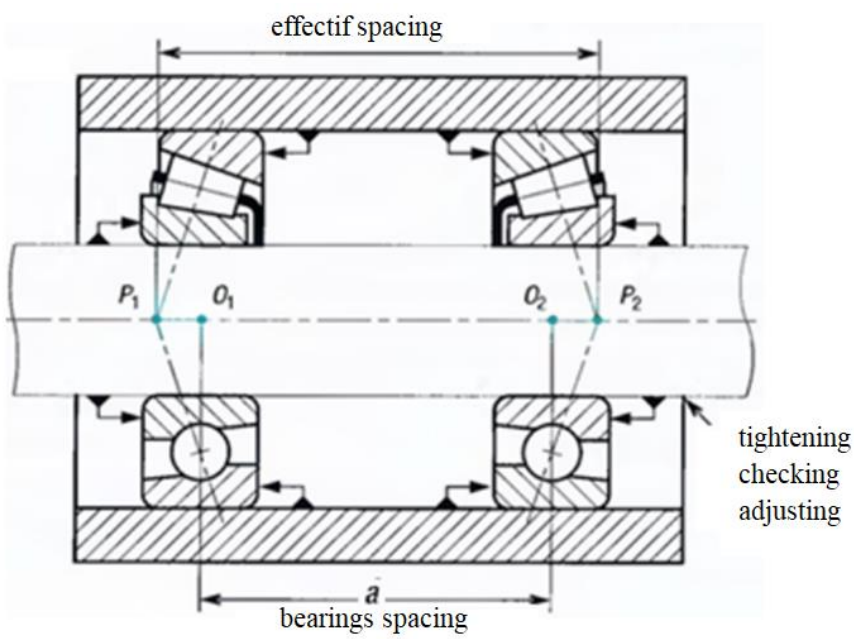


Fig. IV.1. 15: Mounting of a rotating shaft and fixed housing (X).

IV. 1.10. b. *O-mounting or indirect mounting*

- Solution to adopt when the rigidity of the entire connection is sought ⇒ case of the largest effective space between the bearings.
- It is preferable in the case of rotating housing.
- The outer rings, rotating relative to the load, are mounted tightly.
- It is also used with rotating shafts when the transmission components are located outside the connection (cantilever gear).
- The internal clearance adjustment carried out on the inner rings.
- Shaft expansion tends to decrease bearing loads and increase internal clearance in the connection, and vice versa if there is housing expansion.



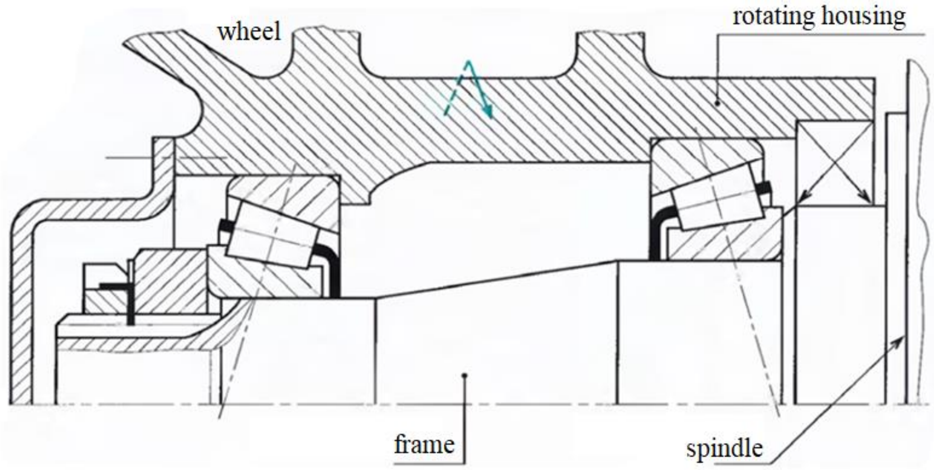


Fig. IV. 1. 16: assembly of a rotating housing and fixed shaft (O).

IV. 1. 11. Applications

Exercise 1

A bearing catalogue specifies a dynamic load capacity C of 6300 daN for a radial contact ball bearing. The bearing supports a load P of 2100 daN.

Let's determine the duration's life of the bearing L_{10} and L_{10H} if the shaft rotation speed is 150 rpm.

Solution

Let us calculating the bearing life, from:

$$L_{10} = \left(\frac{C}{P}\right)^n$$

We have a ball bearing, so *n*=3, hence

$$L_{10} = \left(\frac{6300}{2100}\right)^3 = 27$$

$$L_{10} = 27\,millions \quad of \quad revolutions$$

in hours :

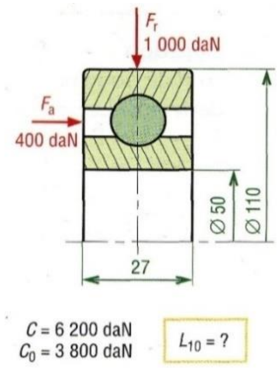
$$L_{10h} = \frac{L_{10}.10^6}{N.60}$$

$$L_{10h} = \frac{27\times 10^6}{150\times 60} = 3000 \quad hours$$

Exercise 2

A radial contact ball bearing with dimensions d = 50, D = 110, B = 27, C = 6200 daN, Cs = 3800 daN, supports a combined load Fa = 400 daN and Fr = 1000 daN.

What service life can be achieved if the shaft rotation speed is 150 rpm?



Solution

$$Fa = 400 daN; \qquad Fr = 1000 daN$$

We know:

$$C = 6200 daN; \qquad Cs = 2800 daN$$

Calculation of the life bearing,

$L_{10} = \left(\dfrac{C}{P}\right)^n$, with *n*=3, for a single-row radial ball bearing.

Calculation of equivalent dynamic load ***P***

$P = X.V.Fr + Y.Fa$; *V*=1: for inner ring rotating.

Calculation of the ration: $\dfrac{Fa}{Cs}$

$$\frac{Fa}{Cs} = \frac{400}{380} = 0.105$$

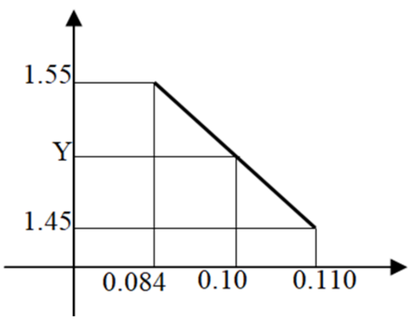
According to Table IV. 1. 2 of the course, this value is situated between 0.084 and 0.110.

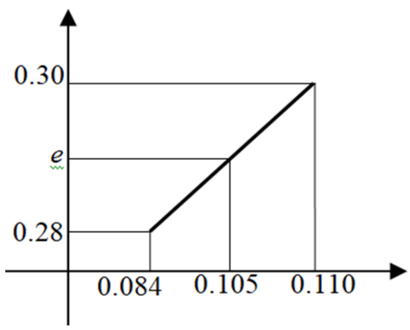
$Y : 1.55 -- 1.45$

and

$e : 0.28 -- 0.30$

To calculate the value of Y and *e* corresponding to 0.105, we use the principle of linear interpolation.





$$\frac{Y-1.45}{0.110-0.105}=\frac{1.55-1.45}{0.110-0.084}$$

$$Y-1.45=\left(0.110-0.105\right)\frac{1.55-1.45}{0.110-0.084}$$

$$Y=1.47$$

In the same way we obtain e

$$\frac{e-0.28}{0.105-0.084}=\frac{0.30-0.28}{0.110-0.084}$$

$$e-0.28=\left(0.105-0.084\right)\frac{0.30-0.28}{0.110-0.084}$$

$$e=0.296$$

Calculation of the ratio $\left(\frac{Fa}{Fr}\right)$

$$\frac{Fa}{Fr}=\frac{400}{1000}=0.4\geq e=0.296$$

where, from Table IV. 1. 2, for a single-row radial ball bearing,

$$P=X.V.Fr+Y.Fa$$

$$P=0.56\times Fr+1.47\times Fa$$

$$P=0.56\times 1000+1.47\times 400$$

$$P=1148daN$$

then

$$L_{10}=\left(\frac{C}{P}\right)^{n}=\left(\frac{6200}{1148}\right)^{3}=157.5 \text{ Millions turns}$$

$$L_{10h}=\frac{L_{10}\times 10^{6}}{60N}=\frac{157.5\times 10^{6}}{60\times 150}=17500 hours$$

IV. 2. Pulleys and belts

Belt drive is suitable for applications where the center distance between rotating shafts is significant, as well as for its simplicity and cost-effectiveness compared to other power transmission solutions such as gears. Belts are generally quiet, easy to replace and, because of their damping capacity and flexibility, they reduce unwanted shocks and vibrations between shafts.

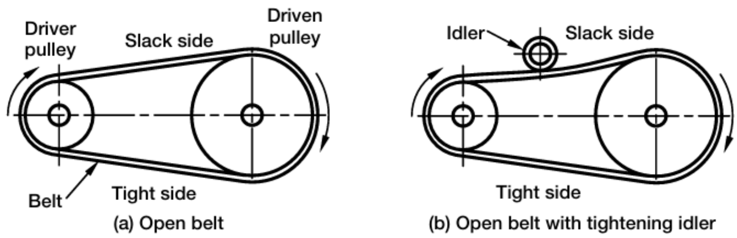


Fig. IV. 2. 1 : Open belt system

IV. 2. 1. Principle: Transmit power by adhesion between two generally parallel, distant shafts (Fig. IV. 2.2). The transmission is carried out with or without change of torque and direction.

θ_s : angle of wrap (smaller pulley) = $\pi - 2\alpha$

θ_L : angle of wrap (larger pulley) = $\pi + 2\alpha$

d_S = diameter of smaller pulley

d_L = diameter of larger pulley

L = belt length

$$L = \sqrt{4C^2 - (d_L - d_S)^2} + \frac{d_L\theta_L + d_S\theta_S}{2}$$

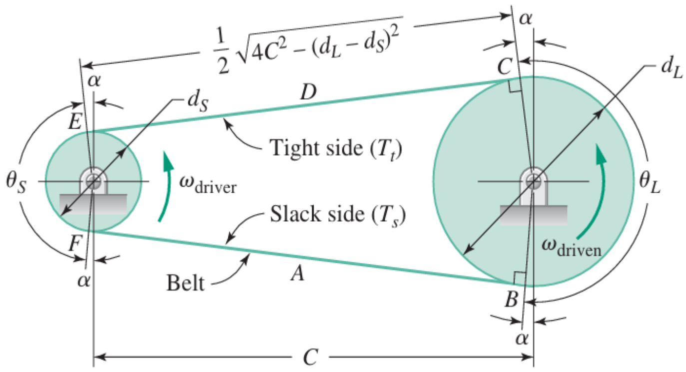


Fig. IV. 2. 2: Geometry and terminology of belt

IV. 2. 2. Types of belts

Belts are commercially available in many different cross-sections.

IV. 2. 2. 1. Flat belts

Very quiet, they allow large reduction ratios, and are mainly used at high speeds under low torques. They absorb torsionally vibrations well, which allows large center distances "C" and long lengths. They have very good efficiency (η ≈98%).

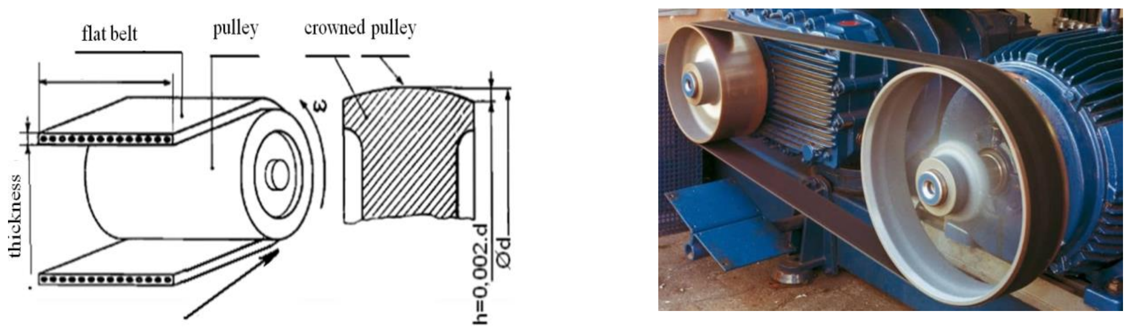


Fig. IV. 2. 3: Flat belt and crowned pulley.

The camber of the pulleys improves the guidance and stability of the belt. To reverse the direction of rotation between the two pulleys, a crossed belt is used (figure IV.2.4a). Shafts are not necessarily parallel, this configuration is called quarter twist belt (figure. IV. 2. 4b).

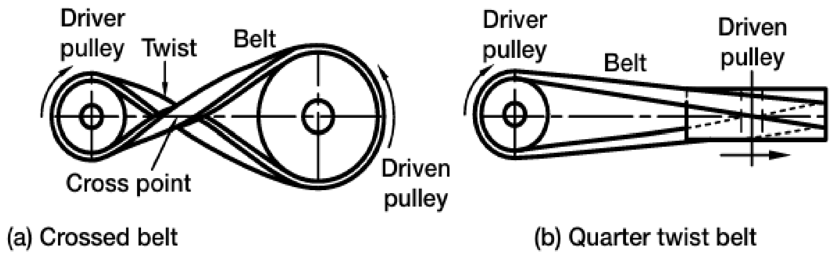


Fig. IV. 2. 4: Cross and quarter twist belt configurations

IV. 2. 2. 1. 1. Kinematic and dynamic study

- **Kinematics** : The linear speed $V = w_m.r = w_r.R$; hence the transmission ratio:

$$\frac{\omega_r}{\omega_m} = \frac{r}{R} \quad (1)$$

- **Dynamics:**

- Balance of the drive pulley/z:

$$C_m - (T_t - T_s).r = 0 \quad (2)$$

- Balance of the receiving pulley/z:

$$C_r - (T_t - T_s).R = 0 \quad (3)$$

hence, the transmission ratio:

$$\frac{\omega_r}{\omega_m} = \frac{r}{R} = \frac{C_m}{C_r} \quad (4)$$

- **Laying tension** $\vec{T_0}$: At rest the belt is tensioned with an equal laying tension:

$$T_0 = \frac{T_t + T_s}{2} \quad (5)$$

T_t is maximum when T_s is minimum. $T_{tmax} = 2T_o$.

- **Ratio between tensions T and t**

1st case: effects of centrifugal force on the belt neglected

$$\frac{T_t}{T_s} = e^{f\theta_S} \quad (6)$$

2nd case: taking into account the centrifugal force on the belt

$$\frac{T_t - \lambda V^2}{T_s - \lambda V^2} = e^{f\theta_S} \quad (7)$$

With λ is the linear mass in kg/m and f is friction coefficient.

Table IV. 1. 4: Coefficient of friction for belt materials and different pulleys.

Belt Material	Pulley Material			
	Wood	Cast Iron/Steel		
		Dry	Wet	Greasy
Leather oak tanned	0.30	0.25	0.20	0.15
Leather chrome tanned	0.40	0.35	0.32	0.22
Rubber	0.32	0.30	0.18	-
Canvas	0.23	0.20	0.15	0.12
Woven cotton	0.25	0.22	0.15	0.12

- **Transmissible power:** In normal operation, we have:

$$P = (T_t - T_S).V \quad (8)$$

hence, the maximum transmittable power is:

$$P_{max} = \left(T_{t,max} - T_{s,\min}\right).V = 2T_o.V \quad (9)$$

IV. 2. 2. 2. Trapezoidal belts

The V-belt is used to increase the contact area and reduce radial force, i.e., to transmit higher power than flat belts. And to transmit significant power, several belts must be used in parallel on the same pulley (with 2; 3; …; 10 grooves).

Quiet and smooth operation, good grip, suitable for transmitting high power; but there is slippage which gives irregular speed ratio; this problem is corrected by poly V belt, it has excellent grip, allows the transmission of high power.

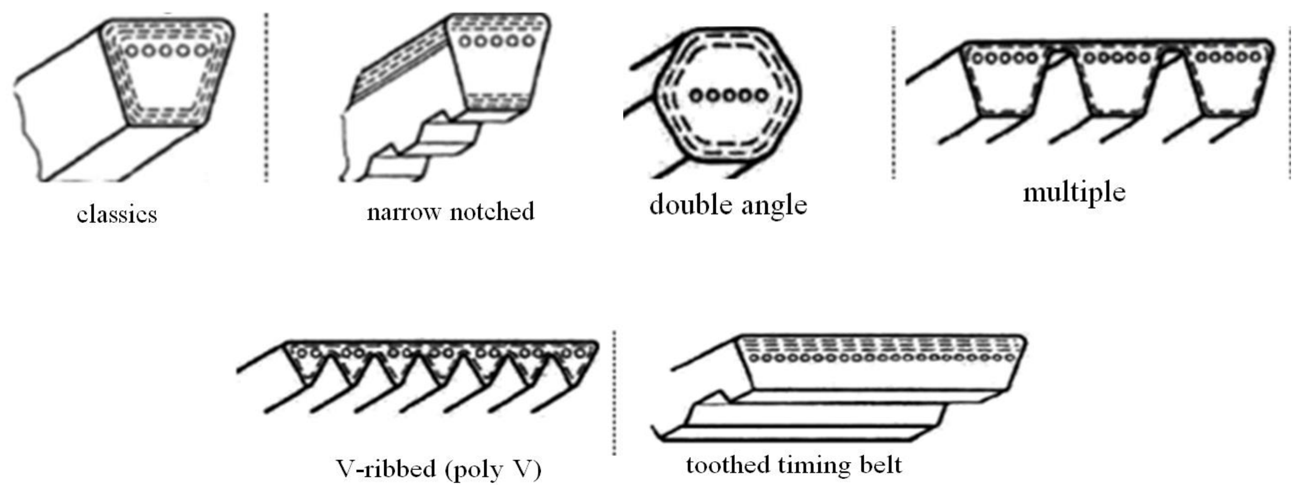


Fig. IV. 2. 5: Different types of V-belts

Fig. IV. 2. 6 : Pulley with one and two grooves

- **Ratio between tensions T_t and T_s**

It is identical to that of flat belts except that d_L and d_S are replaced by " *dp* " and " *Dp* ", primitive diameters of the pulleys, and that the angle "β" intervenes.

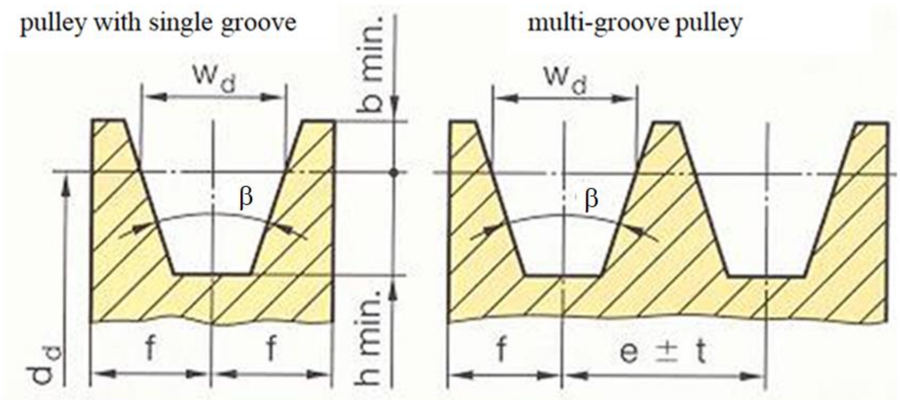


Fig. IV. 2. 7: Geometrical characteristics of V-pulley

$$\frac{T_t - \lambda V^2}{T_s - \lambda V^2} = e^{\frac{f\alpha}{\sin(\beta/2)}} \qquad (10)$$

- **Calculation of trapezoidal belt length:** the primitive length is given by:

$$Lp = 2E + 1.57(Dp + dp) + \frac{(Dp - dp)^2}{4E} \qquad (11)$$

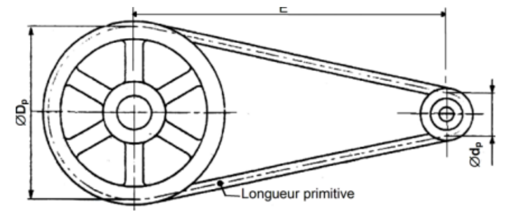
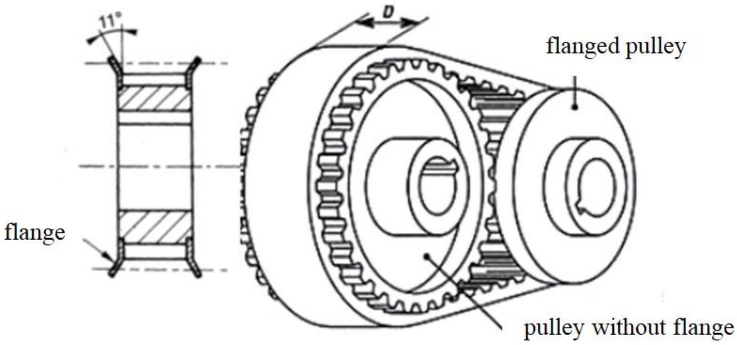


Fig. IV. 2. 8 : Primitive length.

IV. 2. 2. 3. Toothed belts (Timing belts)

Timing belts are components of synchronous drives, a significant class of drives. These drives typically use the positive engagement of two sets of teeth that mesh. As a result, there is no relative motion between the two mesh parts and they do not slip (figure VI. 2. 9).



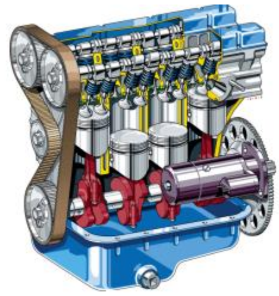


Fig. IV. 2. 9: Pulley and toothed belt

They can be thought of as flat belts with teeth. Unlike other belts, they are effective at low speeds and require lower initial tension. In order to prevent axial slippage of the belt, it is necessary to have flanges on one of the pulleys.

IV. 2. 2. 3. 1. Pulley pitch and outside diameters

Pulley and belt geometry as indicated in figureVI.2.9 shows reference to a Pitch Circle, which is larger than the pulley itself. Its size is determined by the relationship:

$\pi d = pZ_1$: primitive circumference of the small pulley,

where p is the belt tooth spacing (pitch) and $Z_1 = Z_d$ is the number of teeth on the pulley.

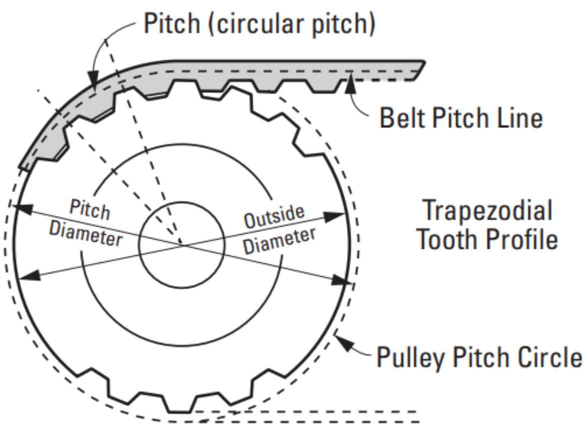


Fig. IV. 2. 10: pulley and belt geometry.

- **Transmission ratio:**

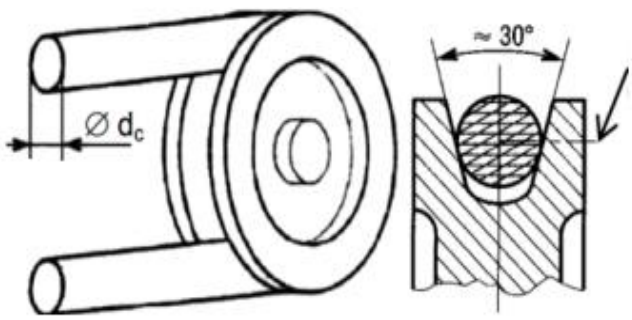
$$\frac{\omega_D}{\omega_d} = \frac{dp}{Dp} = \frac{Z_d}{Z_D} = \frac{C_d}{C_D}$$

Linear speed

$$V = \frac{N_d . p . Z_d}{60}$$

IV. 2. 2. 4. Round belts

They are mainly used in small mechanisms with low power.



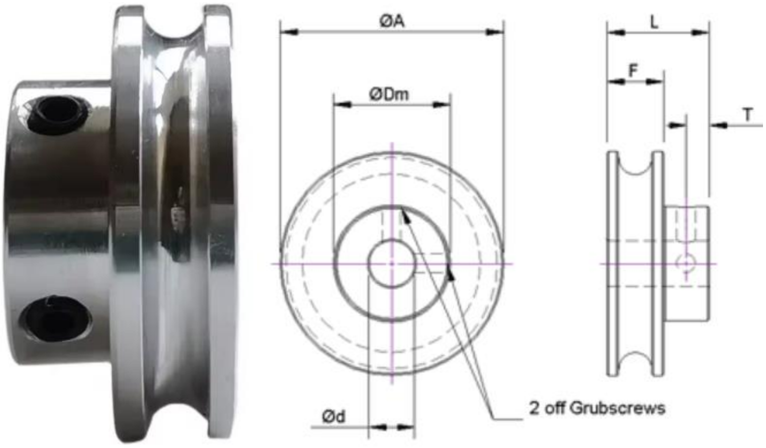


Fig. IV. 2. 11: Pulley and Round belts.

IV. 2. 2. 4. 1. Advantages

- Lighter in weight.
- These offer silent operation.
- Can withstand shock load.
- More efficiency.
- Low maintenance cost.

IV. 2. 3. Belt Composition: Belts are not generally made of a single material, they are made of fabric and cords moulded in rubber and covered in fabric & rubber as shown in Figure IV. 2. 12. These belts are moulded to a trapezoidal shape & are made endless. Except for the circular belts which are very often a synthetic rubber torus.

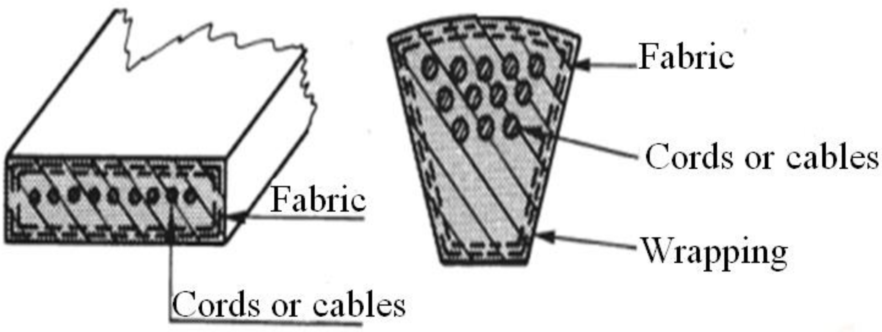


Fig. IV. 2. 12 : Composition des Courroies.

IV. 3. Sprockets and chains

IV. 3. 1. Power transmitting chains

Power transmission by adhesion (friction wheel, pulleys and belts) is inefficient for transmitting high powers (order of magnitude ≤ 100 kW). In addition, the speed ratio is not constant (slipping), and the forces on the bearings are significant. To avoid slipping, steel chains are used.

These chains are used to transmitting power when the distance between shafts is short.

IV. 3. 2. Chain components

The belt is replaced by a set of links, generally made of steel, which mesh with toothed wheels. Sprocket wheels, or just sprocket, are the term for the toothed wheels. The chain transfers motion and power from one shaft to another.

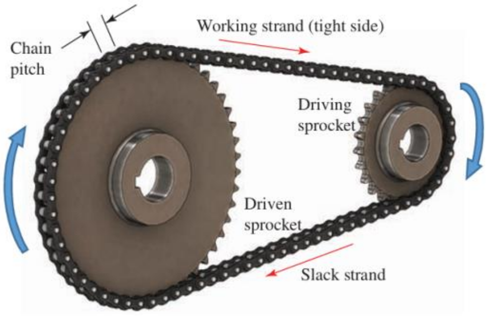


Fig. IV. 3. 1: Chain and toothed wheel (sprocket)

IV. 3. 3. Advantages and disadvantages of chains

IV. 3. 3. 1. Advantages

- It may be used for both long and shorts distance;
- It gives fewer loads on the shafts;
- It transmits more power than belts;
- Constant transmission ratio (no slip);
- Long life spans;
- Possibility of driving several receiving shafts at the same time from the same source;
- Easier assembly and maintenance than gears and lower cost.
- They are mainly used at 'low' speeds; [less than 13 m/s for roller chains (Fig. IV. 3. 2), and less than 20 m/s for silent chains (Fig. IV. 3.3)].

IV. 3. 3. 2. Disadvantages

- The production cost f chain is relatively high;
- The chain drive needs accurate mounting and careful maintenance;

IV. 3. 4. Type of power transmitting chains

Steel is used to make the power transmission chains, which are hardened to prevent wear. Galle chains, roller chains, and inverted tooth chains (silent chains) are the three categories into which these chains fall.

IV. 3. 4. 1. Galle Chain

Low contact surface at the joints, resulting in significant pressure between these surfaces which leads to rapid wear.

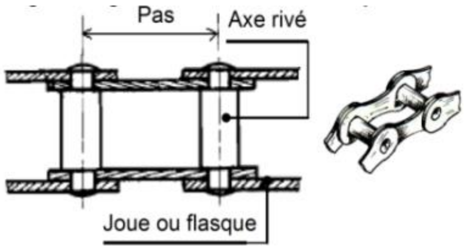


Fig. IV. 3. 2: Galle Chain

IV. 3. 4. 2. Roller chain

Large joint contact surfaces, the rollers roll at the pinion exit, therefore less friction, i.e. very low wear.

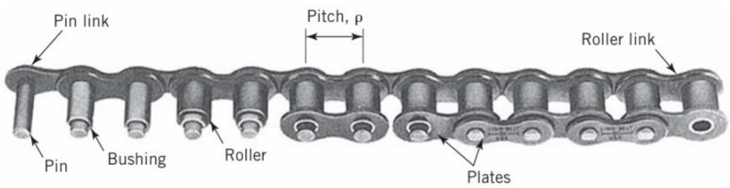


Fig. IV. 3. 3 : Chaine à rouleaux

IV. 3. 4. 2. 1. Characteristics

Pitch: distance between 2 homologous and consecutive points of a wheel.

Primitive diameter: " d_p "

$$\theta = \frac{2\pi}{Z_d} \quad (8)$$

(With Z_d : number of teeth on the pinion)

$$\sin(\frac{\theta}{2}) = \frac{pitch/2}{d_p/2} \quad (9)$$

so :

$$d_p = \frac{Pitch}{\sin(\theta/2)} = \frac{Pitch}{\sin(\pi/Z_d)} \quad (10)$$

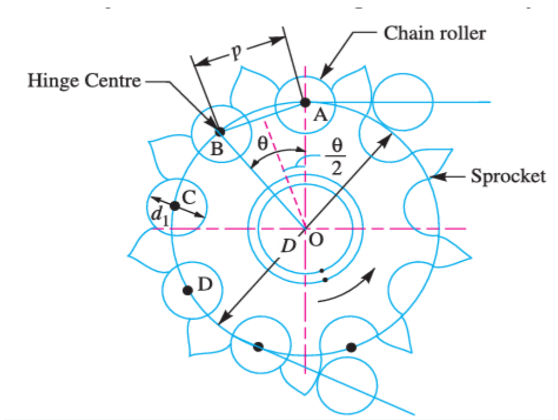


Fig. IV. 3. 4 : Terms used in chain drive

IV. 3. 4. 2. 2. Chain

- Pitch: The chain pitch is equal to the pitch of the wheel and sprocket.
- Chain length:

Primitive length of the chain:

$$L_p = 2E + P\left(\frac{Z_D + Z_d}{2}\right) + \frac{P^2}{E}\left(\frac{Z_D - Z_d}{2\pi}\right)^2 \qquad (11)$$

with:

Z_d: number of teeth on the small sprocket (pinion);

Z_D: number of teeth of the large sprocket wheel;

E: center distance between shafts.

IV. 3. 4. 2. 3. Kinematic calculation

- **Transmission ratio:** It is similar to that of toothed belts.

$$\frac{\omega_D}{\omega_d} = \frac{dp}{Dp} = \frac{Z_d}{Z_D} = \frac{C_d}{C_D} \qquad (12)$$

- **Warp angle** :

$$\theta = \theta_d = 180° - \frac{2}{\sin(\frac{D_p - d_p}{2E})} \geq 120° \qquad (13)$$

- **Linear chain speed** :

$$V = \frac{N_d.P.Z_d}{60} \qquad (14)$$

- **Primitive circumference of the small sprocket**

$$\pi d_p = P.Z_d \qquad (15)$$

IV. 3. 4. 3. Silent chain

Bladed links guide the chain laterally in a groove in the sprocket. Operation is quiet (no play) but very heavy. Long used for timing control in automobile engines.

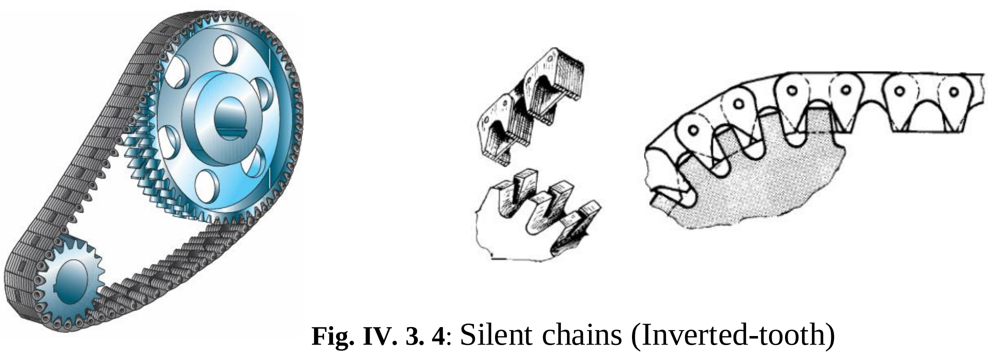


Fig. IV. 3. 4: Silent chains (Inverted-tooth)

Example

An electric motor rotating at 750 rpm drives a pulley 1 with diameter d1. A toothed belt is positioned between pulley 1 and pulley 2. Another toothed belt is positioned between pulley 3 and pulley 4. A table 5, which moves in translation, is fixed to this last toothed belt. The diameters of the pulleys are given in the figure below.

1. Calculate the rotational speed of pulley 3.
2. Determine the rotational speed of pulley 4.
3. Calculate the speed of movement of table 5.

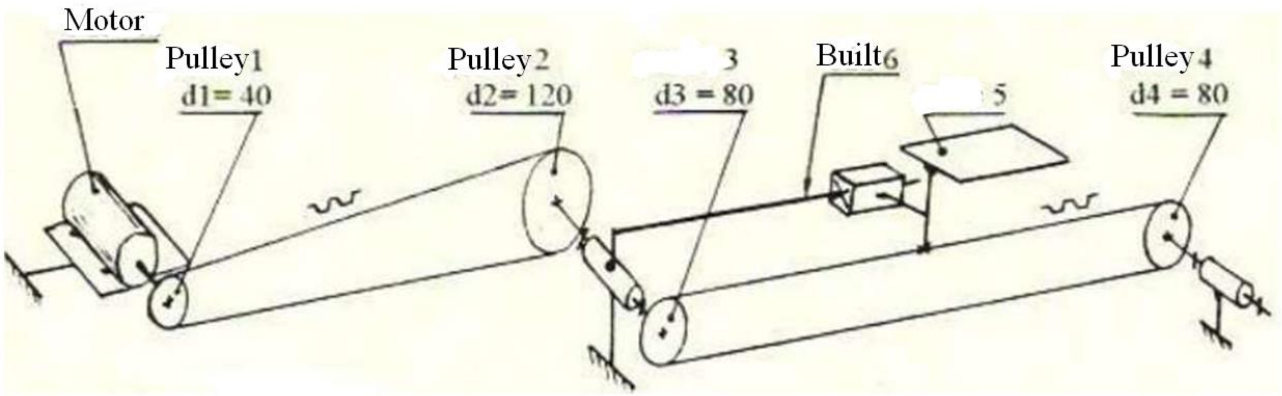


Fig. IV. 3. 5: Transmission with toothed pulley

Solution

given data: $N_m = 750\,rev/\min;\quad d_1 = 40mm;\quad d_2 = 120mm;\quad d_3 = 80mm;\quad d_4 = 80mm$

1. Calculation of the rotational speed of pulley 3.

From the sketch we have: $N_2 = N_3$

ratio of transmission: $r = \frac{N_2}{N_1} = \frac{d_1}{d_2} \Rightarrow N_2 = N_1 \frac{d_1}{d_2} = 750 \frac{40}{120} = 250 rev/\min$

2. Determination of the rotational speed of pulley 4.

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \Rightarrow N_4 = N_3 \frac{d_3}{d_4} = 250 rev/\min$$

3. Calculation of the speed of movement of table 5.

The speed of table (5) is equal to the peripheral velocity of the belt

$$V_5 = \omega_3 \frac{d_3}{2} = \frac{\pi N_3 d_3}{60} = \frac{3.14 \times 250 \times 0.080}{60} = 1.04 m/s$$

Chapter V: Reducers and Gearboxes

V. 1. Kinematic study of a speed reducer

V. 1. 1. Introduction and definition

A speed reducer is the component that transfers power between the motor and the driven component by reducing the rotational speed (ω) and inversely increasing the torque (C).
The motor can be electric, thermal, hydraulic, etc. The main parameter that determines the size of a motor is its power (P). Regardless of the energy source, motors generally deliver a high rotational speed (**ω_i**) for a relatively low motor torque (**C_i**). The driven component can be: an electric generator, a vehicle wheel, a propeller, an industrial machine, etc.
During this transmission, power loss must be as low as possible. It will be characterized by an efficiency η as close as possible to 1:

$$C_o\omega_o = \eta C_i\omega_i \quad (1)$$

o=outlet

i=inlet

In previous chapters, we examined the different types of transmission systems, such as gears, pulley and belt systems, and chains. In what follows, we will focus our study on gear reducers because they offer good efficiency and high power transmission.

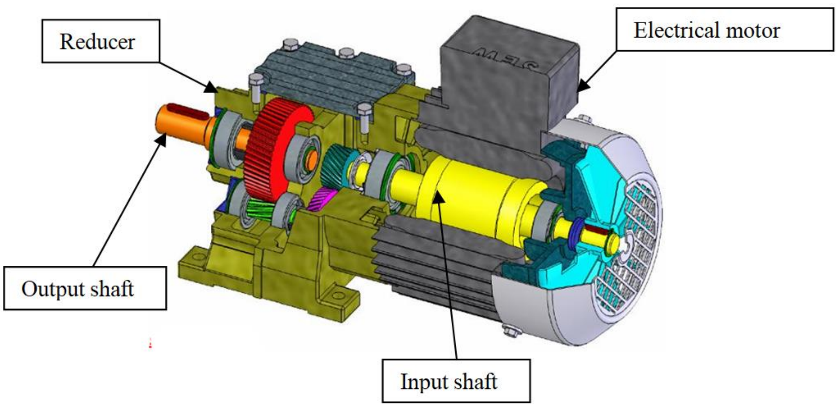
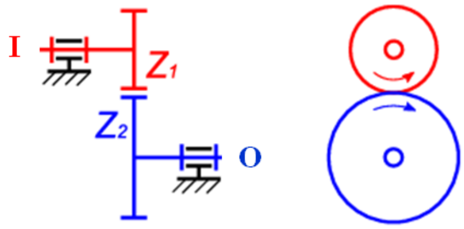


Fig. V. 1. 1: Helical geared reducer

V. 1. 2. Classification of reducers

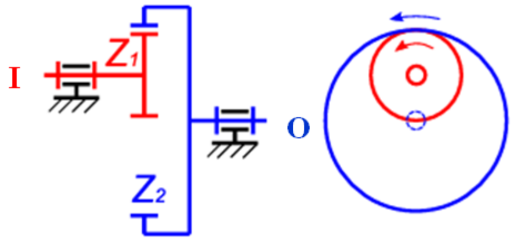
Depending on the orientation of the driving and the driven shafts, reducer can be categorised into the following classes:

V. 1. 2. 1. Spur and helical gears reducers: These types have parallel shafts and may consist of one or more stages of reduction (Figure VI. 2. - VI. 4.)



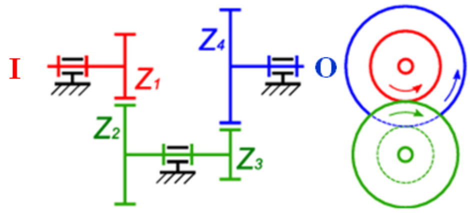
$$\begin{cases} \dfrac{\omega_o}{\omega_i} = -\dfrac{Z_1}{Z_2} \\ \dfrac{C_o}{C_i} = -\eta_{12}\dfrac{Z_2}{Z_1} \end{cases}$$

Fig. VI. 1. 2: One-stage gear reducer (external contact).



$$\begin{cases} \dfrac{\omega_o}{\omega_i} = \dfrac{Z_1}{Z_2} \\ \dfrac{C_o}{C_i} = \eta_{12}\dfrac{Z_2}{Z_1} \end{cases}$$

Fig. VI. 1. 3: One-stage gear reducer (internal contact)



$$\begin{cases} \dfrac{\omega_o}{\omega_i} = \dfrac{Z_1}{Z_2}\dfrac{Z_3}{Z_4} \\ \dfrac{C_o}{C_i} = \eta_{12}\eta_{34}\dfrac{Z_2}{Z_1}\dfrac{Z_4}{Z_3} \end{cases}$$

Fig. V. 1. 4: Two-stage gear reducer.

V. 1. 2. 2. Bevel gears reducer: The axes of the shafts in these reducers are generally at right angles to each other.

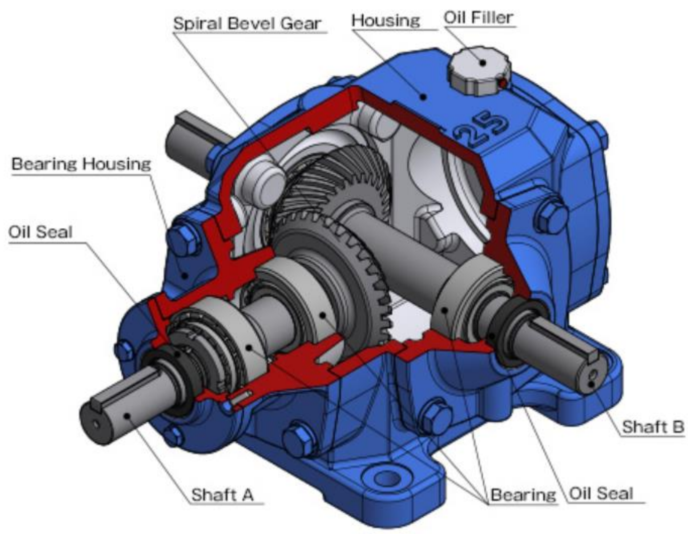


Fig. V. 1. 5: Bevel gears reducer

V. 1. 2. 3. Worm and worm wheel reducer: The axes in these cases are at right angles and non-intersecting.

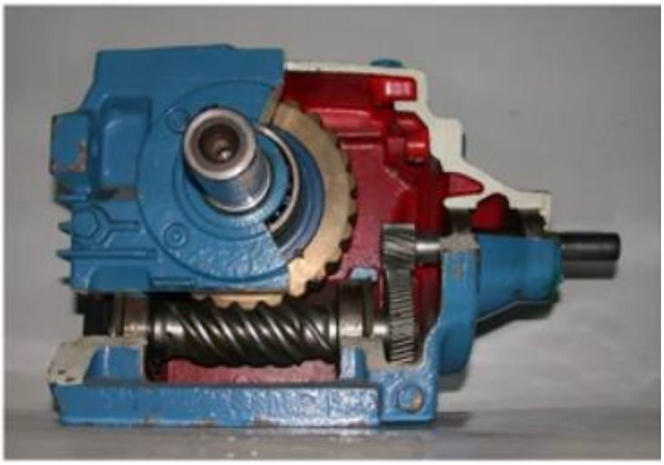


Fig. V. 1. 6: Worm and wormgear reducer

Besides the above common types, reducers may consist of one or more combination of the above classes.

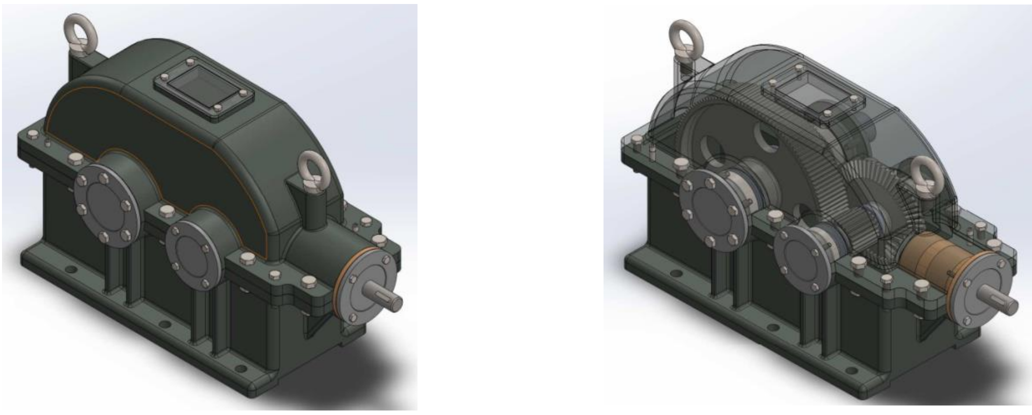


Fig. V. 1. 7: Combination reducer

Other applications can use belt and chain to reduce the speed rotation.

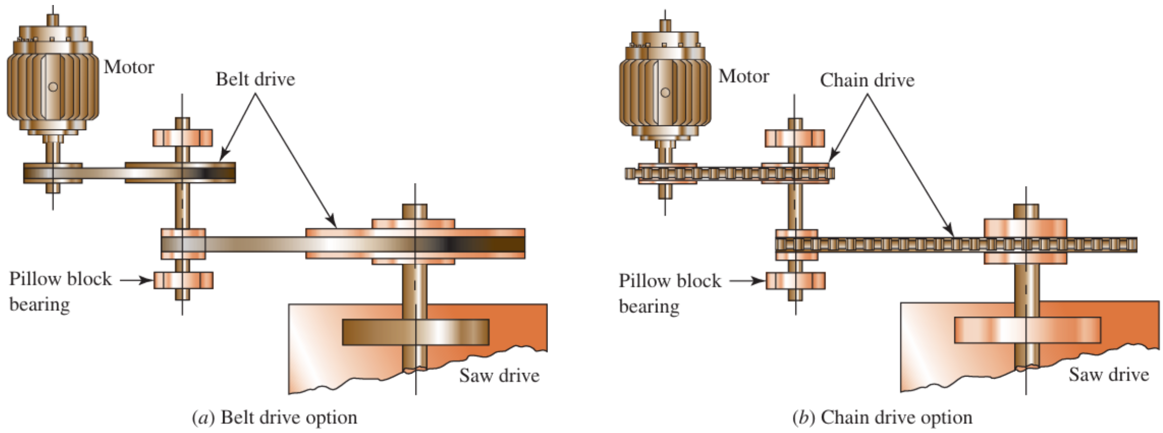


Fig. V. 1. 8: Reducer with belt and pulley (a), chain and sprocket (b)

V. 1. 3. Applications

Exercise 1

Here is an assembly drawing of a speed reducer. The purpose of this reducer is to increase torque.

1. What are the input and output numbers of the reducer?
2. Calculate the transmission ratio of this gearbox.
3. Is it a reversing gearbox?

Data: Z_5=30; Z_{20}= 70; m_5=1mm; m_{11}= 2mm; d_{11}=26mm; d_{21}=162mm.

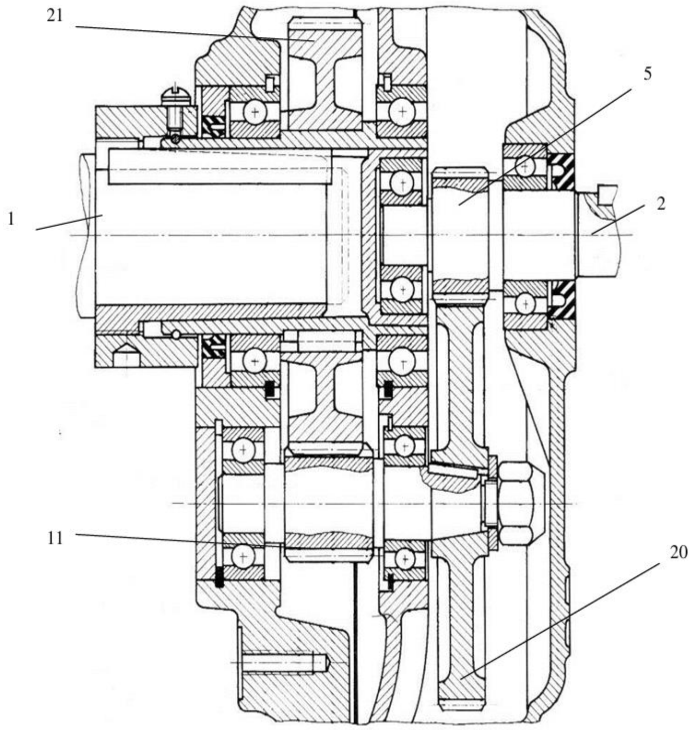


Fig. V. 1. 9: Reducer with spur gear

Solution

1- The input number part is the shaft (2) and output number part is the shaft (1). Because the shaft supports the pinion (5) with a diameter smaller than the wheel gear (20), hence the first speed reduction. The second reduction takes place at the level of the second stage, consisting of the wheel gear (11) and (21).

2. Calculation of the transmission ratio.

we have

$$r_{5/21} = (-1)^2 \frac{Z_5 \times Z_{11}}{Z_{20} \times Z_{21}}$$

calculate of Z_{11} and Z_{21}

$$d_{21} = Z_{21} \times m_{21} = Z_{21} \times m_{11} \Rightarrow Z_{21} = \frac{d_{21}}{m_{11}} = \frac{162}{2} = 81 tooth$$

$$d_{11} = Z_{11} \times m_{11} = Z_{11} \times m_{11} \Rightarrow Z_{11} = \frac{d_{11}}{m_{11}} = \frac{26}{2} = 13 tooth$$

so $r_{5/21} = \frac{30 \times 13}{70 \times 81} = 0.068$

3. In mechanics, a reversible gearbox (or more broadly a reversible reducer) refers to a system where motion can be transmitted in both directions: from input to output, but also from output to input. From this, the system studied is reversible.

Exercise 2

The following diagram represents a classic right-angle drive system, with the specifications:

- The input shaft (1) is subjected to a torque Cm of 50 Nm.
- The gear teeth are straight, m = 2 mm.
- Pitch radius of pinion (2) = 25 mm
- Number of teeth on pinion (1) = 20

1. Calculate the transmission ratio.
2. Calculate the torque on shaft E.

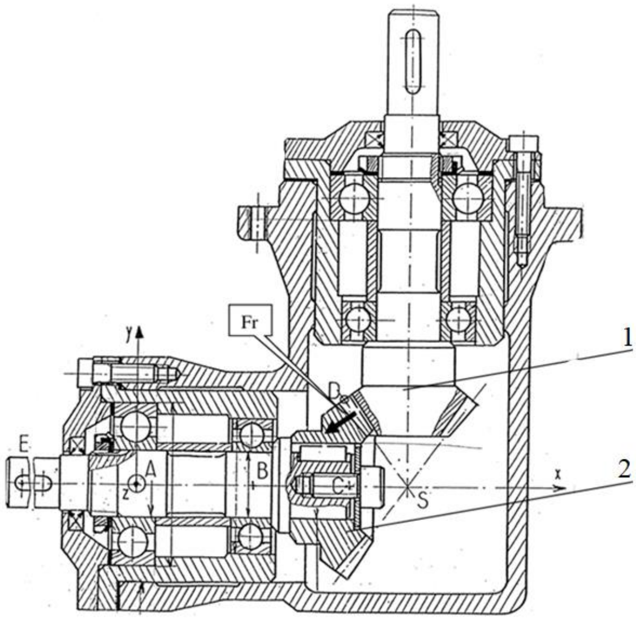


Fig. V. 1. 10: Reducer with bevel gears

Solution

1. The transmission ratio

$$r_{5/21} = (-1)^1 \frac{Z_1}{Z_2} = -\frac{Z_1}{Z_2}$$

Calculate of the number of teeth in the gear (2):

$$d_2 = 2R_2 = 2\times 25 = 50mm$$

in the other hand:

$$Z_2 = \frac{d_2}{m} = \frac{50}{2} = 25teeth$$

so $r_{5/21} = \frac{Z_1}{Z_2} = \frac{20}{25} = \frac{4}{5} = 0.8$

2. Calculation of the torque on the shaft E.

$$r_{5/21} = \frac{Z_1}{Z_2} = \frac{C_m}{C_E} \Rightarrow C_E = \frac{C_m}{r_{5/21}} = \frac{50}{0.8} = 62.5Nm$$

V. 2. Kinematic Study of a Gearbox

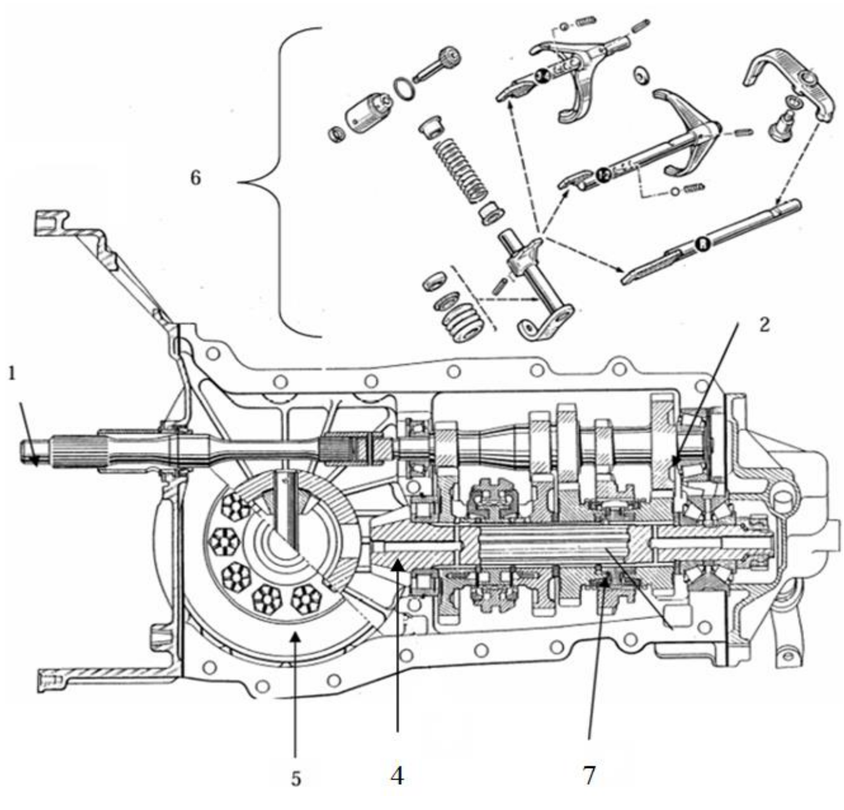
V. 2.1. Introduction and definition

The gearbox transmits the motion from a driving shaft (internal combustion motor, electric, etc.) to a driven shaft (car wheels, ship's propeller, etc.) by modifying torque and speed via a system of gears. Each gear engages a different set of gears, optimizing engine speed according to the situation (starting, climbing, and high speed).

A primary shaft receives power from the engine, transmits it to the secondary shaft via gears, and the output shaft drives the driven shaft.

V. 2. 2. Presentation of mechanical gearbox

The diagram below shows a 4-speed mechanical gearbox with reverse gear.

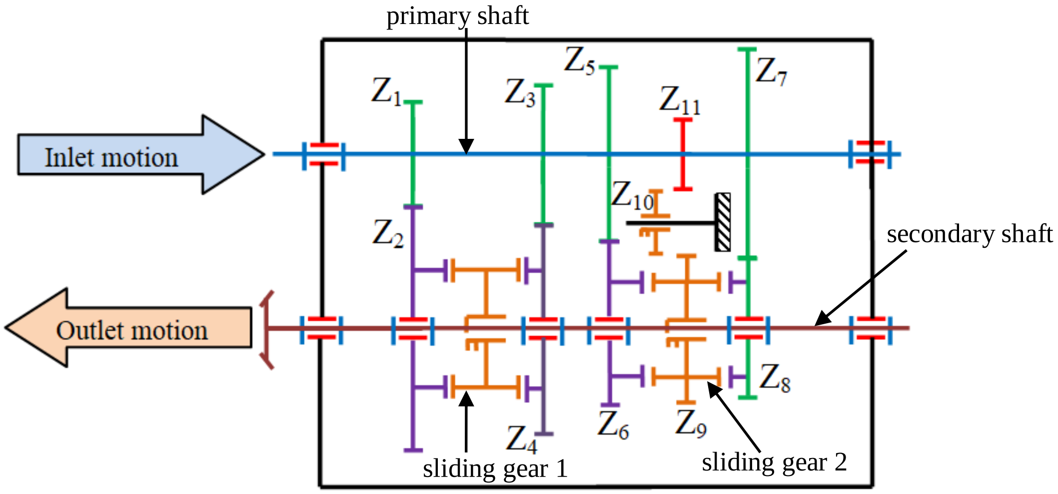


Nomenclature:

1: Input shaft
2: Primary shaft
3: Secondary shaft
4: Drive pinion
5: Ring gear
6: Slides and forks
7: Synchrony

Fig. V. 2. 1: Mechanical Gearbox

Our goal is the kinematic study of this gearbox; to this end, we will present its kinematic diagram. Figure VI.2 shows this diagram.



Z_1=17 teeth
Z_2=44 teeth
Z_3=26 teeth
Z_4=40 teeth
Z_5=40 teeth
Z_6=32 teeth
Z_7=40 teeth
Z_8=24 teeth
Z_9=28 teeth
Z_{10}=10 teeth
Z_{11}=11 teeth

Fig. V. 2. 2: Kinematic diagram of the gearbox

VI. 2. 3. Transmission ratio selection

VI. 2. 3. 1. The first gear ratio

The sliding gear (1) moves to the right. The ration in this case is:

$$r_1 = \frac{Z_1}{Z_2} = \frac{17}{44} = 0.38$$

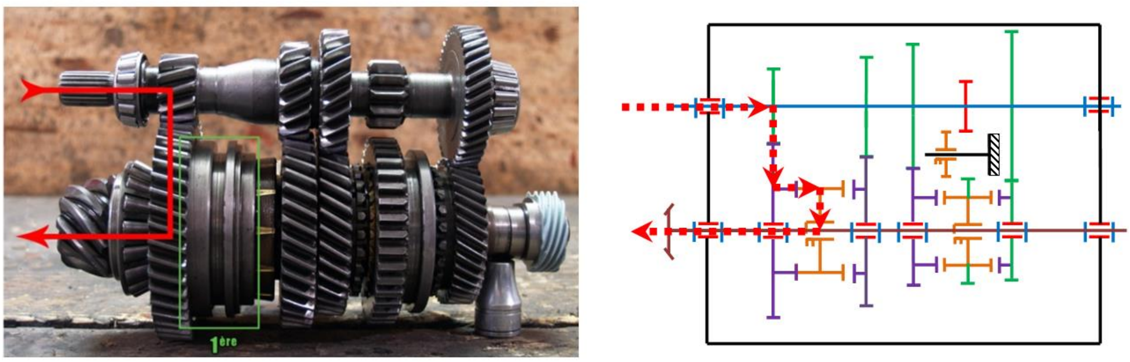


Fig.V. 2. 3: The first gear ratio.

VI. 2. 3. 2. The second gear ratio

The sliding gear (1) moves to the left. The ration in this case is:

$$r_1 = \frac{Z_3}{Z_4} = \frac{26}{40} = 0.65$$

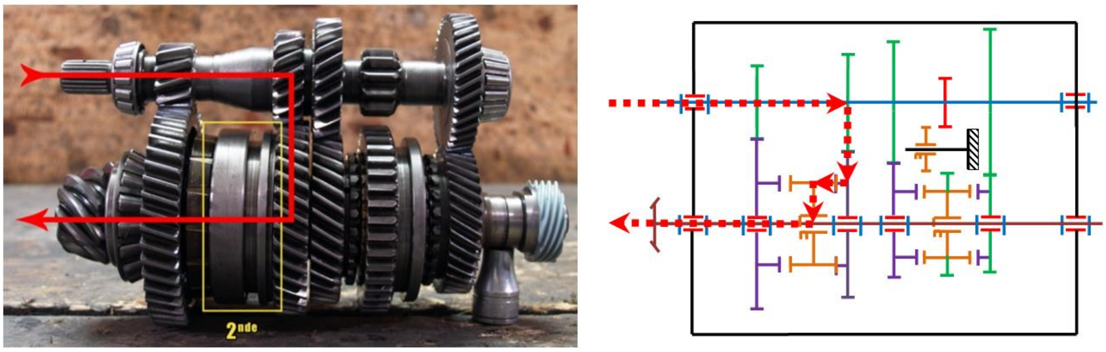


Fig.V. 2. 4: The second gear ratio.

VI. 2. 3. 3. The third gear ratio

The sliding gear (2) moves to the left. The ration in this case is:

$$r_1 = \frac{Z_5}{Z_6} = \frac{40}{32} = 1.25$$

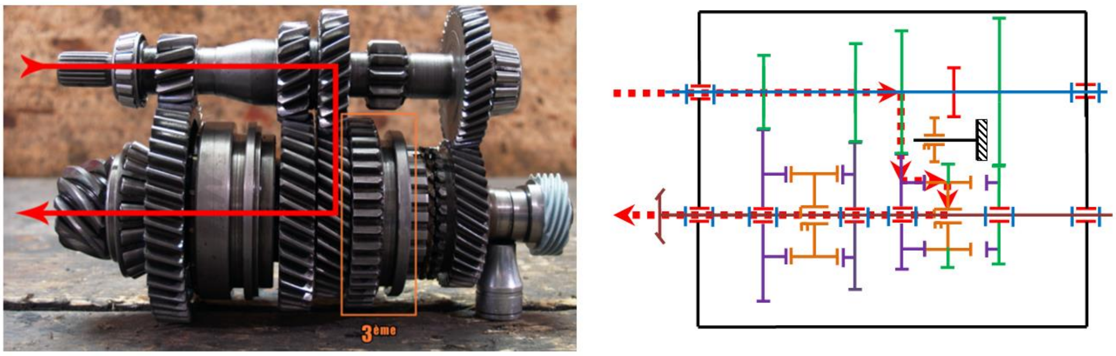


Fig.V. 2. 5: The third gear ratio.

VI. 2. 3. 4. The forth gear ratio

The sliding gear (2) moves to the right. The ration in this case is:

$$r_1 = \frac{Z_7}{Z_8} = \frac{40}{24} = 1.66$$

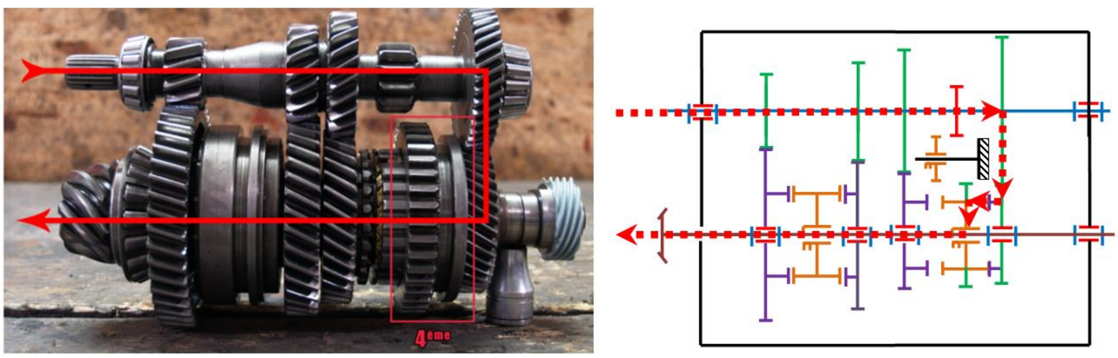
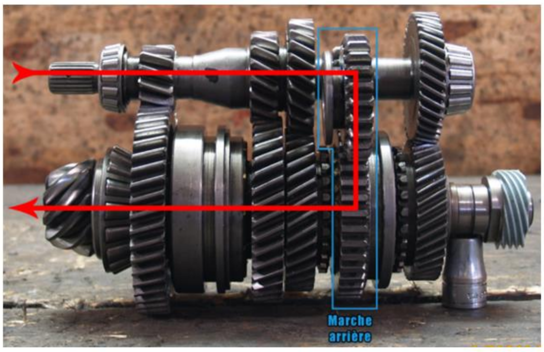


Fig.V. 2. 6: The forth gear ratio.

VI. 2. 3. 5. The reverse gear ratio

The sliding gear (2) is in the middle position, and the idler gear 10 becomes meshes with both gears 11 and sliding gear 2. The ration in this case is:

$$r_1 = (-1)^2 \frac{Z_{11} \times Z_{10}}{Z_{10} \times Z_9} = \frac{Z_{11}}{Z_9} = \frac{11}{28} = 0.392$$



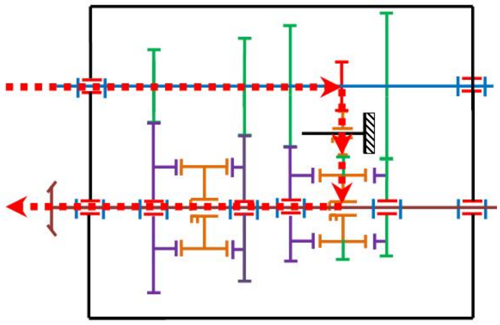


Fig.V. 2. 7: The reverse gear ratio.

VI. 2. 4. The Synchronizer

VI. 2. 4.1. Role of the Synchronizer

The role of the synchronizer is to bring the gears of the selected speed to the same rotational speeds before engaging the clutch.

VI. 2. 4. 2. Building a synchronizer

The synchronous system consists of a set of elements. Figure VI. 2.8 opposite shows the different elements of the synchronization.

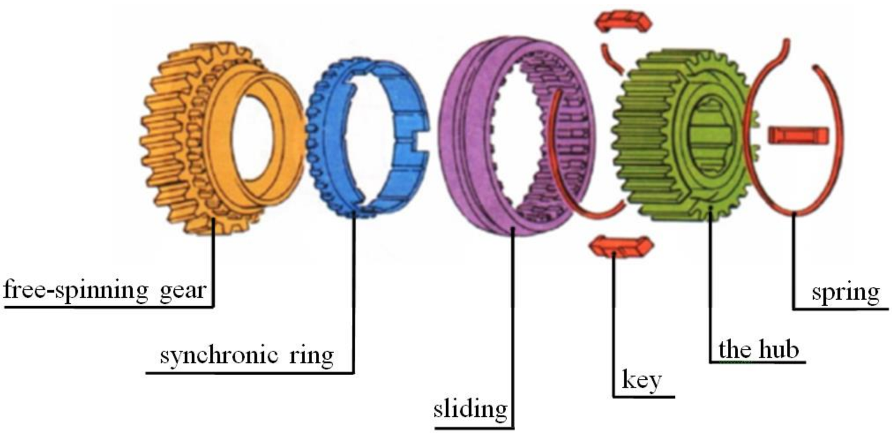


Fig.V. 2. 8: The synchronizer

VI. 3. Concepts of epicyclical gear trains

A gear train is epicyclic if one or more intermediate gears rotate around a shaft that is movable relative to the frame. These gears are called planetary gear "satellites," and the movable shaft is called the planet carrier "satellite carrier." Gears that are not mounted on the planet carrier are called sun gear and ring gear "planets."

Planetary gear trains are extensively used for the power transmission and are the most critical component. Planetary gearboxes are used frequently to match the inertias, lower the motor speed, and boost the torque.

1: Sun gear; 2: Planetary gear; 3: Ring gear; 4: Planet carrier

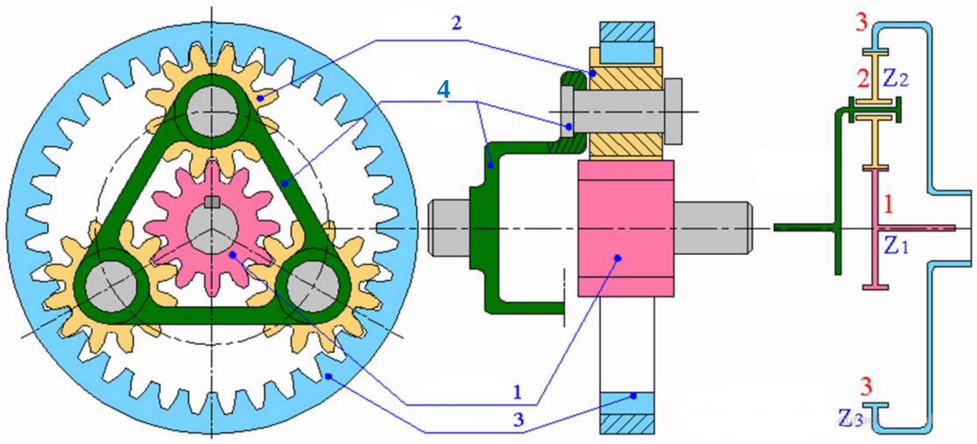


Fig.V. 3. 1: Epicyclical gear train.

VI. 3. 1. Kinematic relationship: Willis formula

Once the input, output, and planet carrier are identified, we work within a frame of reference linked to the planet carrier. In this frame, we see the gears rotating around fixed axes. We can therefore write:

From chapter I we have:

$$r = \frac{N_{outlet}}{N_{inlet}} = \frac{N_o}{N_i} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

The Willis formula is :

$$\frac{N_{Ring\ gear} - N_{Planet\ carrier}}{N_{Sun\ gear} - N_{Planet\ carrier}} = \frac{N_{RG} - N_{PC}}{N_{SG} - N_{PC}} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

V. 3. 2. Typical operating scenarios

V. 3. 2. 1. Ring gear (3) locked

This is the most common operating mode of the simple epicyclic gear train. (1) is the shafts entrance motion and (3) is

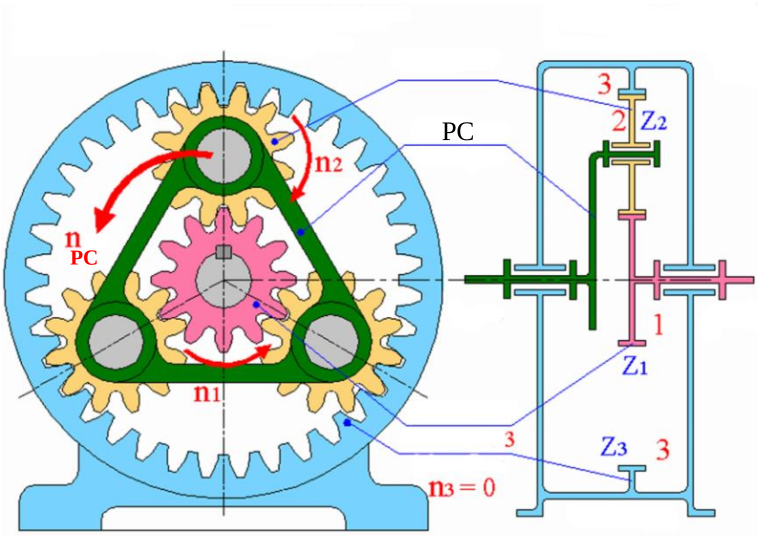


Fig.V. 3. 2: Epicyclical gear train **(**Ring gear (3) locked).

The configuration with planetary gear ring (3) locked is the most used: sun gear (1) at the input and planetary carrier PC at the output.

$$\frac{N_{Ring\ gear} - N_{Planet\ carrier}}{N_{Sun\ gear} - N_{Planet\ carrier}} = \frac{N_3 - N_{PC}}{N_1 - N_{PC}} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$n = 1$$

$$\frac{N_3 - N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1 \times Z_2}{Z_2 \times Z_3} = -\frac{Z_1}{Z_3}$$

$$N_3 = 0$$

$$\frac{-N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1}{Z_3}$$

$$-N_{PC} \times Z_3 = -Z_1 \left(N_1 - N_{PC}\right)$$

$$\frac{N_{PC}}{N_1} = \frac{N_4}{N_1} = \frac{Z_1}{Z_1 + Z_3}$$

V. 3. 2. 2. Sun gear (1) locked

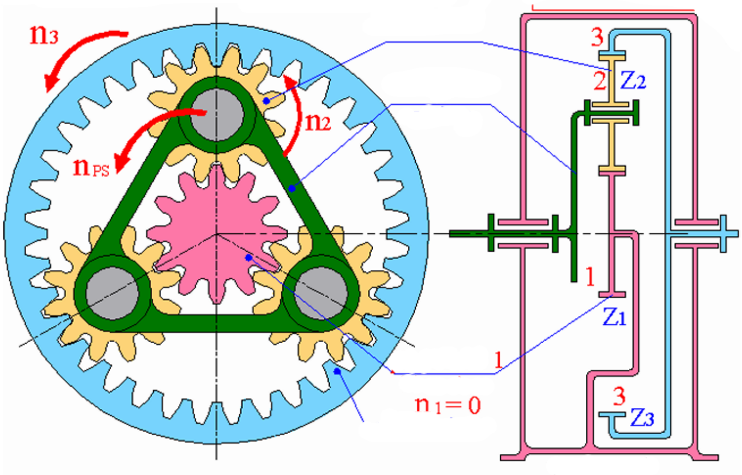


Fig.V. 3. 3: Epicyclical gear train (Sun gear (1) locked)

It's a variant of the blocked planetary (3).

$$\frac{N_{Ring\ gear} - N_{Planet\ carrier}}{N_{Sun\ gear} - N_{Planet\ carrier}} = \frac{N_3 - N_{PC}}{N_1 - N_{PC}} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$n = 1$$

$$\frac{N_3 - N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1 \times Z_2}{Z_2 \times Z_3} = -\frac{Z_1}{Z_3}$$

$$N_1 = 0$$

$$\frac{N_3 - N_{PC}}{-N_{PC}} = -\frac{Z_1}{Z_3}$$

$$N_{PC} \times Z_1 = Z_3 \left(N_3 - N_{PC}\right)$$

$$\frac{N_3}{N_{PC}} = \frac{N_3}{N_4} = \frac{Z_1 + Z_3}{Z_3}$$

V. 3. 2. 3. Planetary carrier (4) locked

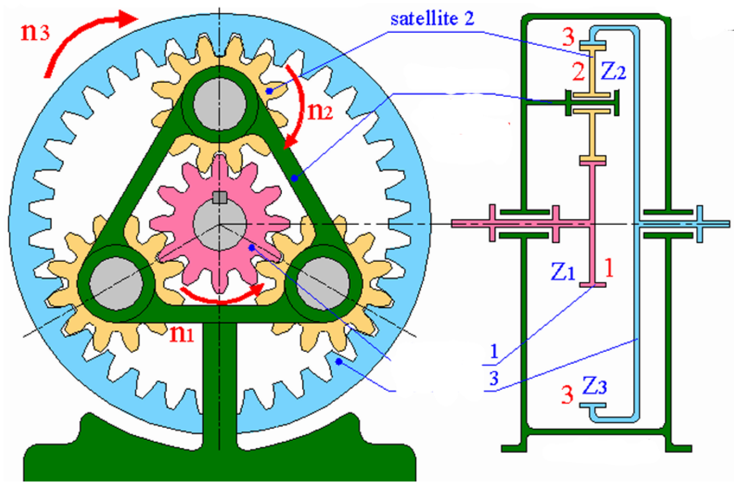


Fig.V. 3. 4: Epicyclical gear train (Planetary carrier (4) locked)

$$\frac{N_{Ring\ gear} - N_{Planet\ carrier}}{N_{Sun\ gear} - N_{Planet\ carrier}} = \frac{N_3 - N_{PC}}{N_1 - N_{PC}} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$n = 1$$

$$\frac{N_3 - N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1 \times Z_2}{Z_2 \times Z_3} = -\frac{Z_1}{Z_3}$$

$$N_{PC} = 0$$

$$\frac{N_3}{N_1} = -\frac{Z_1}{Z_3}$$

V. 3. 3. Applications

Exercise 1

Consider the epicyclic gear train shown in figure 2.

1. Label the parts from (0) to (5).
2. Draw its kinematic diagram.
3. Calculate the overall transmission ratio.

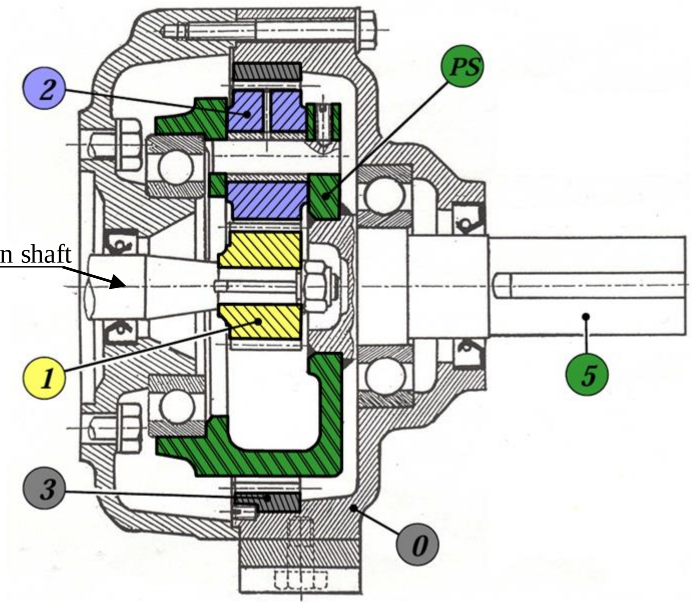


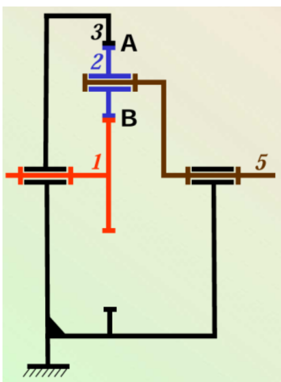
Fig.V. 3. 5: Epicyclic gear train scheme

Solution

1. Naming the parts from 0 to 5

0: Bâti ; 1 : Sun gear ; 2 : Planetary gear; 3:Ring gear; 5: Planetary carrier (shaft)

2.



3. Calculation of the overall transmission

$$\frac{N_{Ring\ gear} - N_{Planet\ carrier}}{N_{Sun\ gear} - N_{Planet\ carrier}} = \frac{N_3 - N_{PC}}{N_1 - N_{PC}} = (-1)^n \frac{Product\ of\ driving\ tooth\ numbers}{Product\ of\ driven\ tooth\ numbers}$$

$$n = 1$$

$$\frac{N_3 - N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1 \times Z_2}{Z_2 \times Z_3} = -\frac{Z_1}{Z_3}$$

$$N_3 = 0$$

$$\frac{-N_{PC}}{N_1 - N_{PC}} = -\frac{Z_1}{Z_3}$$

$$-N_{PC} \times Z_3 = -Z_1 \left(N_1 - N_{PC} \right)$$

$$\frac{N_{PC}}{N_1} = \frac{N_5}{N_1} = \frac{Z_1}{Z_1 + Z_3}$$

Chapter VI: General concepts of couplings, clutches and brakes

VI. 1. Couplings

A coupling allows torque to be transmitted from a driving shaft to a driven shaft, driving the driven shaft at the same speed as the driving shaft. Depending on the technology used, it can compensate for axial, radial, angular, or combined misalignments. It can also offer varying degrees of torsionally rigidity, as well as shock and vibration damping.

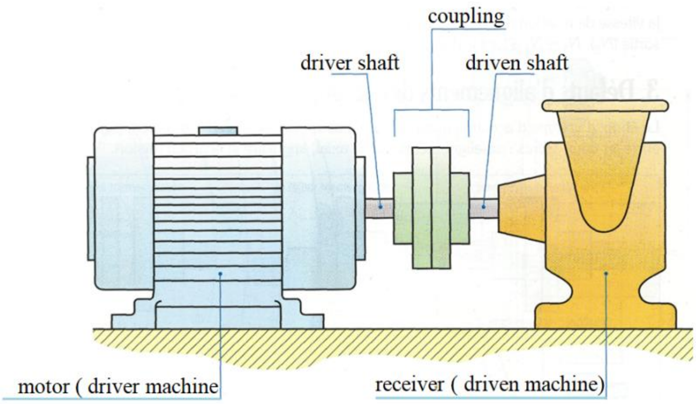


Fig.VI. 1. 1 : The principle of connecting in transmission shafts

They are also used to transmit power between two shafts extending from one another, possibly with misalignment.

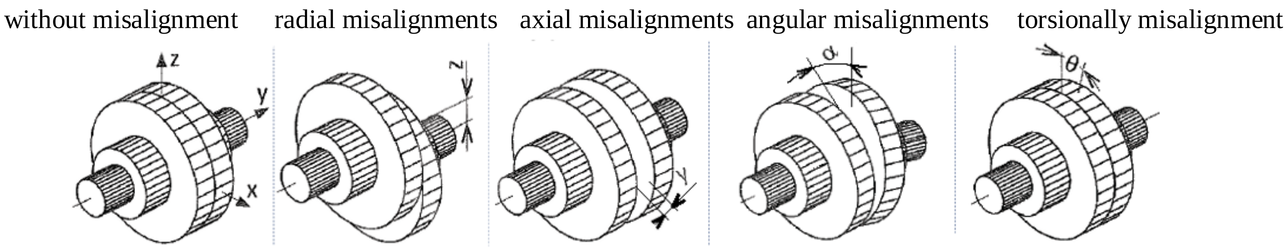


Fig.VI. 1. 2 : Main alignment defects

VI. 1. 1. Pre-selection of a coupling

To select a coupling, the torque available at the output of the drive machine (C) must first be calculated; for this, the following formula is used:

$C = 30P/\pi N$

such as:

P : is the power in (W); N : is the speed of rotation in (rpm).

In a second step, weight this value of the required service factor according to the application in order to define the service torque (see table of service factors below).

The service torque can be calculated as follow:

$$C_{ser} = K_s C$$

Table VI. 1: Values of Ks for different type of loads and training machines

Type of loads	Type of training machines		
	Electric motor Steam turbine	4-6 cylinder fuel-powered engine	1-3 cylinder powered engine
Low loads Low starting torque Smooth operation	1	1.25	1.75
Average loads average starting torque low variation in torque	1.25	1.5	-
Heavy loads Significant impacts Reversal of direction of travel.	1.5	2	2.5

VI. 1. 2. Main types of couplings

Mechanical couplings can be divided into two main families according to the possibility of misalignment.

VI. 1. 2. 1. Rigid couplings

No relative movement between the shafts is possible; the shafts must be perfectly aligned.

- Simple and economical
- Requires perfect alignment of the shafts to be coupled
- Does not filter vibrations

In this type we find for example:

VI. 1. 2. 1. a. Sleeve and pins (figure VI. 1. 3)

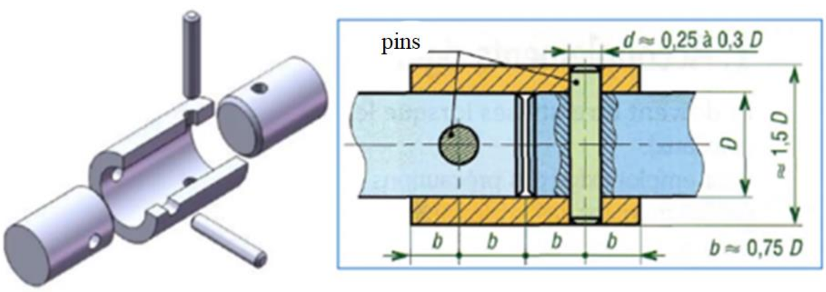


Fig.VI. 1. 3 : Sleeve and pins couple.

VI. 1. 2. 1. b. Sleeves and keys (figure VI. 1. 4)

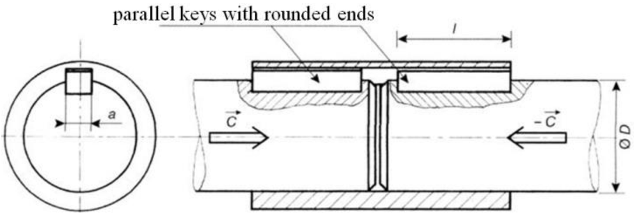


Fig.VI. 1. 4: Sleeve and keys.

VI. 1. 2. 1. c. Plate sleeve couplings

- Widely used, precise,
- Resistant, fairly lightweight,
- Radially bulky,
- They are often shrink-fitted or press-mounted.

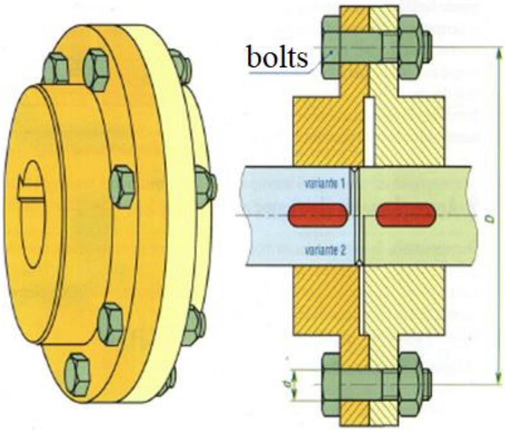


Fig.VI. 1. 5: Plate sleeve coupling

Torque transmission is generally achieved through a series of fitted bolts. In case of overload, the shear strength of the bolts provides some protection.

VI. 1. 2. 2. Elastic couplings

These components consist of two rigid elements connected by one or more intermediate elastic elements (rubber, spring...).

Their role is to overcome the disadvantages of rigid couplings, namely:

- To allow slight variation in the relative position of the axes
- To dampen vibrations.

Examples:

VI. 1. 2. 2. a. Flexible sheath sleeve

Allows for large angular displacements

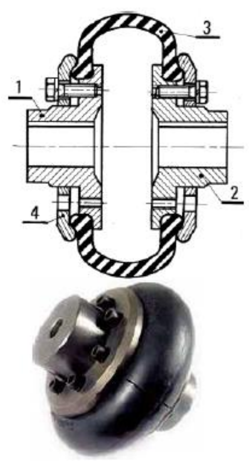


Fig.VI. 1. 6: Flexible sheath sleeve

VI. 1. 2. 2. b. Spit sleeve

The sleeves (4) are made of rubber.

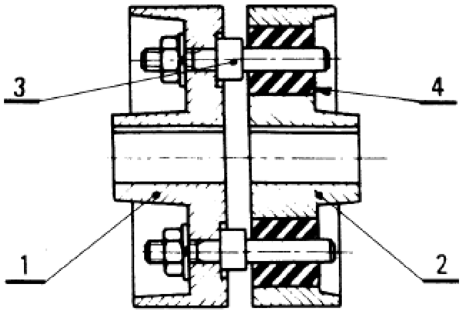


Fig.VI. 1. 7: Spit sleeve

VI. 1. 2. 2. c. Flector coupling

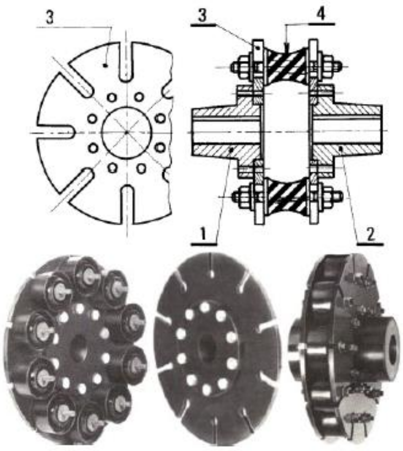
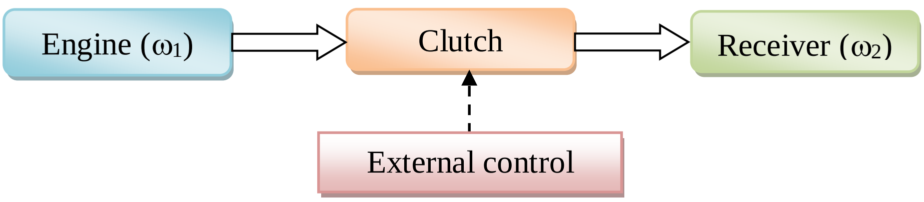


Fig. VI. 1. 8: Flector coupling.

VI. 2. Clutches

VI. 2. 1. Role of clutches

Clutches are designed to transmit power between two shafts without altering torque or speed, with the possibility of making the two shafts connected (engaged $\omega_2=\omega_1$) or independent (disengaged $\omega_2=0$).



VI. 2. 2. Clutch Classification

We can classify the clutches into two types, such as: instantaneous and progressive.

VI. 2. 2. 1. Instantaneous or Interlocking Clutches

- The connection is by means of an interlocking mechanism.
- The clutch can only be operated when the driven and driver shafts are stationary.
- No progressive engagement.

VI. 2. 2. 1.a. Claw Clutch

- Transmission of high torque.

VI. 2. 2. 1.b. Toothed clutches

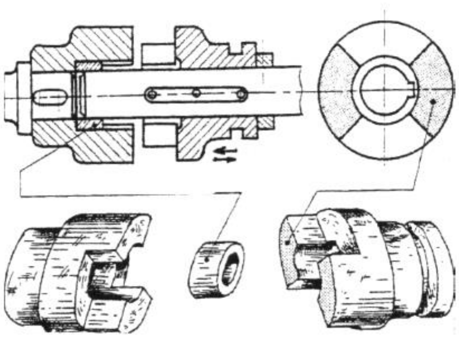


Fig. VI. 2. 1: Claw clutch.

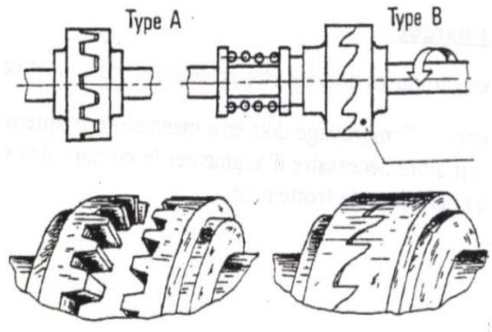


Fig. VI. 2. 2: Toothed clutch.

VI. 2. 2. 2. Progressive Clutch

Progressive clutch is a device for making a smooth, gradual connection between two separate elements rotating at different speeds about a common axis, bringing the two elements to a common angular velocity after the clutch is actuated.

- The connection is achieved through friction.
- The clutch can be operated while in motion.
- Progressive engagement is possible (the transmission drive is gradual).

VI. 2. 2. 2.1. Components

A progressive clutch generally consists of:

a- Contact surfaces; b- Pressure system; c- A control device; d- Sleeve-to-shaft connections

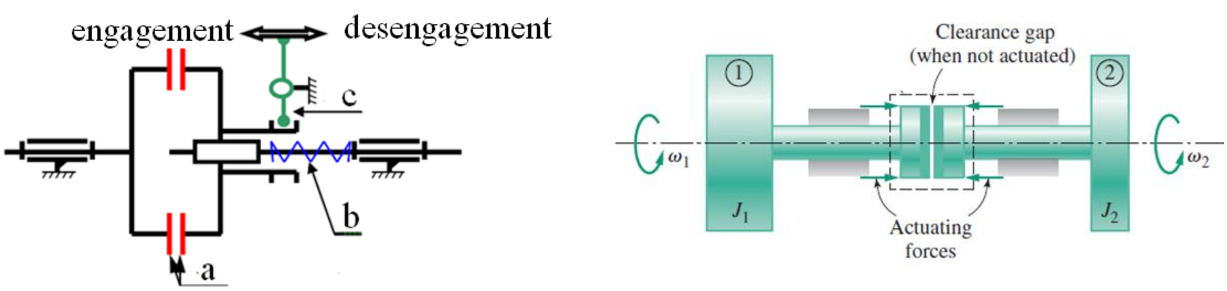


Fig. VI. 2. 3: Progressive clutch

VI. 2. 2. 3. Types of progressive clutch

The progressive clutch can be classified into eight types:

VI. 2. 2. 3.1. Single plate friction clutch

Figure VI. 2. 4 shows a single-plate clutch assembly in an engaged position. It means that the clutch pedal is not pressed by the driver. This condition is referred to as "Clutch Pedal Position-Up." When the pedal position is up, the axial force offered by clutch springs is pressed against the flywheel (i.e., towards the flywheel) with the clutch plate being sandwiched between the pressure plate and flywheel. Therefore, drive is engaged and power is transmitted from the flywheel to the clutch plate due to friction existing between their contacting surfaces. Further, the power is transmitted from clutch plate to clutch shaft through mechanical splines.

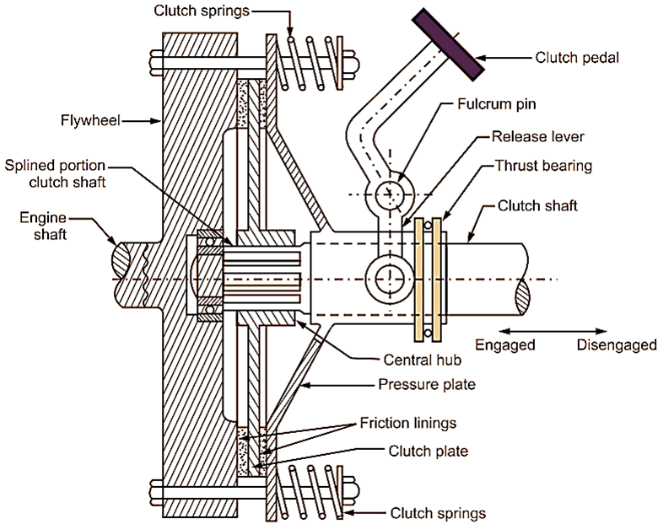


Fig. VI. 2. 4: Single plate friction clutch

VI. 2. 2. 3.2. Multi plate clutch

The Multi Plate Clutch uses multiple clutch plates to make contact with the engine flywheel to transfer power between the engine shaft and the transmission shaft. A multi-plate clutch used in automobiles and machinery where high torque output is required (see Figure VI. 2. 5).

It is a type of clutch that transmits more power from the engine to the transmission shaft of an automobile vehicle and also, makes up for the torque loss due to slippage. Multiple clutches consist of more than three discs or plates so that they can provide more torque output.

Multi-plate clutches are used in heavy vehicles with racing cars and motorcycles for transmitting high torque. As compared to single plate clutches, these are smooth and easy to operate due to their assembly of friction surface's contact. It may be used where the space is very limited.

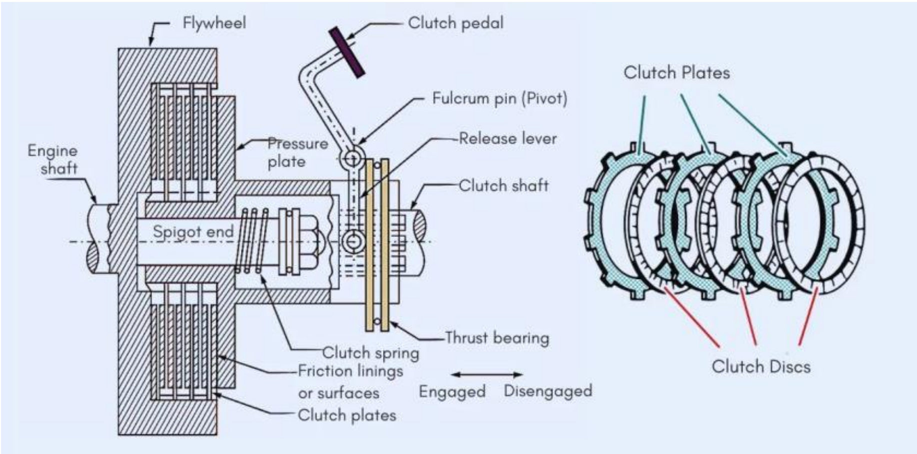


Fig. VI. 2. 5: Multi plate friction clutch

VI. 2. 2. 3.3. Cone clutch

In this type, the contact surfaces are on the fronts of the cones, as shown in the Figure VI. 2. 6. In the engaged position, the male cone is fully inside the female cone so that the friction surfaces are in complete contact. This is done by means of springs, which keep the male cone pressed at all times.

When the clutch is engaged, torque is transmitted from the engine via the flywheel and the male cone to the splined gearbox shaft. For disengaging the clutch, the male cone is pulled out by means of a lever system operated through the clutch pedal, thereby separating the contact surfaces.

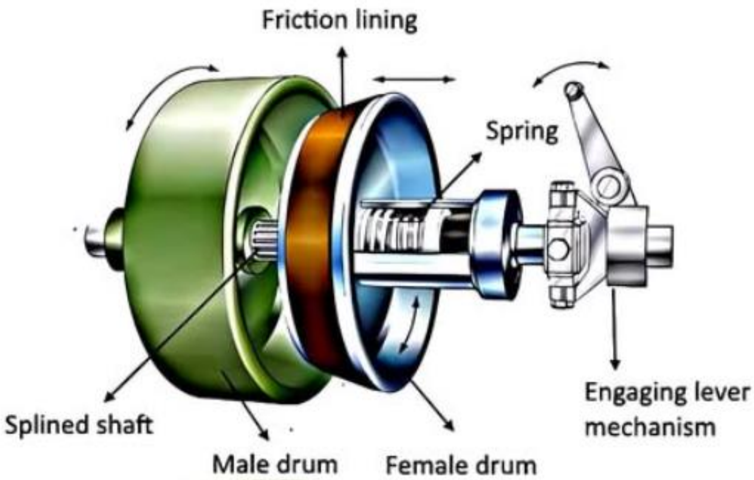


Fig. VI. 2. 6: Cone friction clutch

VI. 2. 2. 3.4. Centrifugal clutch

Centrifugal Clutch is one of the friction type clutches. This type of clutch works on the centrifugal force (see Figure VI. 2. 7). No manual operation is required to operate a clutch.

It is used in fixed (preset) speed applications and in vehicles with continuously variable transmission (CVT) system. No Gearbox is used in such types of vehicles.

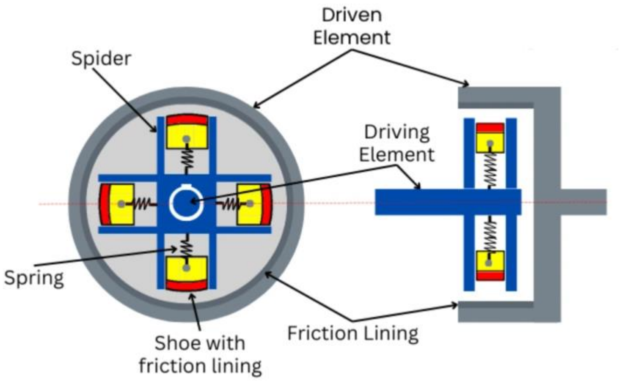


Fig. VI. 2. 7: Centrifugal clutch

VI. 2. 2. 3. 5. Semi-centrifugal clutch

This type of clutch uses lighter pressure plate springs for a given torque carrying capacity, so that the engagement of the clutch in the lower speed range becomes possible, At higher speeds, the extra clamping thrust is supplemented by the centrifugal force (see Figure VI. 2. 8).

The release levers can be centrifugally out of balance because offset bob weights are fastened to their outer ends. This centrifugal force pushes the pressure plate against the driven plate, increasing the clamping tension.

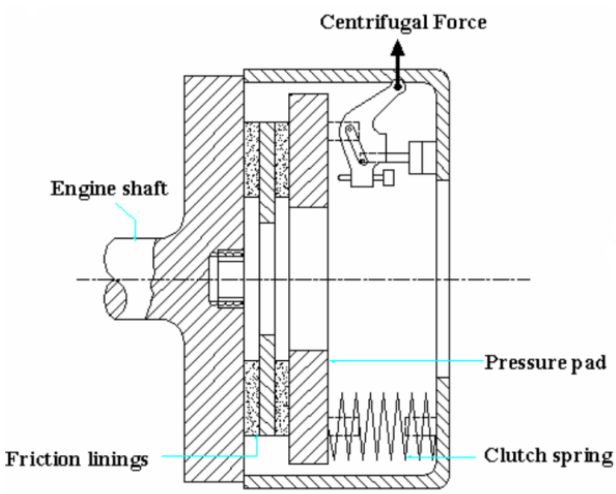


Fig. VI. 2. 8: Semi-centrifugal clutch

VI. 2. 2. 3. 6. Diaphragm clutch

A Belleville (diaphragm) spring supplies the force that holds the friction disc against the flywheel. The spring has tapered fingers pointing inwards from a solid ring. These act as release levers to take up the spring action as the clutch disengages (see Figure VI. 2. 9). As the clutch pedal is pressed, the release bearing pushes against the fingers, which cause the diaphragm to pivot about the inner pivot ring, and outer section moves outwards, and pushes the pressure plates away from friction disc. Spring force varies according to the size and thickness of diaphragm spring.

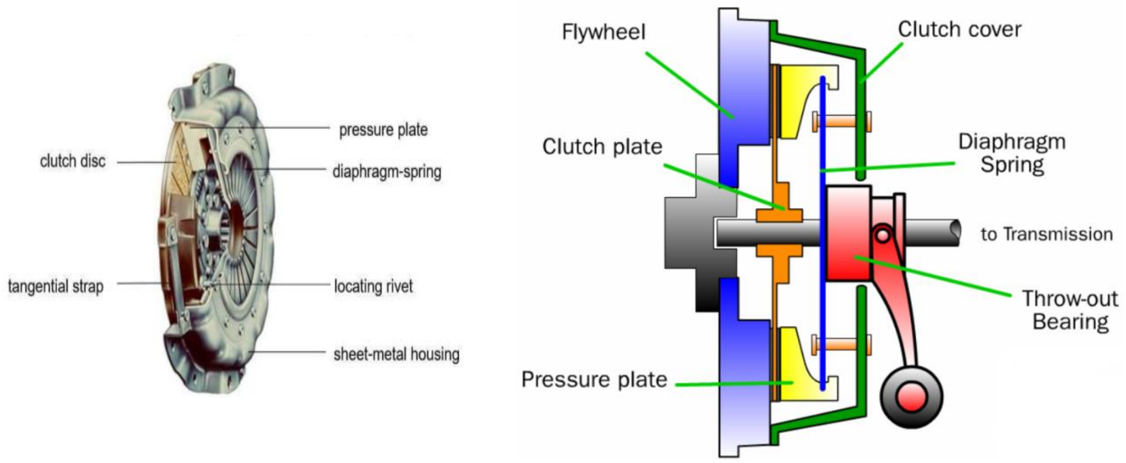


Fig. VI. 2. 9: Diaphragm clutch

VI. 2. 2. 3.8. Electromagnetic clutch

In this system the clutch is controlled by means electric current supplied to the field windings in the flywheel. The flywheel is attached with the field winding, which is given electric current by means of battery, dynamo or alternator. The construction feature of main components is almost similar to the single plate clutch (see Figure VI. 2. 10).

When electric current is supplied to the windings the flywheel will attract the pressure plate and clutch plate is forced between pressure plate and flywheel resulting in engagement.

When the supply to the winding is cut off the clutch is disengaged by releasing the pressure plate due to the force exerted by the helical springs or tension springs.

The electromagnetic clutch presented in figure below is constituted by:

1. Stator body (flywheel)
2. Field coil
3. Armature disc (clutch disc)
4. Return Springs
5. Output hub (pressure plate)

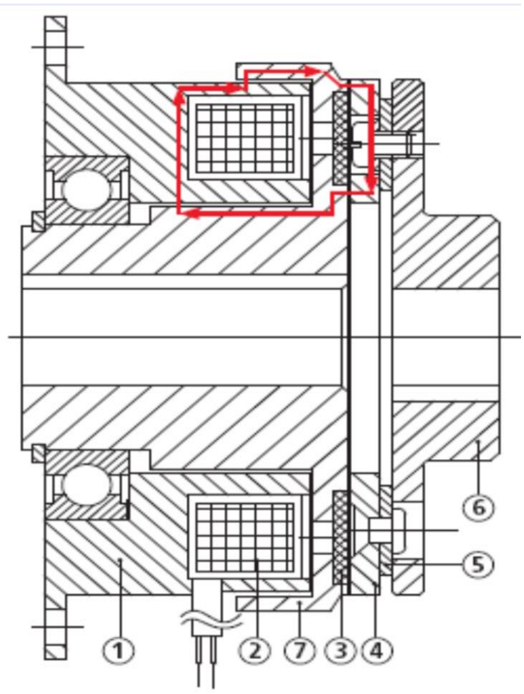


Fig. VI. 2. 10: Diaphragm clutch

VI. 3. The brakes

VI. 3.1. Introduction and definition

A braking system is a mechanical device that stops motion by absorbing kinetic energy from the moving system. It is used to slow or stop a moving vehicle, wheel, or axle, or to prevent motion, which usually happens through friction.

A brake serves a similar function like clutch, except that one of the elements is fixed to the frame, so the common angular velocity is zero following actuation of a brake (see Figure VI.3.1).

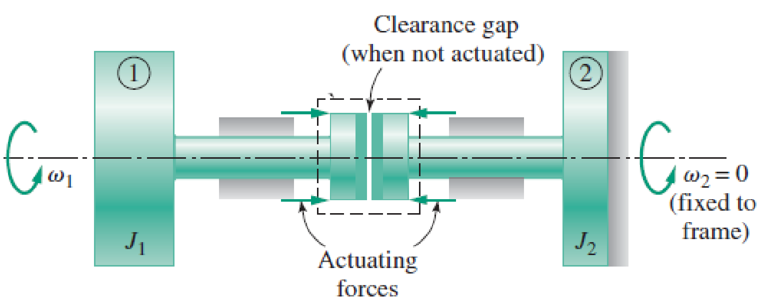


Fig. VI. 3. 1: Brake package

VI. 3. 2. Uses of brakes

Brakes is well known for their use in automotive applications, but are also extensively used in a wide variety of industrial machines including winches, hoists, excavators, tractors, mills, and elevators, and consumer products such as washing machines, garden tractors,, farm tractors, combines.

Although a variety of different types of brakes have been devised, friction brakes are most common. In spite of the adage for friction clutches and brakes it is an essential design ingredient. Selection of a good lining material usually implies selection of a material with a high coefficient of friction that remains essentially unchanged over a wide range of operating conditions.

VI. 3. 3. Brake characteristics

• A brake is powerful if a low control force corresponds to a high braking torque.

• A brake is smooth if the braking torque is proportional to the control force.

Power and smoothness are two characteristics of a brake that depend on its control system. They vary inversely.

VI. 3. 4. Different types of brakes

VI. 3. 4.1. Shoe brakes

The shoe brake is commonly used on trains and trams. The friction between the shoe and the wheel generates a tangential braking force that slows the wheel's rotation. The shoe is pressed against the wheel by a force applied to one end of a lever to which it is rigidly attached, as illustrated in Figure VI.3.2. The other end of the lever pivots around a fixed pivot point O. There are single-shoe brakes and double-shoe brakes (Figure VI.3.3).

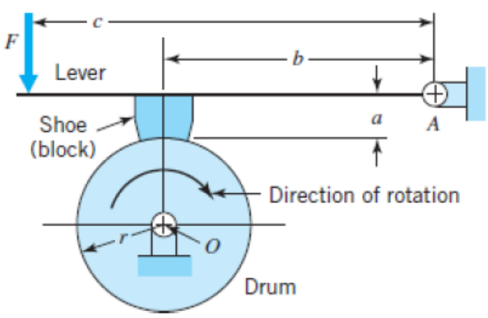


Fig. VI. 3. 2: Single shoe brake

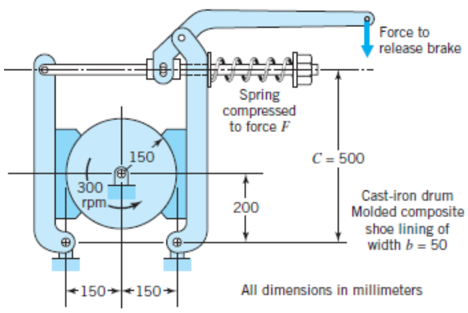


Fig. VI. 3. 3: Double shoe brake

VI. 3. 4. 2. Strap brakes

They are also called band brakes or winding brakes. A flexible band applied to a rotating drum creates a resisting torque through friction (Figure VI.3.4), the value of which depends on:

- The pressing force (*P*) opposing the movement of the drum.
- The coefficient of friction (*f*) between the lining and the drum.
- The radius (R) of the drum.

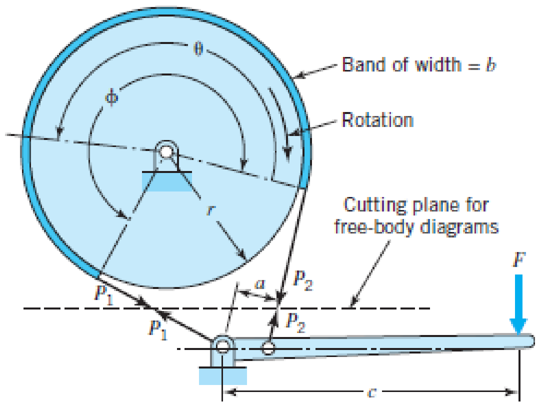


Fig. VI. 3. 4: Strap brakes

VI. 3. 4. 3. Drum brakes (or jaw brakes)

A drum brake consists of two shoes, S_1 and S_2, as illustrated in Figure VI. 3. 5. The outer surface of the shoes is coated with a friction material (usually Ferodo) to increase the coefficient of friction and prevent metal wear. Each shoe is hinged at one end around a fixed pivot point, O_1 and O_2, and contacts a control system at the other end, either mechanical (cam) or hydraulic (Wheel cylinder and piston). As the cam rotates, the brake shoes are pushed outward against the drum rim. The friction between the shoes and the drum produces the braking torque and thus reduces the drum's rotational speed.

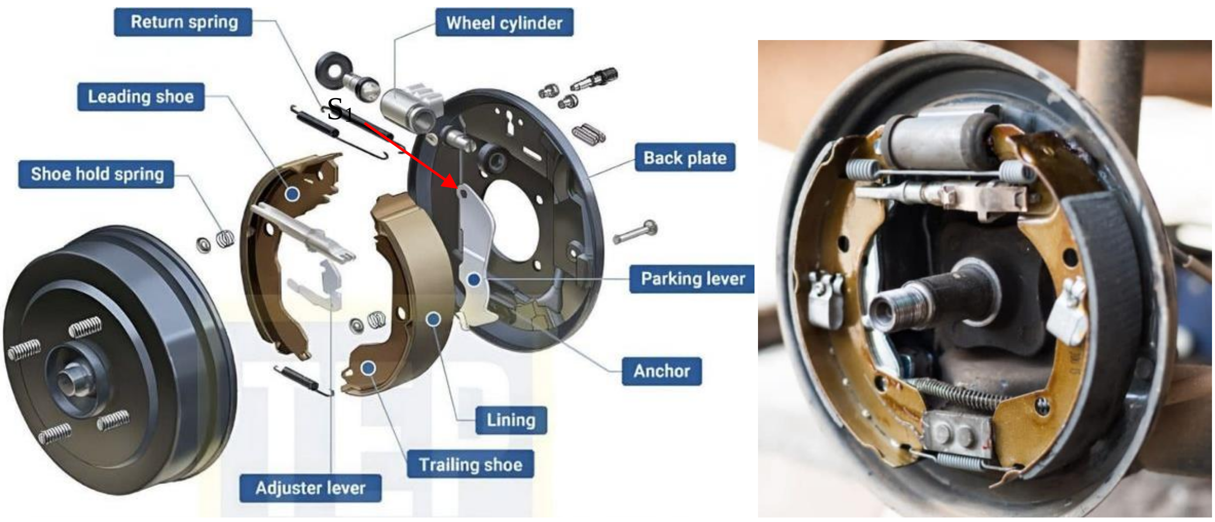


Fig. VI. 3. 5: Drum brakes

VI. 3. 4. 4. Disc brakes

A disc brake consists of a special cast iron disc that rotates with the wheel. It is mounted on a U-shaped caliper that holds two brake pads coated with a high-friction material (see figure VI.6.). When the brakes are applied, one or more cylinders, each containing a moving piston, press the brake pads against the disc. This slows both the disc and the wheel.

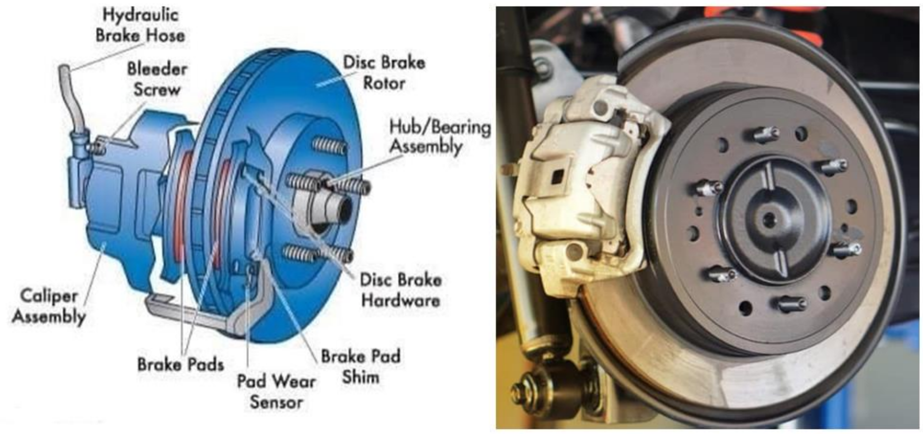


Fig. VI. 3. 6: Disc brakes

VI. 3. 5. Types of braking Systems

Following are the **types of braking systems** used in vehicles:

VI. 3. 5. 1. Mechanical Braking System

The mechanical braking system causes two surfaces to rub against one another, creating friction. In this braking system, a specific force is applied to the pedal, and it is transferred to the final drum by mechanical parts like a fulcrum, springs, and that are used as linkages to transmit force from one point to another, slowing the vehicle.

VI. 3. 5. 2. Hydraulic Braking System

The hydraulic brakes are applied by fluid pressure. The pedal force is transmitted to the brake shoe using a confined liquid through force transmission (see Figure VI. 3. 7).
The master cylinder is connected by tubing to the wheel cylinder on each of the four wheels. This system fills with liquid under light pressure when the brakes are not in operation.

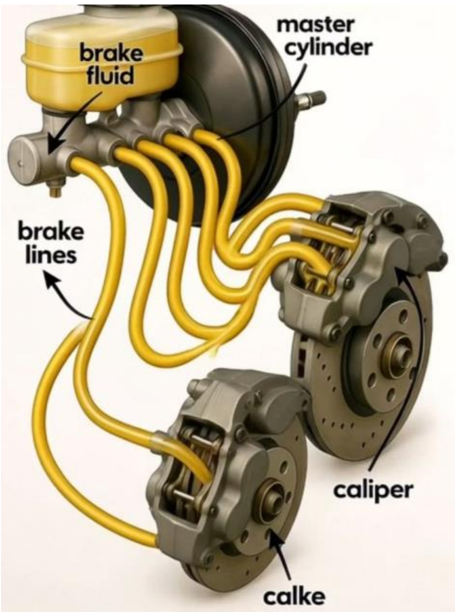


Fig. VI. 3. 7: Hydraulic Braking System

VI. 3. 5. 3. Magnetic Braking system

Electric brakes are also used for some devices like reducer and vehicle, although these are not very popular.

The brake operates when battery current is used to power the electric coil, which activates the mechanism to engage or disengage the brake. Figure VI.3.8 depicts an electromagnetic brake used to brakes a speed reducer. When the coil is energized, an electromagnetic force attracts the plate (20) to release the disc (19), which carries the friction lining, and the system is freed. When the coil is not energized, the spring (21) pushes back the plate, which forces the disc (19) against the fixed plate (18), and consequently, the system is braked.

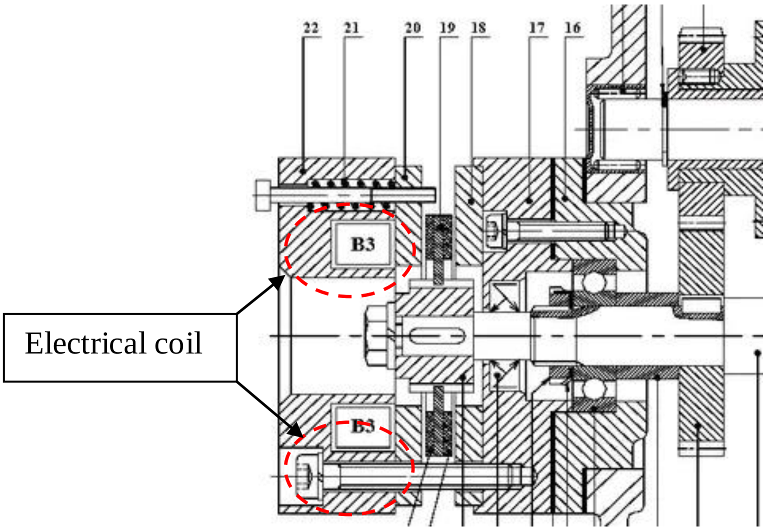


Fig. VI. 3. 8: Magnetic Braking system

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