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Steel Structures 2

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Level:

- 3rd Year: Engineer specializing in Construction and Sustainable Development.

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FOREWORD

This handout, entitled “**Steel Structures 2**”, introduces the fundamental principles required for the analysis and design of steel structural members. It focuses on the mechanical behavior of steel elements, the identification of common steel products and sections, and the verification of members subjected to different loading and stability conditions.

It is primarily intended for third-year engineering students specializing in **Construction and Sustainable Development (CSD)**.

The main **objectives** are to:

- Consolidate students’ knowledge of the principles of steel construction;
- Introduce the mechanical and geometric characteristics of structural steel members;
- Develop the ability of students to analyse and design steel members subjected to tension, compression, buckling, combined effects, and lateral-torsional buckling;
- Prepare students to apply the basic design rules used in metal construction.

Successful completion of this module requires **prior knowledge** in:

- Structural mechanics;
- Strength of materials;
- Structural analysis;
- Building construction;
- Basic principles of steel structures;
- Structural design concepts.

This handout is structured into **nine** chapters. The **first** chapter presents general concepts related to steel construction, including its main fields of application, advantages, disadvantages, and the importance of considering durability, stability, fire protection, and maintenance from the earliest design stages.

The **second** chapter introduces steel as a structural material. It deals with its composition, classification, mechanical properties, tensile behavior, standardized grades, and the usual physical and elastic coefficients required in elementary design calculations.

The **third** chapter is devoted to steel products and sections. It presents the main families of steel profiles used in metal construction and explains how their geometric characteristics influence resistance, stiffness, stability, and detailing.

The **fourth** chapter deals with elastic buckling phenomena. It highlights the importance of stability in steel structures and introduces the main forms of instability that may affect compressed or bent members.

The **fifth** and **sixth** chapters focus respectively on the design of members under pure tension and pure compression. They present the fundamental concepts, resistance checks, and practical procedures required to verify common steel members.

The **seventh** and **eighth** chapters address, respectively, the design of members subjected to simple bending and combined bending. In the case of combined bending, axial compression and bending moments interact, which helps students understand the importance of combined verification in real structural situations.

The **last** chapter is dedicated to lateral-torsional buckling of steel members. It explains the behavior of beams under bending when lateral restraint is insufficient and presents the essential concepts required for safe design.

This handout is not intended to be definitive; it may be revised and improved according to constructive feedback from readers, both in terms of content and presentation. Any suggestions or comments are therefore welcome, and I hope that this work will be a useful pedagogical resource for students in civil engineering.

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Preface

The importance of steel construction in modern civil engineering is scarcely challenged today. Steel has become one of the most efficient structural materials used in buildings, industrial halls, bridges, roofs and other engineering works due to its high strength-to-weight ratio, quick erection, industrial fabrication quality and great architectural flexibility. However, a safe and economical design of steel structures requires a clear understanding of material behavior, member resistance, stability phenomena and practical verification rules.

Good knowledge of structural mechanics, strength of materials, structural analysis, building construction & fundamental design principles of structures are required for the study of steel structure. In this context, the module “Steel Structures 2” is meant to support students in developing both theoretical understanding and practical competence in the analysis and design of steel members subjected to different loading and stability conditions.

This handout, titled “Steel Structures 2” has been prepared for the support of third year engineering students specialising in Construction and Sustainable Development. It introduces the basic concepts of steel construction and takes students through the main aspects required for the design of steel structural members in a step-by-step manner. It contains mechanical behavior of steel, steel products and sections, elastic instability phenomena, members under tension and compression, simple and combined bending and lateral-torsional buckling.

Particular attention is given to the engineering meaning of the design procedures rather than to the mechanical application of formulae. The intention is to make students familiar with the behavior of steel members under load, the potential impact of instability on structural safety and the use of suitable verification methods for practical design purposes.

This handout is intended as pedagogical support for lectures, tutorials and personal study. It may be useful to students in consolidating their knowledge and preparing themselves for more advanced applications in steel design and structural engineering. As with any teaching document it is open to improvement and constructive comments or suggestions from readers are always welcome.

Chapter 1

General Concepts

Learning objectives:

- ↳ At the end of this chapter, the student should be able to:
- Explain the role of steel construction in civil engineering,
 - Identify the main fields of application of steel structures,
 - Distinguish their principal advantages and limitations,
 - Understand why durability, stability, fire protection and maintenance must be considered from the earliest design stage.

1.1. Introduction:

Steel constructions are one of the main structural systems of modern civil engineering due to their high mechanical efficiency, fast construction and wide adaptability for various architectural and functional requirements. Steel combines high tensile and compressive resistance with relatively low self-weight, and unlike many conventional structural materials, is particularly suitable for structures where strength, stiffness and construction speed must be simultaneously achieved. Hence steel has become a strategic material in projects where long spans, controlled fabrication quality, ease of transportation and the possibility of future modification or extension are required.

From the engineering point of view the growing use of steel structures is related also to the industrial character of their production. Structural members are usually manufactured in workshops under controlled conditions, allowing for precise dimensions, better quality assurance and more reliable assembly on site. Moreover, steel structures can be efficiently combined with composite systems, bolted or welded connections and prefabricated construction methods, thus increasing efficiency of execution and reducing on-site uncertainties. These characteristics are the reason for the wide application of steel construction in fields where the structural performance and operational flexibility are of paramount importance.

Typical applications of steel structures (Figure 1.1) are:

- Industrial buildings with expansive interior spaces created by steel frames to accommodate equipment installation and heavy service loads.
- Storage and warehouse buildings – needing long-span roof systems and flexible internal layouts for logistic and operational efficiency.
- Civil engineering works, especially bridges, where the high strength-to-weight ratio of steel permits a reduction of the dead load over large distances.
- Parking structures. They are advantageous due to fast erection, repetitive structural grids and potential for future expansion/adaptation.
- Facilities for sports including stadiums and gymnasiums where large column-free spaces and architecturally expressive roof forms are often required.

- Educational and public buildings, for which steel can allow rapid construction and greater flexibility of functional change over time.
- High-rise buildings, in which steel framing can reduce structural weight and improve construction speed when compared with heavier alternatives.
- Aircraft hangars and other large enclosures where very long spans and high clearances are critical for operational use.
- Agricultural buildings, which often require economical structural systems, rapid installation, and flexible space organization.
- Crane and industrial runway supporting structures. Steel in these provides the necessary resistance, stiffness and detail capacity against the dynamic loads due to their operation.

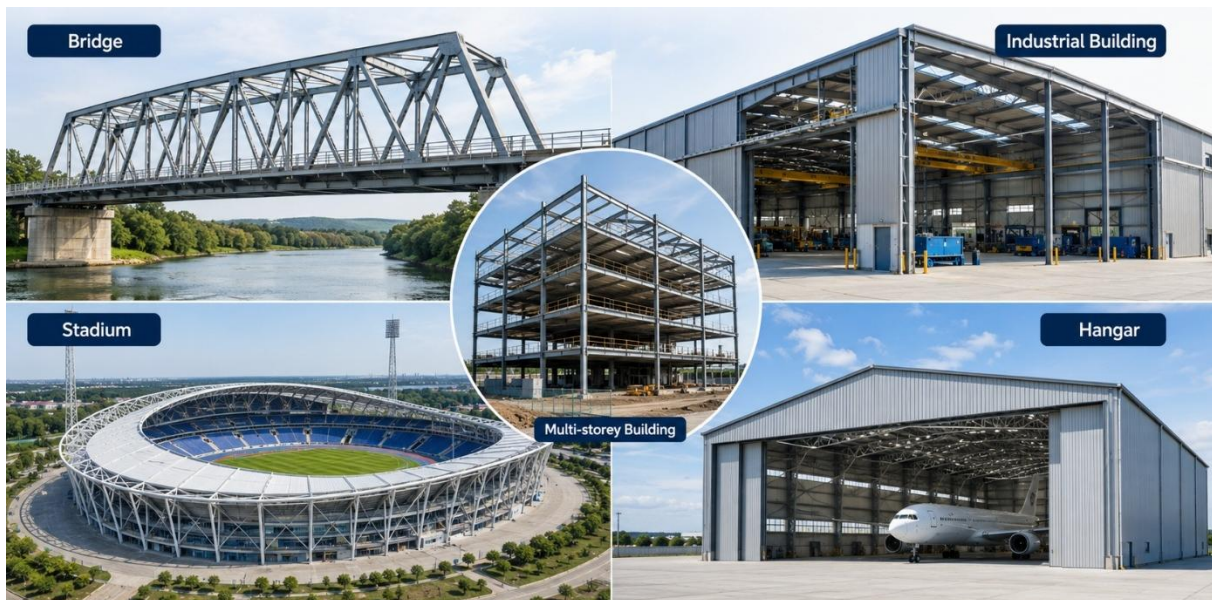


Figure 1.1. Applications of steel structures [1].

1.2. Advantages of steel structures:

Steel is one of the most used materials in structural engineering due to its mechanical, constructive and economic advantages. These benefits are related not only to the resistance of the material itself but also to the efficiency of fabrication, transportation, assembly and maintenance during the service life of the structure. Steel structures can ensure high structural reliability and be adapted to a very wide range of engineering applications, provided they are properly designed and protected.

- Light weight, high strength. Steel is capable of withstanding large internal forces and, at the same time, has relatively small cross-sectional dimensions. This property permits long spans, reduces the dead load transmitted to the foundations and allows optimisation of the global structural system.
- Dependable, predictable mechanical performance. Steel is a material that is manufactured industrially, and it has homogeneous properties, a well-defined stress-strain behaviour and standardised grades. This predictability is beneficial for the analysis, design and safety verification of structures.
- Fast to fabricate and erect. As a large number of steel elements are pre-fabricated prior to arriving on site, assembly operations can be carried out rapidly, often resulting in shorter construction schedules and less disruption than with slower wet construction processes.
- Quick and easy to assemble, disassemble and reinforce. Steel members are assembled efficiently by bolted and welded connections, which can be disassembled, replaced, or reinforced if necessary. This feature is especially suitable for rehabilitation, vertical extension and adaptive reuse projects.
- High-dimensional accuracy. Steel components are usually manufactured by industrial cutting, drilling and welding procedures giving good geometric accuracy. This accuracy improves the fit-up during erection and controls the accumulative tolerances in the finished structure.
- Compatible with prefabrication and modular construction. Steel lends itself well to standardisation of production and modular fabrication. This in turn facilitates better quality control, less material waste and improved site organization.
- Reusable and recyclable Structural steel can be reclaimed, recycled and in some cases reused in other projects. This is resource efficient and supports more sustainable approaches to construction when lifecycle considerations are taken into account.
- Architectural Flexibility & Aesthetic Value. Thanks to its resistance and slenderness, steel makes possible elegant structural forms, light roofs, transparent facades and expressive architectural solutions difficult to achieve with bulkier materials.

- Waterproofness of well executed elements. Well detailed and protected joints and members of steel assemblies can provide good impermeability to water and gases and this is an advantage in tanks, industrial facilities and some special structures.

Steel is therefore often considered an efficient material, not only for its mechanical performance, but also for the possibility of an integrated construction process based on precision, speed and adaptability. Its full potential is realised when design, fabrication, erection and maintenance are regarded as a coherent engineering system.

1.3. Disadvantages of steel structures:

Despite its many advantages, steel is not free from limitations.. During the design phase and throughout the life cycle of the structure many technical and economic constraints have to be carefully considered. Not accounting for such limitations could endanger the durability, serviceability, safety, or general economy of the project. Therefore, the design of steel structures should always be accompanied by adequate protective measures, detailing provisions and inspection strategies.

- Corrosion susceptibility. In aggressive environments protected steel may oxidise and slowly lose section thickness, which can reduce both durability and load-carrying capacity. Protective coatings, galvanisation, drainage detailing and regular maintenance are therefore needed.
- Risk of instability. Steel members are often slender and efficient and may experience local buckling, lateral-torsional buckling or overall instability when subjected to compression and bending. This requires stringent verification of slenderness of member, bracing systems and boundary conditions.
- Fire resistance and high temperature resistance reduced. Steel is not combustible but its mechanical properties considerably decrease with increasing temperature. In the event of fire, the loss of stiffness and strength can lead to premature deformation or collapse without adequate fire protection.
- Susceptible to fatigue. Steel elements and in particular their connections can develop fatigue damage under repeated or fluctuating stresses with time. This problem is of particular importance in bridges, crane-supporting structures, towers and other systems subjected to cyclic loadings.

- Qualified fabrication and execution requirement. The structural performance of steel is very much a function of the quality of welding, bolting, alignment and erection procedures. Poor workmanship or lack of control can cause residual stresses, geometric imperfections or connection defects.
- Maintenance needs. Many steel structures require regular inspection and maintenance to maintain corrosion protection, fire protection systems and connection integrity, unlike some materials that can tolerate long periods with limited intervention.
- Possible cost sensitivities. Although steel can be economical in many instances, the total cost can be relatively high depending on market fluctuations, complexity of fabrication, transport requirements, and the need for corrosion and fire protection treatments.

In conclusion, steel is a high-efficient structural material, the advantages of which can be fully exploited only with a proper control of its main disadvantages. Thus, a good steel design has to balance structural efficiency with durability, stability, fire safety, quality of execution and lifecycle maintenance requirements. This balance is what makes steel construction still one of the most efficient answers in modern structural engineering.

1.4. Conclusion:

This chapter has shown that steel construction is valued for its strength, its lightness, its speed of execution, its adaptability and its industrial quality. But these advantages have to be paid for by corrosion protection, fire resistance, stability control, execution quality and maintenance. Knowing this balance prepares the student for the chapters on material and member design that follow.

Chapter 2

Steel as a Structural Material

Learning objectives:

- ↳ At the end of this chapter the student should be able to: define structural steel; describe the composition and basic classification of structural steel; interpret the main results of the tensile test; identify the meaning of standardized grades such as S235, S275 and S355; use the usual elastic and physical coefficients in elementary steel-design calculations.

2.1. Introduction:

An adequate understanding of the material used in steel construction is necessary for safe and rational structural design. In steel structures, the designer is not dealing with a geometric form only; he is dealing with a material whose mechanical response, manufacturing route and standardised properties directly govern strength, stiffness, durability and structural reliability.

That is why this chapter introduces steel as a structural material and reviews the basic concepts required for the design and analysis to follow. It is considered in terms of its composition, general classification, main mechanical properties, tensile behaviour, standardised grades and the usual physical and elastic coefficients that are used in structural engineering calculations.

The object is not to increase the scope beyond the essentials of structural steel, but to give a technically clear basis of understanding of the behaviour of steel under loading and the reason for the selection of particular grades in metal construction practice.

2.2. Definition:

Steel is a metal material based on iron, obtained by metallurgical processes that intentionally add carbon in controlled proportions. From the engineering point of view, steel can be defined as an alloy, the mechanical performance of which can be adapted to the composition, thermal treatment and manufacturing conditions in order to satisfy structural, mechanical or technological requirements.

In comparison to pure iron, steel has much greater strength and a more useful balance between stiffness, ductility, toughness, and manufacturability. This explains its widespread use in construction and industry where a material is required to withstand load efficiently, yet maintain workability during fabrication, welding, shaping and assembly.

These properties make steel useful in a very wide range of applications (Figure 2.1) such as the following:

- Buildings, industrial halls, bridges and other civil engineering works.
- Manufactured products, including automotive, railway, shipbuilding and aeronautical components and transportation systems.

- Mechanical tools and equipment, for example hammers, drills, machine parts, and all kinds of industrial devices.

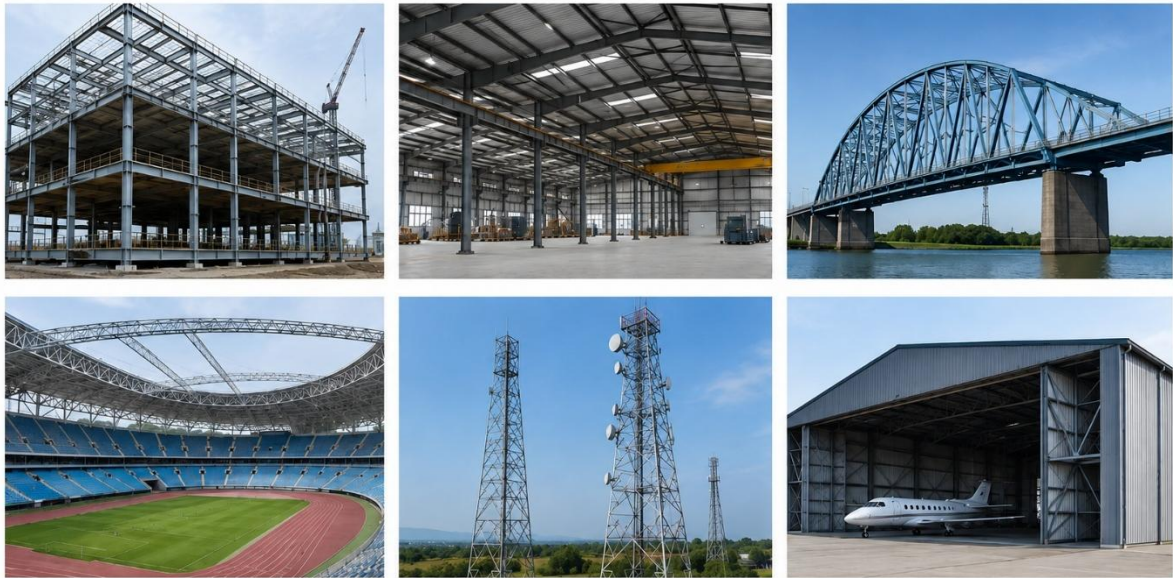


Figure 2.1. Steel applications [2].

2.3. Composition of steel:

Structural steel is mainly iron, generally at least about 95% of the total mass, with carbon present in much smaller proportions, typically less than 1% for ordinary construction steels. Carbon is added in relatively small quantities, but it has a decisive influence on the behavior of steel. It strongly influences strength, hardness, ductility and weldability.

In practical metallurgy steel, too, contains minute quantities of other elements. These may be either residual constituents from the manufacturing process or additions for the improvement of specific properties. Deoxidation and strength elements such as manganese and silicon may be added but phosphorus and sulphur are normally limited because they may reduce toughness, ductility or weldability if added in excess.

The carbon content is still one of the simplest and useful criteria for the steel classification. Generally, the increase of percentage of carbon increases hardness and strength of the material but reduces ductility and suitability for some fabrication operations. For structural applications, low-carbon steels are preferred because they provide a good compromise between mechanical resistance and fabrication performance.

2.3.1 Classification of steels according to carbon content:

The carbon content has a significant influence on the properties, fabrication behavior, and typical fields of application of steel (Table 2.1).

Table 2.1. Classification of steels according to carbon content and their use [3]

Type of steel	Carbon content (%)	Typical use
Mild steel	$0.10 < C < 0.20$	General structural construction
Medium-mild steel	$0.20 < C < 0.30$	Construction and common fabricated members
Medium-hard steel	$0.30 < C < 0.40$	Rails and forged mechanical components
Hard steel	$0.40 < C < 0.60$	Tools and wear-resistant parts

From the structural engineering standpoint, the steels used in buildings and bridges are therefore generally low-carbon steels, since these grades ensure adequate weldability, good ductility, and reliable behavior under static and, in many cases, dynamic loading.

2.4. Types of steel used in metal construction:

In metal construction, different categories of steel are used, depending on the required strength class, corrosion resistance, fabrication performance and service conditions. Here is a summary of the most popular families:

- General-purpose structural steels, used for most frames, trusses, columns, beams and ordinary welded or bolted members.
- Stainless steels. Alloying elements, including chromium, are added to these steels, which are used when high corrosion resistance and a durable architectural appearance are desired.

- Heat-treated steels, which are strengthened and hardened by controlled thermal processes such as quenching and tempering to meet more stringent mechanical requirements.

The choice of these steel families is a compromise between structural and environmental aspects. For typical building construction, general purpose structural steels continue to be the most economical answer. However, higher performance steels may be justified in aggressive atmospheres, special requirements of durability or highly loaded structural components.

2.5. Mechanical properties of structural steel:

The suitability of steel for structural use is mainly related to a number of mechanical properties that describe its behaviour under load. These properties are experimentally determined and form the basis for the selection of steel grades, design checks and serviceability assessment.

The basic mechanical properties of structural steel are:

- **Strength:** the ability of steel to resist the applied actions without attaining the point of yielding, excessive plastic deformation or rupture.
- **Elasticity:** the characteristic of the material to recover the original dimensions upon removal of the load, if the elastic limit has not been exceeded.
- **Plasticity:** ability to undergo permanent deformation (after yielding) without immediate fracture.
- **Ductility:** capacity to undergo large tensile deformations prior to failure. This property is important for warning prior to failure and for the redistribution of internal forces.
- **Resilience or toughness:** the ability to absorb energy, especially under impact or dynamic loading, before fracture.
- **Hardness:** the resistance of the steel surface to indentation, abrasion or localised plastic deformation.

The combination of high strength with adequate ductility and toughness is usually most important for structural engineers. A steel grade which is strong but not sufficiently

ductile may be brittle, while a steel with good ductility and toughness is generally more dependable in actual structural situations, especially where stress concentrations, accidental overloads or temperature effects can occur.

2.6. Tensile test:

The tensile test is the fundamental laboratory test used to characterize the mechanical behaviour of steel in uniaxial tension. It is carried out on a standardised specimen, typically with a reduced gauge length in the central section, which is exposed to a gradually increasing axial tensile force until fracture.

During the test the applied load and the corresponding elongation are continuously measured. These measurements can be used to determine engineering stress and engineering strain, which are generally reported as:

$$\sigma = \frac{F}{A_0}, \quad \varepsilon = \frac{\Delta L}{L_0} \quad (2.1)$$

where A_0 is the initial cross-sectional area and L_0 is the initial gauge length.

The resulting stress-strain curve gives direct access to the main parameters used in steel design such as the yield strength, the ultimate tensile strength, the elastic modulus and the elongation at fracture.

Besides the numerical values, the tensile test shows the order of physical phenomena, through which the specimen passes: elastic elongation, yielding onset, plastic flow, strain hardening, necking, and final fracture. It thus serves as the reference test for characterising the mechanical response of structural steel (see Figure 2.2).

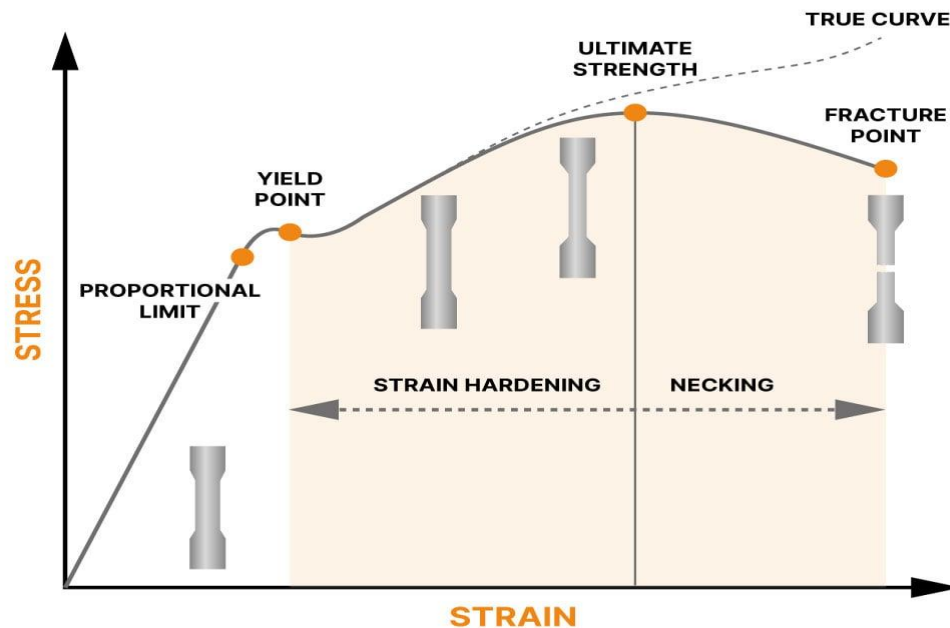


Figure 2.2. Typical stages of a tensile test on a steel specimen [5].

2.6.1. Stress-Strain Diagram:

At room temperature, the engineering stress-strain curve of ordinary structural steel is typically divided into four main phases (Figure 2.4). Each phase corresponds to a distinct mechanical behavior and has direct significance for structural analysis and design.

- **Domain OA** - Elastic domain: in this initial part of the curve, stress is proportional to strain and the material obeys Hooke's law, $\sigma = E\varepsilon$. Deformation is fully recoverable when unloading occurs. This is the normal working range adopted in most elastic design methods. The two characteristic parameters associated with this stage are the modulus of elasticity E and the yield strength f_y .
- **Domain AB** - Yield plateau or flow phase: after the elastic limit is reached, the steel may exhibit a nearly horizontal plateau in which strain increases considerably while stress remains approximately constant. The material begins to deform plastically, which means that part of the deformation becomes permanent.
- **Domain BC** - Strain hardening phase: once the yield plateau ends, the stress rises again with increasing strain. This phenomenon, known as strain hardening, reflects the capacity of the material to mobilize additional resistance through plastic deformation. The maximum stress attained in this region is the ultimate tensile strength f_u .

- **Domain CD** - Necking and rupture phase: after the maximum load is reached, a localized reduction of the cross-section develops, known as necking. Although deformation continues to increase, the engineering stress decreases until fracture occurs. This final phase illustrates the ductile character of structural steel.

The stress-strain diagram is therefore essential for distinguishing between elastic behavior, plastic behavior, and failure. It also explains why steel is regarded as a highly reliable structural material: before rupture, it generally exhibits significant warning in the form of yielding and large deformation.

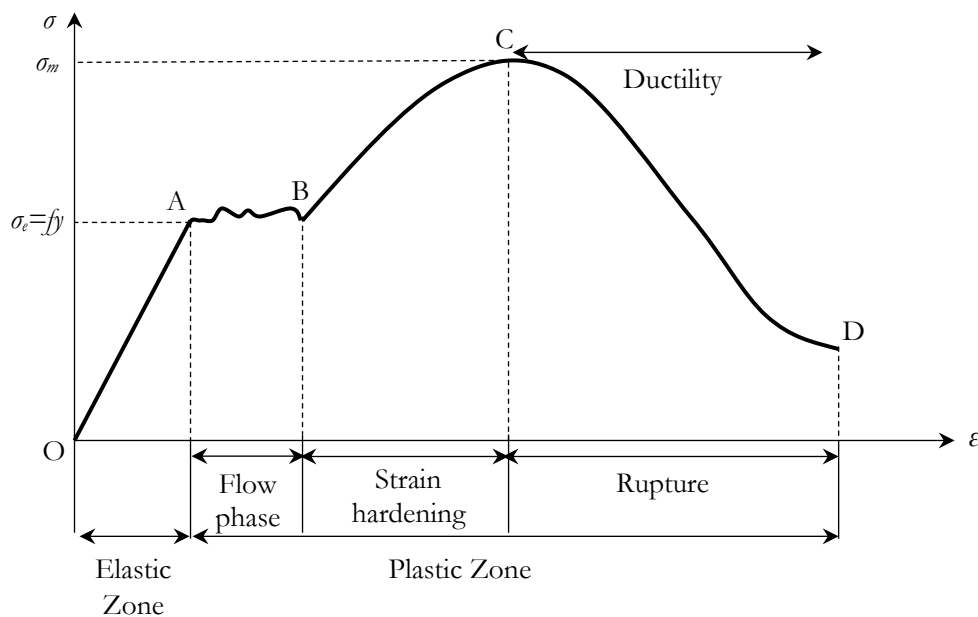


Figure 2.4. Idealized engineering stress-strain curve of steel [6].

2.7. Resilience (Toughness):

Resilience is often mentioned in structural steel along with toughness, and is the ability of a material to absorb energy when under impact or sudden loading without fracturing. In engineering practice this property is of particular importance when structures may be subject to dynamic effects, accidental actions, low temperatures or stress concentrations.

A steel with good toughness does not fail suddenly under shock loading, but can absorb a large amount of energy before fracture. Such behaviour is highly desirable in bridges, crane supporting structures, offshore components and other systems where loading may be variable or severe.

The impact resistance of steel is normally assessed with resilience tests such as the Charpy V-notch test where the absorbed fracture energy is determined. Toughness generally drops with falling temperature. In standardised steel grades, one of the parameters used is the specified rupture energy at a given temperature.

2.8. Ductility:

Ductility is the property of steel to plastically deform to a considerable extent before rupture. It is one of the most important qualities of structural steel as it provides a visible warning prior to failure and allows a redistribution of stresses within indeterminate structural systems.

In the tensile test the ductility is often determined by the elongation at fracture and sometimes by the reduction of the cross-sectional area at the necked region. High ductility improves the ability of structural members and connections to accommodate local stress concentrations, fabrication tolerances and exceptional actions without sudden brittle failure.

In structural engineering, ductility is also closely related to safety. A ductile material does not fail immediately after the first local yielding. Instead, it continues to deform which gives the structure reserve capacity and makes damage more detectable during service or overload situations.

2.9. Standardized grades of structural steels:

As regards the structural steels, this course refers to the European standards, in particular the EN 10025 family, the most used also in the French practice. These standards set out the mechanical and technological requirements which must be met by structural steels to enable them to be used safely in engineering applications.

A structural steel grade is a symbolic designation, usually consisting of the letter S, which stands for structural steel, followed by a number that is the minimum specified yield strength f_y , in MPa, for the smallest thickness range. Further letters can then be added to specify the manufacturing route, toughness class or other supplementary requirements.

The principal supplementary symbols frequently encountered are:

- **N:** normalized or normalized rolled steel.
- **M:** thermo-mechanically rolled steel.
- **Q:** quenched and tempered steel.

The grade may also include a symbol related to impact toughness and test temperature. For general-purpose structural steels, the most common quality levels are JR, J0, and J2, which correspond to a specified absorbed energy of 27 J measured at different temperatures, as shown in Table 2.2.

Table 2.2. Standardized grades of structural steels [6]

Symbol	Test temperature (°C)	Rupture energy (J)
JR	+20	27
J0	0	27
J2	-20	27

Accordingly, a designation such as S235J2N may be interpreted as follows:

- **S:** structural steel.
- **235:** minimum specified yield strength f_y , equal to 235 MPa for the relevant thickness range.
- **J2:** minimum absorbed impact energy of 27 J at (-20 °C).
- **N:** normalized rolling condition.

Structural steels are thus defined not only by their strength level, but also by guaranteed service properties such as yield strength f_y , tensile strength f_u , elongation at fracture, and impact toughness. These standardized guarantees are essential for safe structural design and for the control of material quality in fabrication workshops and on site.

2.10. Characteristics of standardized structural steels:

In structural engineering practice, the grades S235, S275, and S355 are among the most frequently used steels. Their designation refers primarily to the minimum specified

yield strength in MPa, and each grade corresponds to a different balance between strength, economy, and field of application.

- S235 steel is widely used in ordinary building construction and in many common steel members where moderate strength is sufficient.
- S275 steel provides a somewhat higher yield strength and may be selected when improved resistance is required or when plastic analysis and more demanding structural checks are involved.
- S355 steel offers a higher level of strength and is commonly used in civil engineering structures, bridges, heavily loaded members, and situations where the reduction of member weight is advantageous.

The selection of the grade must nevertheless remain compatible with fabrication constraints, especially welding, detailing, availability, and the expected service environment. A higher-strength steel is not automatically the best solution; the optimum choice depends on the overall structural context and the design objectives.

2.11. Design values of the usual coefficients of steel:

In addition to strength-related parameters, structural calculations require a number of physical and elastic coefficients that characterize the general response of steel. These values are commonly adopted in design practice for preliminary sizing, analytical modeling, and verification of structural members (Table 2.3).

Table 2.3. Physical and elastic coefficients characterizing the response of steel

Property	Symbol	Typical value	Unit	Description
Modulus of elasticity (Young's modulus)	E	2.1×10^5	N/mm ²	Measures the axial stiffness of steel under tension or compression.
Shear modulus	G	8.1×10^4	N/mm ²	Relates shear stress to shear strain

				and is used in torsion and shear calculations.
Poisson's ratio	ν	0.30	-	Represents the ratio of lateral strain to longitudinal strain in the elastic range.
Coefficient of thermal expansion	α	12×10^{-6}	$/^{\circ}\text{C}$	Expresses the linear dimensional variation of steel per degree Celsius.
Mass density	ρ	7850	kg/m^3	Mass per unit volume of steel; useful for self-weight evaluation.

These coefficients are used many times in steel design. For instance, the modulus of elasticity is involved in the calculations for deflection and buckling, the shear modulus is employed in problems of torsion and shear deformation, the coefficient of thermal expansion is needed for temperature effects, and the mass density is applied to estimate self-weight. In practice these values are taken as normal engineering constants for common structural steels.

In summary, steel offers a high level of mechanical performance with standardised and well-documented properties. Therefore, it is necessary to have a clear understanding of its composition, tensile behaviour, toughness, ductility, grades and physical constants for analysis and design of steel structures.

2.12. Conclusion:

In conclusion, steel combines a high level of mechanical performance with standardized and well-documented properties. A clear understanding of its composition, tensile behavior, toughness, ductility, grades, and physical constants is therefore indispensable for the analysis and design of steel structures.

Chapter 3

Steel products and sections

Learning objectives:

- ✎ At the end of this chapter, the student should be able to recognize the main families of steel products and sections, read the basic notation of common profiles, relate the shape of a section to its structural function, and understand how geometric characteristics influence resistance, stiffness, stability, and detailing.

3.1. Introduction:

Structural design in steel begins with the correct choice of a section. The shape of a member determines not only its resistance, but also its stiffness, stability behavior, ease of connection, fabrication method, and cost. This chapter therefore introduces the principal steel products and profiles used in metal construction before later chapters deal with their verification under different internal forces.

3.2. Geometric characteristics of steel products:

Steel products used in structural engineering are manufactured in standardized shapes and dimensions so that their mechanical response, fabrication requirements, and connection details can be assessed reliably during design. For this reason, the geometric characteristics of steel sections are generally provided in updated catalogues and design manuals issued by manufacturers or standardization bodies.

These catalogues do not merely indicate nominal dimensions. They also provide the section properties required for structural verification, such as the cross-sectional area, mass per unit length, moments of inertia about the principal axes, elastic and plastic section moduli, radii of gyration, torsional properties, and, when relevant, the location of the centroid and shear center. In practical design, these parameters are essential for checking resistance in tension, compression, bending, shear, torsion, and overall stability.

Accordingly, the engineer selects a steel product not only on the basis of its shape, but also according to the structural function to be fulfilled, the load path, the required stiffness, the connection arrangement, fabrication constraints, and economic considerations. The main families of steel sections commonly encountered in metal construction are presented in Table 3.1.

Table 3.1. Main families of steel sections

Section family	Common notation	Typical structural role
Angle sections	$L\ 1 \times 1 \times t$	Bracing members, truss components, secondary framing
I sections	IPE h, IPN h	Beams and flexural members
H sections	HEA h, HEB h,	Columns, heavily loaded beams, portal frames

	HEM h	
U sections	UAP h	Channels, edge beams, secondary members
Hollow sections	SHS, RHS, CHS	Columns, trusses, architectural and torsion-sensitive members
Other profiles	C, Ω , Z	Cold-formed purlins, cladding rails, light secondary systems

3.2.1. Angle sections (L-Shaped Sections):

Angle sections are among the simplest and most frequently used steel products. Their L-shaped cross-section may be formed with equal legs or unequal legs, depending on the required structural function and connection detail (Figure 3.1). Because of their simplicity and ease of fabrication, angle sections are widely used in lattice structures, bracing systems, towers, roof trusses, transmission structures, and other secondary steel components.

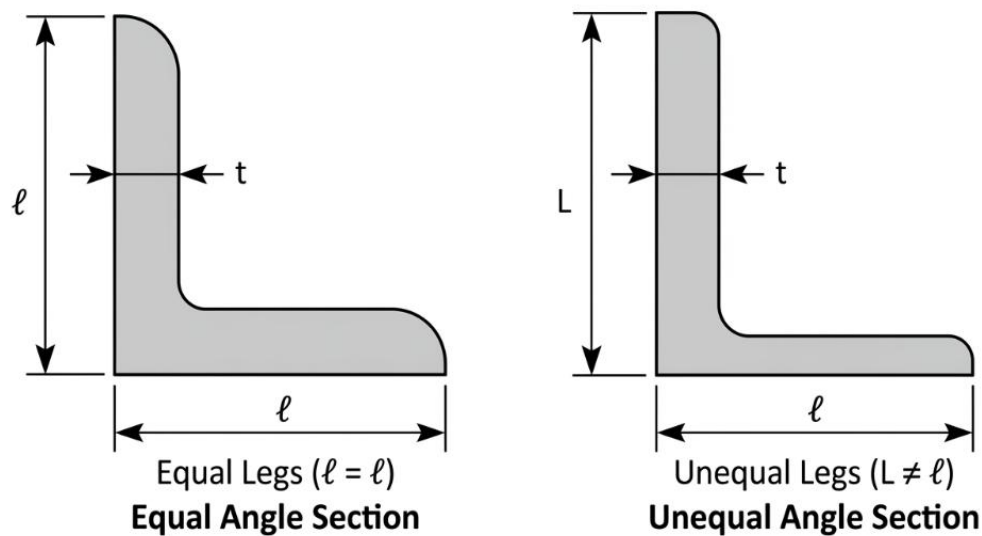


Figure 3.1. Angle sections (L-Shaped Sections) [3].

From a mechanical point of view, angle sections are open and generally unsymmetrical or only partially symmetrical sections. This means that their centroid is not located at the intersection of the outer legs, and bending may be accompanied by torsional effects when the load is not applied through the relevant shear center. For this reason, the behavior of angle sections must be examined carefully when they are subjected to compression or combined bending and axial force.

Angles are often connected by one leg to a gusset plate or by both legs to the adjoining members. Their use is therefore particularly advantageous in trussed systems, where members mainly resist axial forces and where connection simplicity is a major design criterion.

Notation : $L\ l \times l \times t$ (mm). **Example:** $L\ 40 \times 40 \times 4$ mm.

3.2.2. I Profiles:

I-shaped sections are designed to provide high bending efficiency by concentrating most of the material in the flanges, far from the neutral axis, while the web primarily resists shear and keeps the flanges apart. This geometry allows a high moment of inertia to be achieved with a relatively moderate self-weight, which explains the widespread use of I sections in floor beams, roof beams, bridge girders, and many other flexural members.

Two common families are frequently distinguished in catalogues: IPN and IPE sections. In practice, IPE sections are widely used for building structures because of their efficient geometry and convenient detailing, whereas IPN sections belong to an older standardized family that may still be encountered in certain applications and existing structures (Figure 3.2).

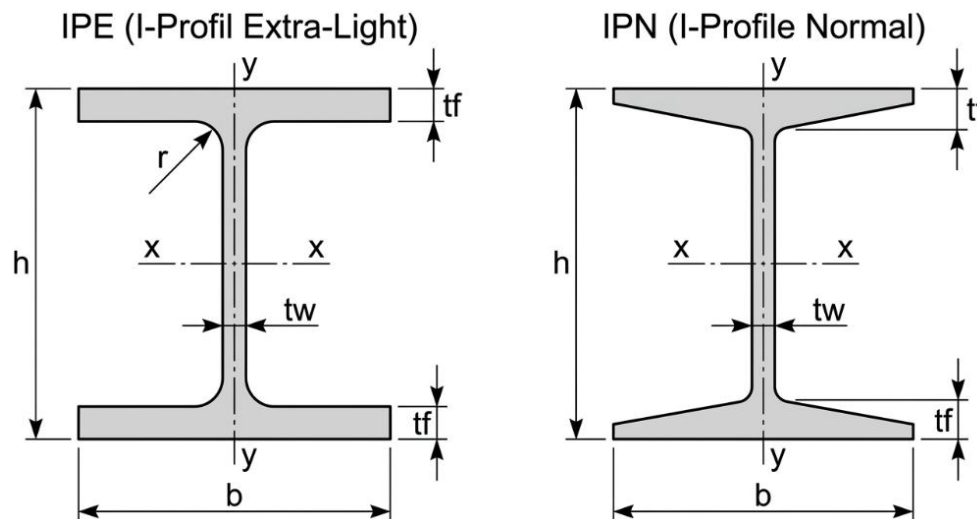


Figure 3.2. I Profiles [4].

The principal geometric relation usually retained in introductory courses is that the flange width is of the same order as half the nominal depth. In design, however, the exact

tabulated dimensions of the web thickness, flange thickness, root radius, area, and section modulus must always be taken from the corresponding catalogue.

Common range: $80 \text{ mm} \leq h \leq 600 \text{ mm}$.

Notation: IPE h (mm), IPN h (mm). **Example:** IPE 200.

3.2.3. H Profiles (H-Shaped Sections):

H-shaped sections are characterized by wider flanges and a more compact overall geometry than ordinary I sections (Figure 3.3). They are particularly suitable for members subjected to important axial loads, bending moments, or combined actions. For this reason, H profiles are extensively used in building frames, portal structures, industrial halls, and civil engineering works where both stiffness and load-carrying capacity are required.

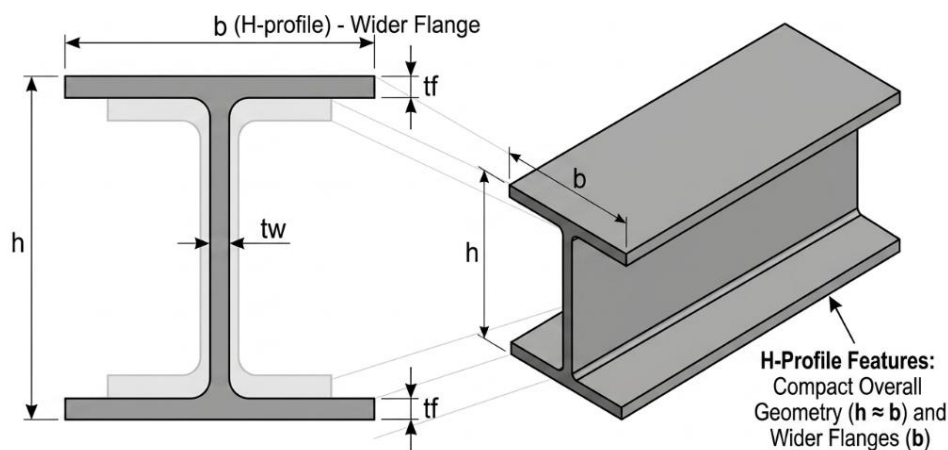


Figure 3.3. H Profiles [4].

The main European series are HEA, HEB, and HEM. For the same nominal depth, these families correspond to progressively heavier sections, with increasing thicknesses and mass per unit length. As a result, HEA profiles are generally associated with relatively lighter applications, HEB profiles with intermediate and versatile structural use, and HEM profiles with highly loaded columns and primary members.

Because of their favourable distribution of material about both principal axes, H profiles are well suited to compression members such as columns, particularly when overall buckling and connection rigidity are important design considerations.

Notation: HEA h (mm), HEB h (mm), HEM h (mm).

- **HEA** $h < b$
- **HEB** $h = b$
- **HEM** $h > b$

In many practical documents, the series distinction reflects the relative heaviness of the section for a given nominal depth.

3.2.4. U Profiles (U-Shaped Sections):

U-shaped or channel sections are open profiles composed of a web and two flanges oriented in the same direction (Figure 3.4). These sections are frequently used as purlins, side rails, secondary beams, edge members, lintels, and components of built-up members.

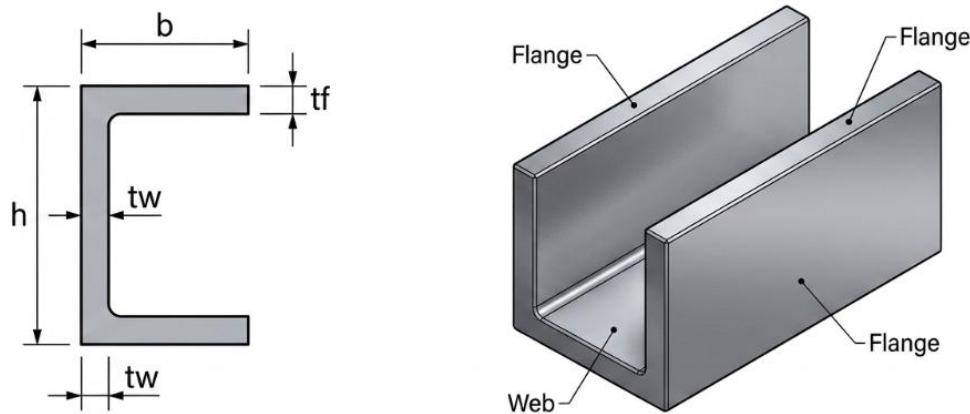


Figure 3.4. U Profiles [3].

Compared with closed sections, U profiles are easier to connect and fabricate, especially when bolted or welded to plates and stiffeners. However, because they are open sections, they may be more sensitive to torsion and warping when subjected to eccentric loading. Their performance must therefore be evaluated carefully in members where the line of action of the load does not coincide with the shear center.

Channel sections can also be combined back-to-back or face-to-face in order to form built-up members with improved symmetry and higher resistance. Such arrangements are common in columns, lattice girders, and other fabricated steel components.

Notation: UAP h (mm). Example: UAP 80.

3.2.5. Hollow sections:

Hollow sections are closed steel products available in square, rectangular, and circular forms (Figure 3.5). Their closed geometry gives them a high torsional stiffness compared with open sections of similar mass, which makes them especially attractive in structures where torsion, multi-directional bending, or architectural appearance are important.

Square hollow sections (SHS) and rectangular hollow sections (RHS) are widely used in columns, truss members, façades, masts, and lightweight space structures. Circular hollow sections (CHS) are particularly efficient in compression and in members subjected to uniform bending in different directions, since the material is distributed symmetrically about the centroid.

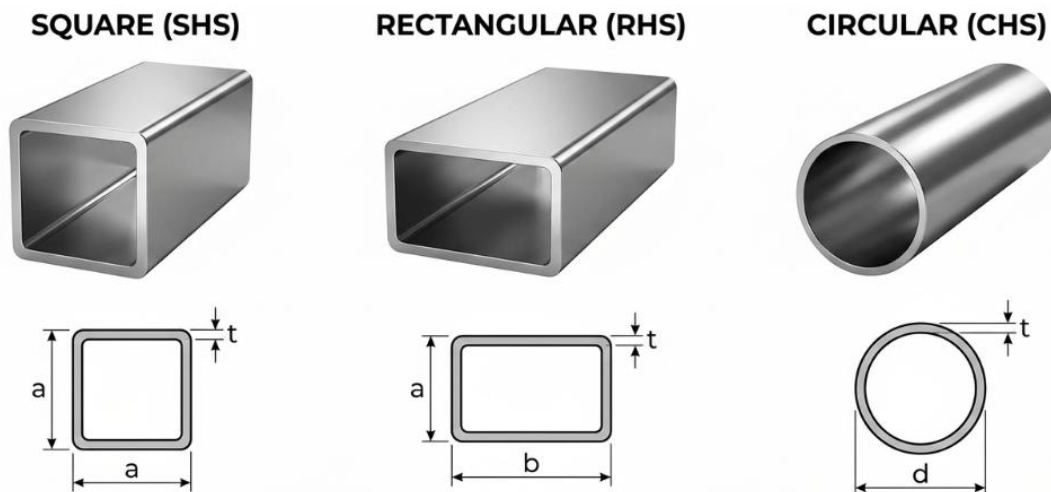


Figure 3.5. Hollow sections [3].

Another practical advantage of hollow sections lies in their clean appearance and relatively favourable surface-to-volume ratio. These characteristics make them popular in exposed steelwork and architecturally expressive structures. Nevertheless, the design of welded and bolted connections for hollow sections may be more demanding than for open sections, especially when access to the interior is limited.

3.2.6. Other profiles:

In addition to the conventional hot-rolled sections, other profiles are commonly used in steel construction, particularly for secondary framing and lightweight systems. Among the most common are C profiles, omega profiles, and Z profiles (Figure 3.6).

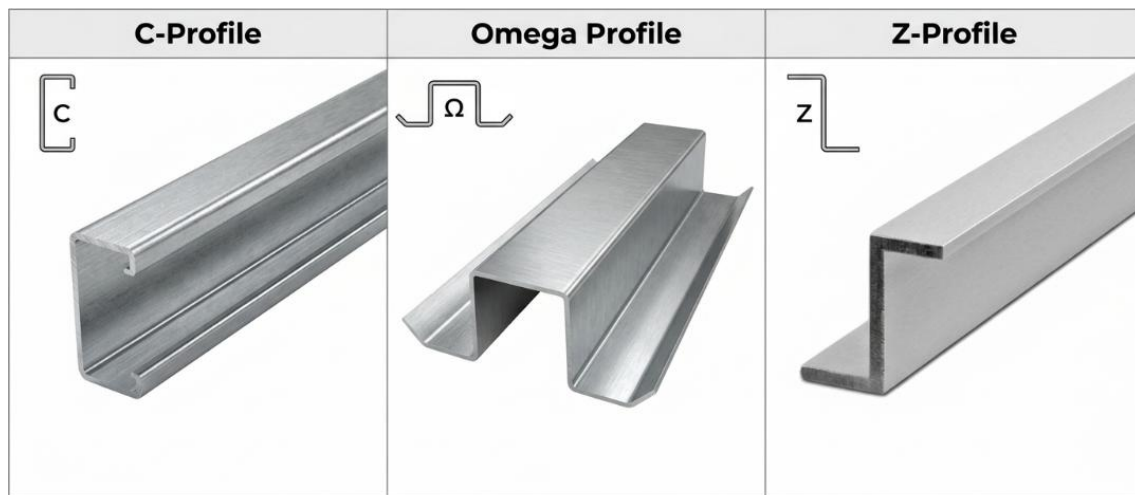


Figure 3.6. Other profiles: C profiles, Ω profiles, and Z profiles [4].

C and Z sections are widely employed as purlins, girts, cladding supports, and cold-formed secondary members. Their structural efficiency derives from the folding of thin steel plates into shapes that increase bending stiffness while maintaining a low self-weight. Because these elements are generally thin-walled, local buckling, distortional buckling, and connection detailing become more significant than in hot-rolled sections.

Omega profiles are mainly used in light steel construction, façade support systems, and finishing applications where ease of fixing and reduced weight are important. Although these shapes are not usually intended for the primary load-bearing system of a building, they play an essential role in the stability and serviceability of the envelope and secondary structural components.

3.3. Conclusion:

The choice of a steel product or section must always be related to its structural role, the nature of the internal forces, fabrication and erection constraints, and the requirements of economy and detailing. Consequently, a sound understanding of section geometry is a fundamental step in steel design, since geometric characteristics directly influence resistance, stiffness, stability, and constructability.

Chapter 4

Elastic Instability Phenomena

Learning objectives:

- ✎ At the end of this chapter, the student should be able to distinguish the main forms of elastic instability in steel structures, explain the physical meaning of buckling, lateral-torsional buckling and local plate instability, identify the parameters governing critical loads or stresses, and propose suitable restraint and stiffening measures.

4.1. Introduction:

One of the fundamental requirements of structural design is not only to offer adequate strength against the applied loads but also to assure the stability of the structure during its service life. In practice, a structure should be able to resist actions without rapid changes of configuration, excessive displacements or loss of equilibrium. This is especially true in steel construction where members tend to be slender and highly efficient, and where failure may be dominated by instability rather than by material yielding alone.

Elastic instability phenomena are of great importance in the analysis of steel structures, because steel members are generally characterised by high strength combined with relatively small thicknesses and long unsupported lengths. The structural response under such conditions is very sensitive to geometry, stiffness, support conditions and initial imperfections. This implies that a member may reach its load-carrying capacity well before the yield strength of the material is fully mobilised.

The present chapter is concerned with the three main types of elastic instability that are met in metal construction, namely overall buckling of compressed members, lateral buckling by torsion of bent beams and local plate instability, which is generally called in this book warping (buckling). These three mechanisms are different but they all have the same basic idea that is a structural element can become unstable due to its geometry and stiffness, even if the material is still within its elastic limit.

4.2. Origin of elastic instability phenomena:

The design of steel structures requires verification at two complementary levels: the global stability of the structural system as a whole and the local stability of each individual member or plate element. Under all relevant combinations of permanent, variable, thermal, and accidental actions, the engineer must ensure that the equilibrium configuration remains stable and that the resulting stresses and deformations stay within acceptable design limits.

From a mechanical point of view, instability appears when the deformed configuration of a member begins to influence its internal forces to such an extent that the initial equilibrium path can no longer be maintained. The phenomenon is therefore

closely linked to geometric nonlinearity, second-order effects, and the amplification of unavoidable imperfections such as residual stresses, fabrication tolerances, slight out-of-straightness, or load eccentricity.

4.2.1. The case of small deformations:

When deformations remain small, their influence on the internal force distribution is negligible. In such a situation, a first-order analysis is generally sufficient, and structural verification may be carried out by checking that stresses do not exceed the admissible or design resistance of the material. For compact and stocky members, this approach is often adequate because stiffness is high enough to prevent significant geometric amplification.

4.2.2. The case of large deformations:

When deformations become appreciable, however, the initial geometry of the structure can no longer be considered unchanged during loading. The deformed shape modifies the line of action of forces and generates additional moments, commonly described as second-order effects. This situation is typical of slender compression members, laterally unrestrained beams, and thin plate elements subjected to compression or shear.

In these circumstances, specific stability checks are required. Depending on the type of member and the governing stress state, the compressed regions of the structure may exhibit one of the following elastic instability modes (Figure 4.1):

- Buckling of compressed members;
- Lateral buckling by torsion of beams subjected to bending;
- Warping (buckling), that is, local plate instability of thin elements.

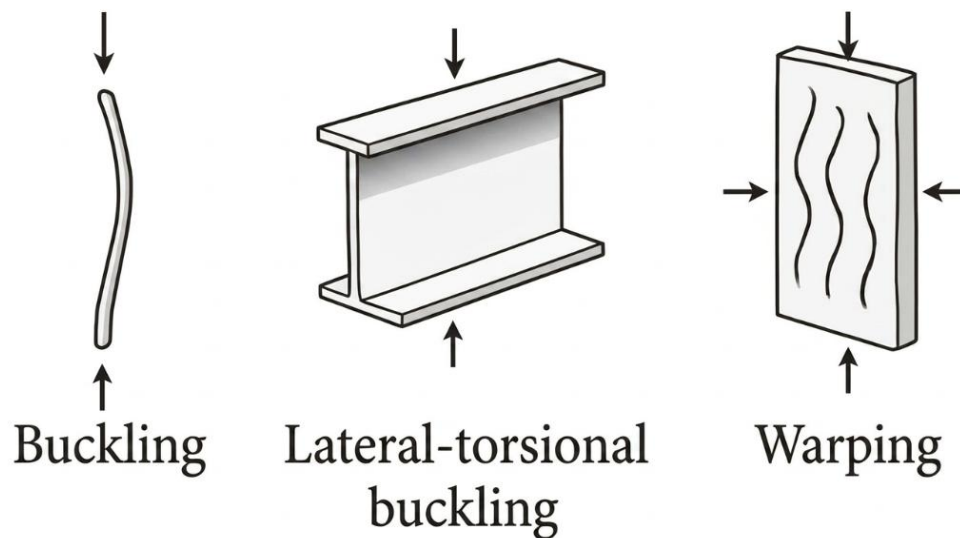


Figure 4.1. Elastic instability modes [2].

4.3. Buckling:

Buckling is an elastic instability phenomenon affecting slender members subjected predominantly to axial compression. It corresponds to a sudden loss of stability in which a member that was initially straight develops a significant lateral deflection once the compressive force reaches a critical value. The phenomenon is global in nature because the entire member participates in the change of shape (Figure 4.2).

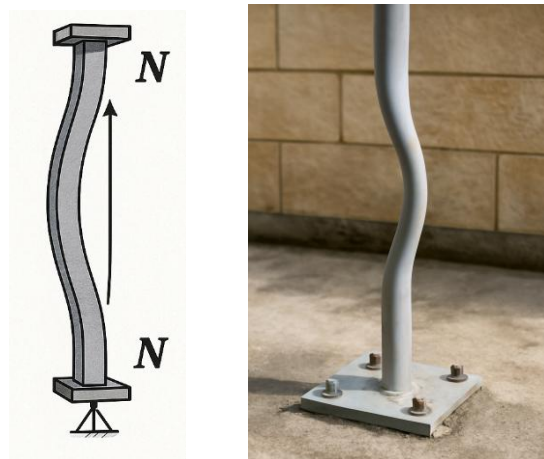


Figure 4.2. Buckling phenomenon [8].

4.3.1. Principle of buckling:

When a column or compression member is loaded gradually, it first responds elastically and remains approximately aligned with the direction of the applied force. As long as the member is sufficiently stiff and the load remains below the critical threshold,

the straight equilibrium configuration is maintained. In reality, however, no member is perfectly straight and no load is applied with perfect centricity. Small imperfections are always present.

Beyond a certain critical load, denoted P_{cr} , those imperfections are amplified and the member develops a rapidly increasing lateral displacement. The resulting bending moment, created by the axial load acting on the deflected shape, further increases the deformation. Buckling is therefore a stability problem governed not only by the intensity of the compressive force, but also by the flexural stiffness EI , the member length, and the end restraint conditions (Figure 4.3).

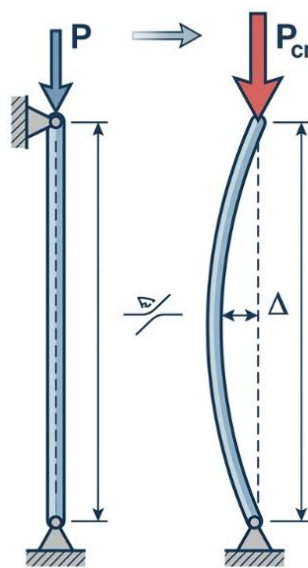


Figure 4.3. Lateral deflection associated with column buckling [8].

4.3.2. Euler's critical buckling load:

For an ideal prismatic column that is initially straight, perfectly elastic, and subjected to concentric compression, Euler established the classical expression of the elastic critical load. For a member with pinned ends, the critical load is written as follows:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (4.1)$$

In this expression, E is the modulus of elasticity of the material, I is the second moment of area of the cross-section about the relevant buckling axis, and L is the free length of the member. The formula shows that the elastic critical load increases with flexural rigidity and decreases very rapidly as the member length increases.

For practical design, support conditions are introduced through an effective length factor K , leading to the more general form:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad (4.2)$$

The parameter K accounts for the restraint provided by the supports and by the connected structure. Typical values used in elementary analysis are:

$K = 1.0$ for a pinned-pinned member,

$K = 0.5$ for a fixed-fixed member,

$K = 2.0$ for a cantilevered member with one end fixed and the other free.

It should be noted that Euler theory is valid mainly for slender members that buckle while the material still behaves elastically; for stockier members, yielding and residual stresses reduce the applicability of the purely elastic model.

4.3.3. Types of buckling:

In structural engineering practice, buckling may appear under different combinations of internal actions. The most elementary case is pure compression, but many real members act as beam-columns and are therefore subjected simultaneously to axial force and bending. For this reason, it is useful to distinguish between simple buckling and buckling with bending.

4.3.3.1. Simple buckling:

Simple buckling occurs when a member is subjected essentially to a normal compressive force acting through its centroidal axis. Once the critical load is reached, the member bends laterally in a direction perpendicular to the compression axis. The response is dominated by axial compression combined with induced flexure resulting from the deflected shape. This case is representative of centrally loaded slender columns and truss members in compression as shown in Figure 4.4.

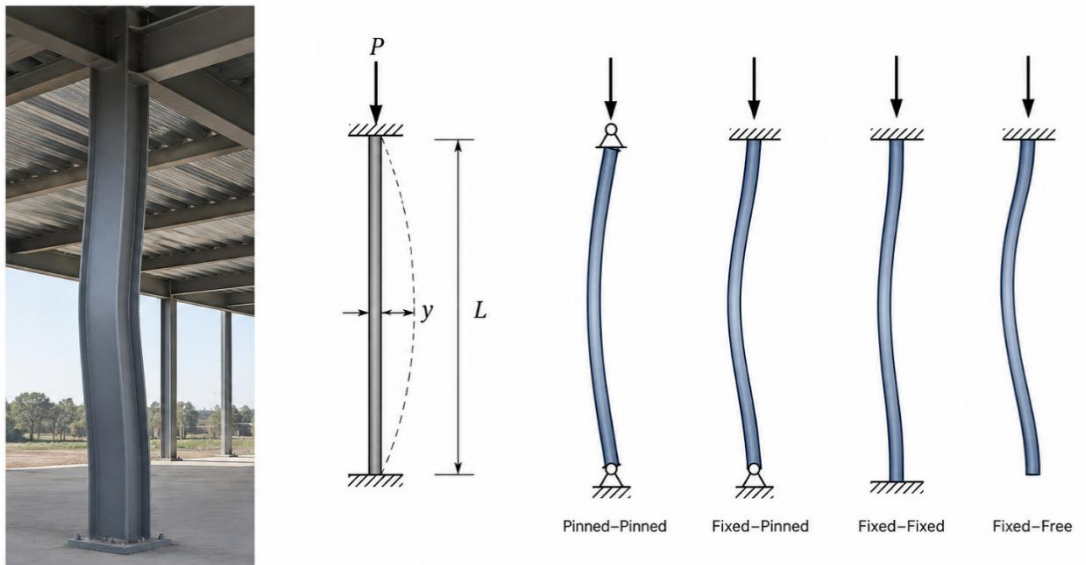


Figure 4.4. Simple buckling [1].

4.3.3.2. Buckling with bending:

Buckling with bending concerns members subjected simultaneously to axial compression and bending moment. In this situation, the applied bending moment produces an initial curvature or stress gradient, while the axial force tends to amplify the lateral displacement. The interaction between N and M leads to a beam-column behavior that is more severe than the isolated action of either effect taken separately. This form of instability is common in frame members, portal columns, and structural elements subjected to eccentric loading (see Figure 4.5).

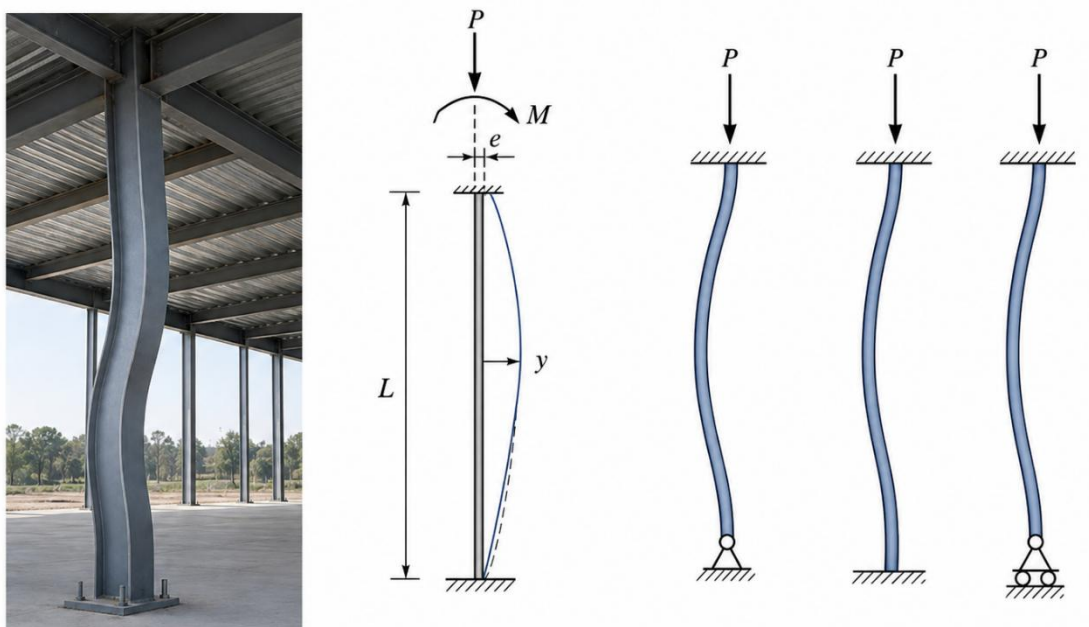


Figure 4.5. Buckling with bending [1].

4.4. Lateral buckling by torsion:

Lateral buckling by torsion, more commonly referred to in engineering literature as lateral-torsional buckling, is an instability phenomenon that primarily affects beams bent about their major axis when the compression flange is not adequately restrained. It is particularly significant for open thin-walled sections such as I-sections and channel sections, whose torsional rigidity is relatively low.

Unlike column buckling, which is associated mainly with axial compression, lateral buckling by torsion develops in members subjected predominantly to bending. The instability combines a lateral displacement of the member with a simultaneous rotation of the cross-section about its longitudinal axis. The phenomenon therefore reflects the coupling between flexural and torsional behavior (Figure 4.6).

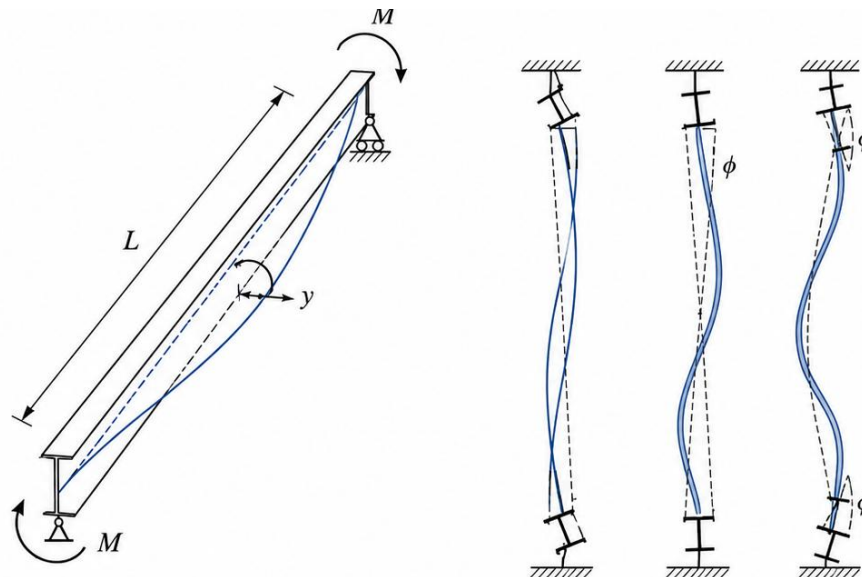


Figure 4.6. Lateral buckling by torsion [2].

4.4.1. Principle of lateral buckling by torsion:

When a beam carries an increasing bending moment, the compression flange tends to move laterally because it is the most destabilised part of the section. If no lateral restraint is provided, a small out-of-plane displacement may develop. As the load increases, this displacement interacts with the low torsional stiffness of the section, producing rotation and further lateral movement.

Beyond a critical bending moment M_{cr} , the initially stable equilibrium path is lost and the deformation amplifies rapidly. The risk is particularly pronounced when the unbraced length is large, when the beam is loaded on the compression side, or when the cross-section is open and torsionally flexible. In practical steel design, floor diaphragms, purlins, secondary beams, and bracing systems are often used specifically to control this instability mode.

4.4.2. Expression of the critical moment:

For a simply supported beam with an open section, subjected to bending and idealised support conditions, the elastic critical moment may be represented in simplified form by the following expression:

$$M_{cr} = \frac{\pi^2 EI_{\tilde{z}}}{(KL)^2} \sqrt{1 + \frac{KL^2 GI_t}{\pi^2 EI_{\tilde{z}}}} \quad (4.3)$$

In this relation, E denotes the modulus of elasticity,

G the shear modulus,

$I_{\tilde{z}}$ the moment of inertia associated with the relevant lateral bending axis,

I_t the torsional constant,

L the beam length,

K a coefficient reflecting support and restraint conditions.

The expression highlights the fact that lateral-torsional stability depends simultaneously on bending stiffness, torsional stiffness, and member length.

From a design standpoint, the critical moment is not a pure material parameter; it is a structural parameter. It varies with end restraints, moment distribution, loading position, and section type. For that reason, design codes generally provide reduction factors and verification procedures rather than relying exclusively on a single closed-form expression.

4.4.3. Factors influencing lateral buckling:

The occurrence and severity of lateral buckling by torsion are influenced by several structural parameters, among which the most important are the following:

- The unbraced length between lateral restraints, since the critical moment decreases rapidly as the free length increases;
- The location of the centroid and the shear center, which governs the coupling between bending and torsion;
- The distribution of bending moment along the beam, because non-uniform bending may be either more or less favourable than constant moment;
- The presence of floor systems, diaphragms, or discrete bracing devices capable of restraining the compression flange;
- The type of section, especially whether it is open, closed, doubly symmetric, monosymmetric, or torsional stiff.

4.4.4. Prevention measures:

In practice, the prevention of lateral buckling by torsion is based on improving either restraint conditions or section stiffness. Effective measures include:

- Reducing the unbraced length by introducing intermediate lateral supports or continuous restraint of the compression flange;
- Selecting sections with greater torsional rigidity, such as closed or box-type sections when appropriate;
- Using floor slabs, purlins, diaphragms, or secondary framing members to provide lateral restraint;
- Adopting a structural arrangement that limits destabilising load positions and reduces the susceptibility of the compression flange to lateral movement.

4.4.5. Physical interpretation:

Physically, lateral buckling by torsion can be interpreted as the loss of stability of a bent beam in which longitudinal bending and torsion act in a coupled manner. The beam does not merely bend sideways; it also twists. This combined mechanism distinguishes it clearly from axial buckling of columns. It is therefore a characteristic instability mode of flexural members rather than purely compressed members.

4.5. Warping (Buckling):

Warping, as presented in this chapter, corresponds essentially to the local buckling of thin plate elements such as webs, flanges, walls, or stiffened panels. In modern steel design terminology, this behavior is often referred to as plate buckling or local instability. It differs from global member buckling because the entire member does not necessarily lose equilibrium; instead, only a thin component of the section deforms out of its middle plane.

This phenomenon is especially relevant in slender plates subjected to compressive stress, shear stress, or combined stress states. Because steel structures frequently include thin-walled components designed for efficiency, local instability must be checked carefully to avoid premature loss of stiffness and reduced resistance of the section (Figure 4.7).

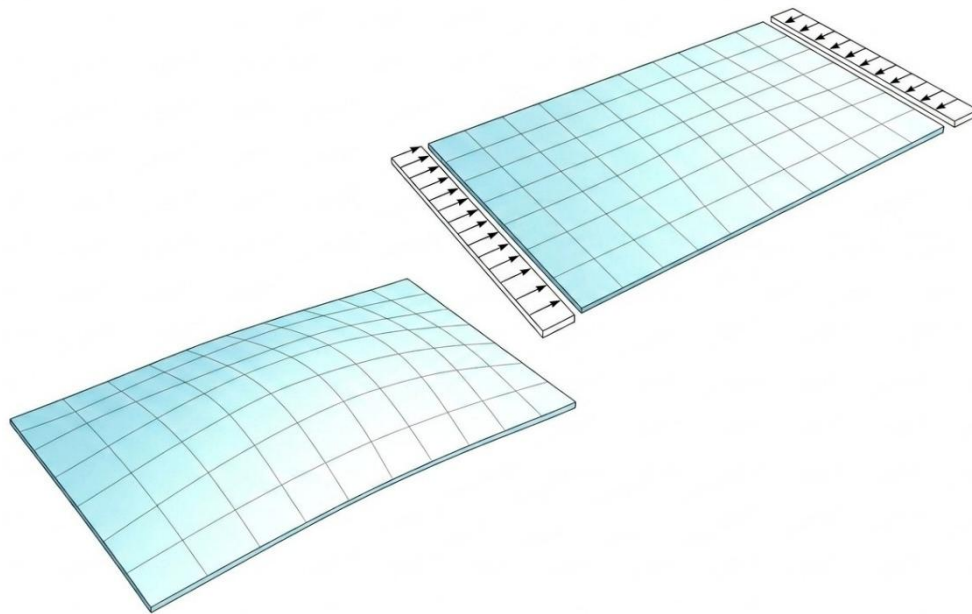


Figure 4.7. Warping (local buckling) [5].

4.5.1. Principle of warping:

When a thin plate is loaded gradually, it initially resists through in-plane membrane action and remains approximately flat. As long as the stress level is below the critical threshold, the plate behaves elastically and the middle surface does not experience significant out-of-plane deformation.

Once the critical stress σ_{cr} is reached, however, the flat configuration becomes unstable and the plate suddenly develops a wave-like out-of-plane deformation. This

deformation indicates a local loss of stiffness rather than immediate material fracture. For this reason, local buckling may occur while the steel is still far from rupture and, in some cases, even before full yielding of the plate has been reached.

4.5.2. Critical warping stress:

For a rectangular plate of thickness t , length a , and width b , with edges that are either simply supported or restrained, the critical compressive stress may be expressed in simplified form as follows:

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b} \right)^2 \quad (4.4)$$

In this equation, E is the modulus of elasticity,

ν is Poisson's ratio,

k is a coefficient that depends on the boundary conditions,

The expression shows that local plate stability is highly sensitive to the width-to-thickness ratio: the thinner and wider the plate element, the more vulnerable it becomes to buckling.

4.5.3. Types of warping:

Several manifestations of local plate instability may be distinguished according to the structural configuration and the stress field (Figure 4.8):

- Local buckling, which affects a limited part of a section such as a web or flange element;
- Global panel buckling, in which a larger plated component or panel loses stability as a whole;
- Shear buckling, often observed in beam webs subjected to high shear forces;
- Distortional buckling, a more complex mode involving rotation and translation of plate elements, frequently encountered in cold-formed thin-walled profiles.

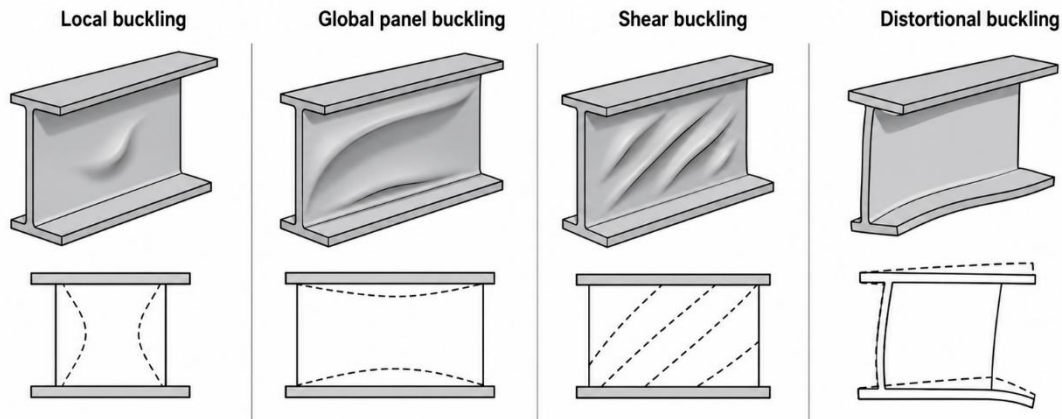


Figure 4.8. Types of warping [5].

4.5.4. Factors influencing warping:

The critical stress associated with local instability is governed by several parameters, notably:

- The plate thickness, since thinner plates possess lower out-of-plane stiffness;
- The support and restraint conditions along the edges, which strongly affect the coefficient k ;
- The geometric proportions of the plate and the width-to-thickness ratio;
- The nature and distribution of stress, whether uniform compression, bending compression, shear, or combined loading;
- The presence, spacing, and effectiveness of longitudinal or transverse stiffeners.

4.5.5. Prevention measures:

To reduce the risk of local warping or plate buckling, the designer may adopt several practical measures:

- Increase plate thickness where local slenderness is excessive;
- Introduce longitudinal or transverse stiffeners in order to shorten the effective buckling width or length of the plate;
- Optimise the cross-sectional geometry so that thin elements are better supported and local stress concentrations are limited;
- Use fabrication details and assembly methods that improve plate restraint and reduce distortion.

4.5.6. Physical interpretation:

From a physical standpoint, warping represents a local elastic instability that may be followed by a post-buckling phase in which the plate continues to carry part of the load. This means that local buckling does not always imply immediate collapse. Nevertheless, it reduces stiffness, modifies stress redistribution within the section, and may compromise the performance of the member if not considered properly in design.

4.6. Conclusion:

The safety of structures is not only dependent on resistance but also on the ability of members and plate components to maintain their equilibrium under load and thus the study of instability phenomena is necessary in the design of steel structures. The main modes of elastic instability are buckling, lateral-torsional buckling and warping. All of these are governed by the interaction between geometry, stiffness, loading and restraint conditions.

Buckling is the general instability of members under compression; lateral buckling by torsion occurs in beams under bending, when there is not enough lateral restraint; and warping is the local instability of thin plate elements. All three phenomena, despite the fact that they differ in their mechanical expression, show the possibility of a steel component becoming unstable at stress levels below the nominal strength of the material.

A good understanding of these mechanisms allows the engineer to more accurately estimate critical loads and moments, choose suitable cross-sections, specify effective bracing and stiffening systems, and include second-order effects in the design process. The control of instability is therefore a regulatory requirement and a basic condition for structural performance, safety and durability in modern steel construction.

Chapter 5

Simple Tension

Learning objectives:

- ✦ At the end of this chapter, the student should be able to identify structural members subjected primarily to axial tension, explain their mechanical behavior, select the relevant resistance checks, determine the gross and net sectional areas, and assess the design resistance of common bolted tension members such as plates and angles. The emphasis is placed on the engineering meaning of the formulae, the correct interpretation of geometric parameters, and the practical design sequence used in steel structures.

5.1. Introduction:

Simple tension is one of the fundamental loading cases in steel structures. Although a tension member is not normally governed by overall buckling, its safety depends strongly on the quality of the connected zone, the presence of bolt holes, and the effective transfer of force. This chapter introduces the practical checks required to verify plates and angle members subjected to axial tensile force.

5.2. Definition and structural meaning of simple tension:

A member is said to be subjected to simple tension when the resultant internal action is essentially an axial normal force acting along the centroidal axis of the member and directed outwards at its ends. In an ideal tension member, bending moment, shear force, and torsion are either absent or sufficiently small to be neglected in the primary design model. This assumption is acceptable when the member is straight, the load path is clear, and the connections transmit the force without introducing significant eccentricity.

In steel structures, tension members are usually efficient because the whole cross-section can contribute to resisting the axial force. Unlike compression members, they are not governed by global buckling. Nevertheless, their design is not limited to checking the gross area. Local weakening caused by bolt holes, stress concentration near connections, and unequal force transfer in connected angles must also be considered. Therefore, the design of a tension member is fundamentally a comparison between the applied design tensile force and the lowest resistance associated with all relevant limit states (Figure 5.1).

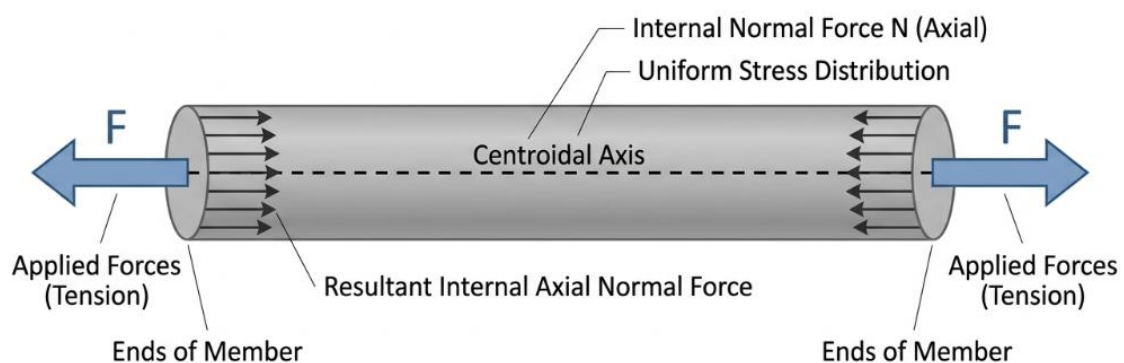


Figure 5.1. Idealized axial tension model for a straight steel member [7].

5.3. Main fields of application:

Tension members are widely used in steel structures because they provide stiffness, stability, and force transfer with relatively small quantities of material. Their role is particularly important when the structural system is triangulated or when horizontal actions must be resisted through axial load paths (Figure 5.2).

- **Truss members:** diagonals, verticals, or bottom-chord segments may work mainly in tension depending on the load arrangement and the global equilibrium of the truss.
- **Stability systems:** wind bracing, roof bracing, and façade bracing frequently include slender tension members that resist horizontal actions and limit lateral displacement.
- **Suspension and hanger systems:** hangers, tie rods, and suspended supports transmit loads by direct axial tension from a supported component to a primary structural element.
- **Tie members:** ties are used to connect structural parts, prevent spreading, and ensure global continuity in frames, arches, and roof systems.

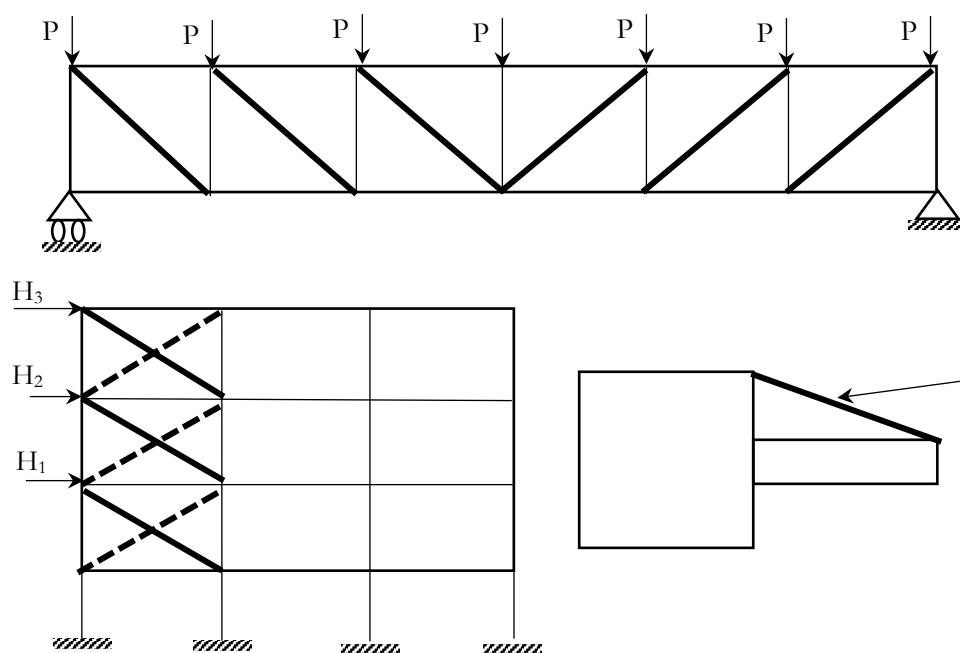


Figure 5.2. Typical uses of tension members in steel structures [2].

5.4. Mechanical behavior and design assumptions:

The behavior of a steel tension member may be described in three main stages. First, the response is elastic and the axial stress is approximately proportional to the

applied load. Second, yielding begins when the average stress in the gross section approaches the yield strength. Third, if the member is connected by bolts or contains holes, fracture may occur through a net section where the effective resisting area is smaller than the gross area. The designer must therefore ensure that yielding of the gross section and rupture of the net section are both prevented under the design tensile force.

The following assumptions are commonly adopted for the simplified design model:

- The member is straight and the axial force is applied through, or very close to, the centroidal axis of the member.
- The material is homogeneous and its mechanical properties are represented by the yield strength f_y and the ultimate tensile strength f_u .
- The gross cross-sectional area A is used for the yielding check, whereas the net area A_{net} is used for the fracture check.
- Connection eccentricities, prying effects, secondary bending, and fatigue are outside the basic scope of simple tension unless they are explicitly introduced by the design problem.

5.5. Design principle and resistance format:

The design requirement is expressed by comparing the design tensile action with the available tensile resistance of the member. The applied design tensile force is denoted by N_{sd} , depending on the notation used in the course. The member is safe in tension if:

$$N_{sd} \leq N_{t,Rd} \quad (5.1)$$

where $N_{t,Rd}$ is the design tensile resistance. In practice, the tensile resistance is governed by the minimum of the plastic resistance of the gross section and the ultimate resistance of the net section:

$$N_{sd} \leq N_{t,Rd} = \min[N_{pl,Rd}, N_{ult}] \quad (5.2)$$

This minimum-value approach is essential because a member may have a large gross area but a weak connection zone due to bolt holes. Conversely, a member with limited gross area may yield before fracture occurs at the connection.

5.5.1. Plastic resistance of the gross section:

The plastic resistance corresponds to the capacity of the full gross cross-section to resist axial yielding. It is evaluated as follows:

$$N_{pl,rd} = \frac{Af_y}{\gamma_{m0}} \quad (5.3)$$

Where A is the gross cross-sectional area,

f_y is the yield strength of steel,

γ_{m0} is the partial safety factor associated with the resistance of the gross section.

In the present course convention, $\gamma_{m0} = 1.10$.

5.5.2. Ultimate resistance of the net section:

The ultimate resistance represents the fracture capacity of the reduced section after deducting bolt holes. The net section is more critical near connections because the holes interrupt the stress flow and reduce the available resisting width. A common design expression is:

$$N_{ult} = 0.9 \frac{A_{net} f_u}{\gamma_{m2}} \quad (5.4)$$

where A_{net} is the net sectional area, f_u is the ultimate tensile strength, and γ_{m2} is the partial safety factor for net-section rupture. In the present course convention, $\gamma_{m2} = 1.25$.

5.6. Determination of the net section:

The gross area A is the total cross-sectional area of the member before any deduction. The net area A_{net} is obtained by subtracting the material removed by bolt holes along a possible line of fracture. In tension design, it is not sufficient to consider only one obvious section. The designer must examine all realistic rupture paths and retain the smallest value of A_{net} , because the smallest net area gives the lowest fracture resistance.

5.6.1. Holes arranged on a straight line perpendicular to the force:

When the holes are arranged in one transverse row perpendicular to the tensile force, the critical net section is usually a straight line passing through the holes (Figure 5.3). If n holes of diameter d_0 are intercepted by the rupture line and the plate thickness is t , the net area may be written as:

$$A_{net} = A - n \times d_0 \times t \quad (5.5)$$

The value of d_0 must correspond to the hole diameter, not simply the nominal bolt diameter. This distinction is important because the drilled or punched hole is normally slightly larger than the bolt.

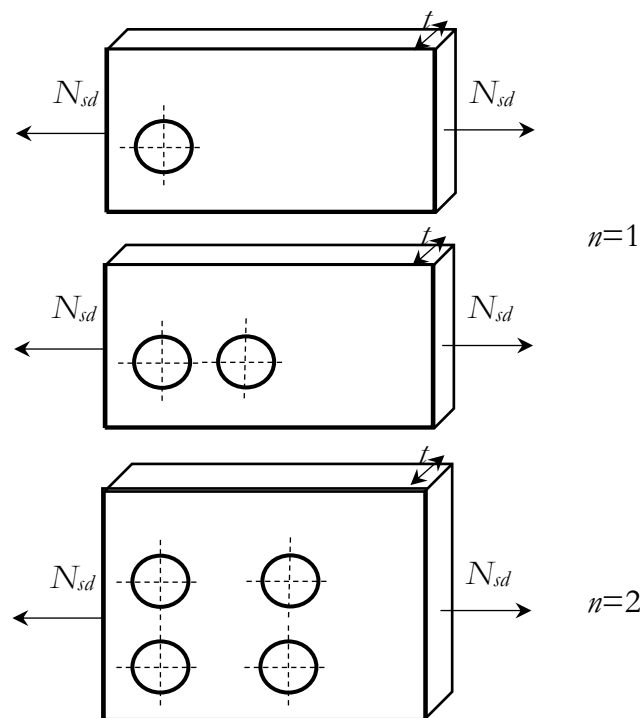


Figure 5.3. Net section through a transverse row of bolt holes [6].

5.6.2. Holes arranged in staggered or zigzag pattern:

When holes are staggered, the potential rupture line may follow a broken path from one hole to another. In this case, the net area is not obtained by a simple straight deduction only. A stagger correction is introduced because the inclined path between holes increases the rupture length and therefore partially compensates for the loss of area

caused by the holes. For a broken line, a simplified expression for the net width is generally written in the form:

$$A_{net} = A - n \times d_0 \times t + \sum \frac{S_i^2}{4P_i} \times t \quad (5.6)$$

where S_i is the stagger spacing measured parallel to the direction of the force transfer between consecutive holes, and P_i is the gauge spacing measured perpendicular to the force between the same holes (Figure 5.4). The designer should calculate A_{net} for each possible rupture path and select the smallest value.

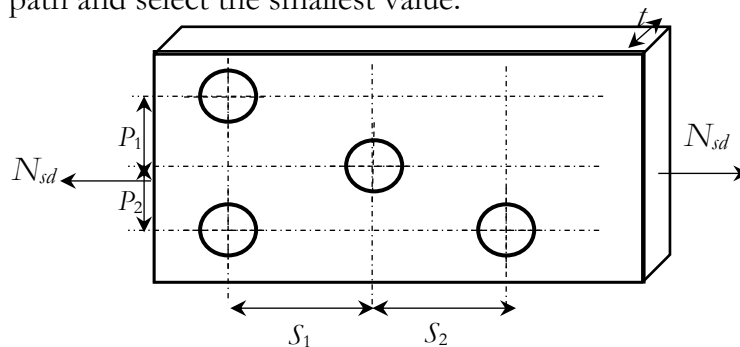


Figure 5.4. Staggered holes and broken rupture path [6].

5.7. Design of angles connected by one leg:

Angles are common tension members in trusses and bracing systems. However, when only one leg of an angle is connected to a gusset plate, the tensile force is not transferred uniformly to the entire cross-section. The unconnected leg is less directly engaged, and the stress distribution becomes non-uniform. This phenomenon is often described as shear lag. For this reason, the resistance of angles connected by one leg is usually reduced by coefficients that depend on the number of bolts, their spacing, and the connection geometry (Figure 5.5).

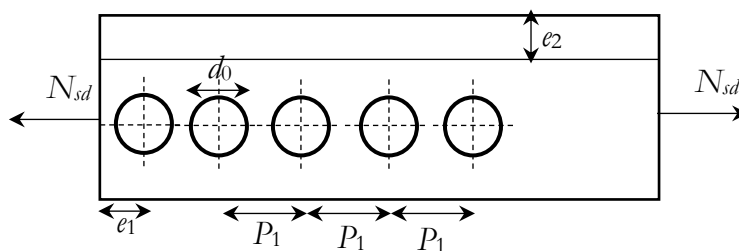


Figure 5.5. Angle connected by one leg with several bolts.

5.7.1. Angle attached by one bolt:

With a single bolt, the force transfer is highly localised. The connected leg carries the force into the bolt region, while the unconnected leg contributes less effectively. This type of connection may be sensitive to eccentricity, local bearing, edge distance, and tear-out. The design must therefore be conservative and must verify that the hole, the edge distance, and the available net section are sufficient. The parameters d_0 , t , e_1 , and e_2 must be clearly identified before the resistance is calculated (Figure 5.6).

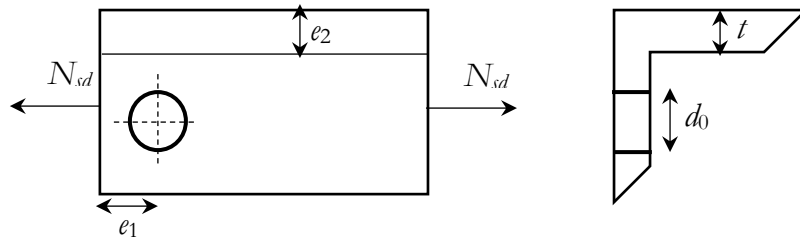


Figure 5.6. Angle connected by one bolt.

$$N_{ult} = \frac{2(e_2 - 0.5d_0) \times t \times f_u}{\gamma_{m2}} \quad (5.7)$$

where d_0 : diameter of bolt hole.

t : thickness of the plate or angle leg.

5.7.2. Angle attached by two bolts:

A two-bolt connection provides a better distribution of the tensile force than a single-bolt connection, because the load is transferred through two bearing zones. Nevertheless, the angle remains connected by one leg only, so shear lag is still present. The reduction coefficient β_2 is used to account for this non-uniform force transfer. Its value depends on the bolt pitch p_1 in relation to the hole diameter d_0 (Figure 5.7).

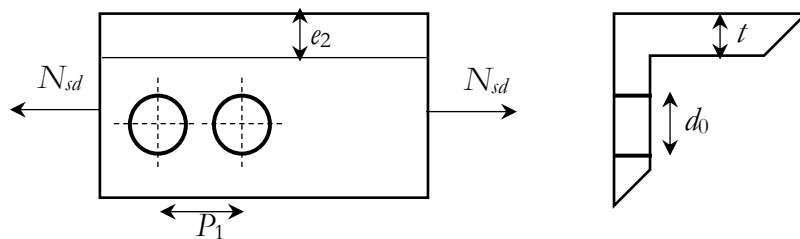


Figure 5.7. Angle connected by two bolts.

$$N_{ult} = \beta_2 \frac{A_{net} f_u}{\gamma_{m2}} \quad (5.8)$$

5.7.3. Angle attached by three bolts or more:

When three or more bolts are used in the direction of the applied force (Figure 5.8), the connection becomes more effective because the load transfer length is increased. The shear-lag effect is reduced but not completely eliminated. The coefficient β_3 is therefore used. In comparison with β_2 , β_3 normally gives a more favorable resistance because the member is better engaged over a longer connection length.

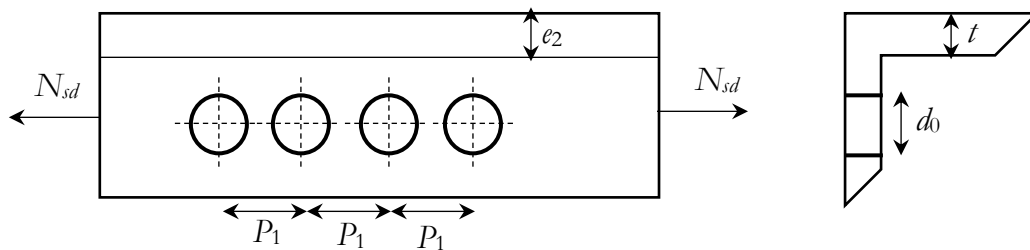


Figure 5.8. Angle connected by three bolts or more.

$$N_{ult} = \beta_3 \frac{A_{net} f_u}{\gamma_{m2}} \quad (5.9)$$

5.7.4. Coefficients β_2 and β_3 :

Table 5.1 gives the course values of the coefficients β_2 and β_3 as a function of the longitudinal pitch P_1 . For intermediate values, linear interpolation is used.

Table 5.1. Values of the coefficients β_2 and β_3

Parameter P_1	$P_1 \leq 2.5 d_0$	$P_1 \geq 5 d_0$
β_2	0.40	0.70
β_3	0.50	0.70

For any intermediate value of P_1 , the coefficient may be obtained from:

$$\beta_2 = 0.4 + x \Rightarrow \frac{x}{P_1 - 2.5d_0} = \frac{0.3}{2.5d_0} \Rightarrow x = \frac{0.3}{2.5d_0}(P_1 - 2.5d_0) \quad (5.10)$$

This interpolation reflects the gradual improvement of load transfer as the connection length increases. The designer must not use the maximum coefficient when the pitch is small, because this would overestimate the contribution of the unconnected part of the angle.

5.8. Conclusion:

A tension member is a structural element whose primary internal action is an axial force directed outwards at the ends. Its design is generally simpler than the design of compression members because global buckling is not the governing phenomenon. However, the presence of connections makes the design more subtle. The engineer must check both gross-section yielding and net-section rupture, identify the most critical rupture path, and apply suitable reductions for non-uniform force transfer in angles connected by one leg. The governing resistance is the smallest relevant design resistance, and the design is satisfactory only when the applied design tensile force does not exceed this value.

5.9. Application Exercises:

This part presents a set of application exercises devoted to the verification of steel members subjected to axial tension. The proposed work emphasizes the determination of gross and net cross-sections, the influence of bolt holes and staggered failure lines, the calculation of design tensile resistance, and the verification of tension members encountered in truss and bracing systems.

Exercise No. 01

Determine the net cross-section of the steel plate shown in the statement.

The member is a flat plate subjected to a tensile design force N_{sd} .

The plate thickness is $t = 6$ mm and the bolt-hole diameter is $d_0 = 24$ mm. The transverse dimensions and possible rupture paths are indicated in the original figure, with four possible lines of fracture numbered 1, 2, 3, and 4 (Figure 5.9).

The objective is to evaluate the effective net area by considering the most unfavorable rupture line through the bolted region.

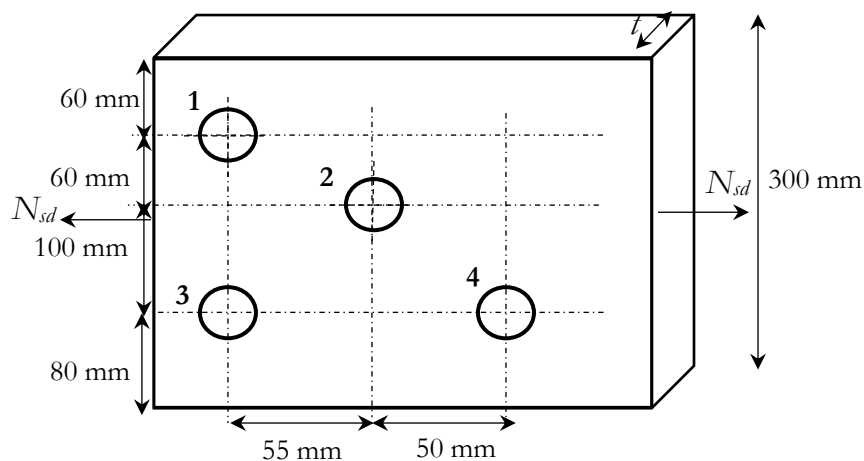


Figure 5.9. Steel plate.

Expected technical work:

- Identify all possible rupture paths across the bolted plate.
- For each rupture path, subtract the appropriate hole areas from the gross section.
- When the rupture path is staggered, apply the correction term associated with staggered holes.

- Compare the obtained net areas and retain the smallest value as the governing net section.

Exercise No. 02

Verify the tensile resistance of a bracing diagonal made of double equal angles.

The diagonal member of a truss system is composed of double equal-angle sections L 70 × 70 × 7, with a total gross area $A = 18.80 \text{ cm}^2$ (Figure 5.10).

The steel grade is defined by $f_y = 235 \text{ N/mm}^2$ and $f_u = 360 \text{ N/mm}^2$.

The member is subjected to a design tensile force $N_{sd} = 13950 \text{ daN}$. The connection is made with three bolts of hole diameter $d_0 = 22 \text{ mm}$, spaced at $P_1 = 50 \text{ mm}$.

The objective is to verify whether the member satisfies the required tensile resistance condition.

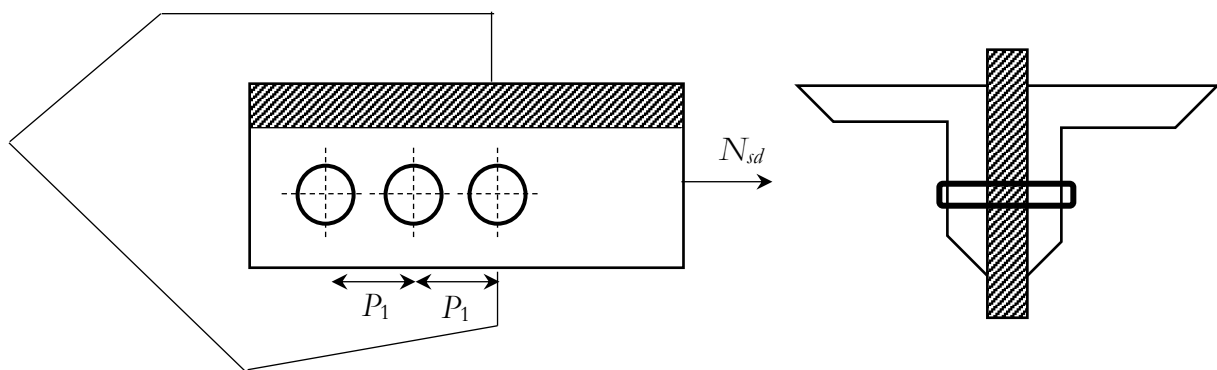


Figure 5.10. Bracing diagonal made of double equal angles.

Expected technical work:

- Determine the gross cross-sectional area of the double-angle member.
- Evaluate the net cross-section A_{net} by accounting for the bolt holes in the connected legs.
- Calculate the design resistance in yielding of the gross section.
- Calculate the design resistance in fracture of the net section.
- Compare the design tensile force N_{sd} with the governing tensile resistance and conclude on the safety of the member.

Exercise No. 03

Verify the resistance of the tension members of the truss system (Figure 5.11).

The truss is defined by the nodes A, B, C, D, and E, with panel lengths of 1.0 m, 1.0 m, 1.5 m, and 1.5 m as indicated in the original statement. The members are equal-angle sections L $60 \times 60 \times 6$ with a gross area $A = 6.91 \text{ cm}^2$.

The connection is made with three bolts of hole diameter $d_h = 16 \text{ mm}$, spaced at $P_1 = 30 \text{ mm}$. A load $P = 20 \text{ kN}$ is applied to the structure. The steel grade is $f_y = 235 \text{ N/mm}^2$ and $f_u = 360 \text{ N/mm}^2$.

The objective is to identify and verify the tensile members of the truss.

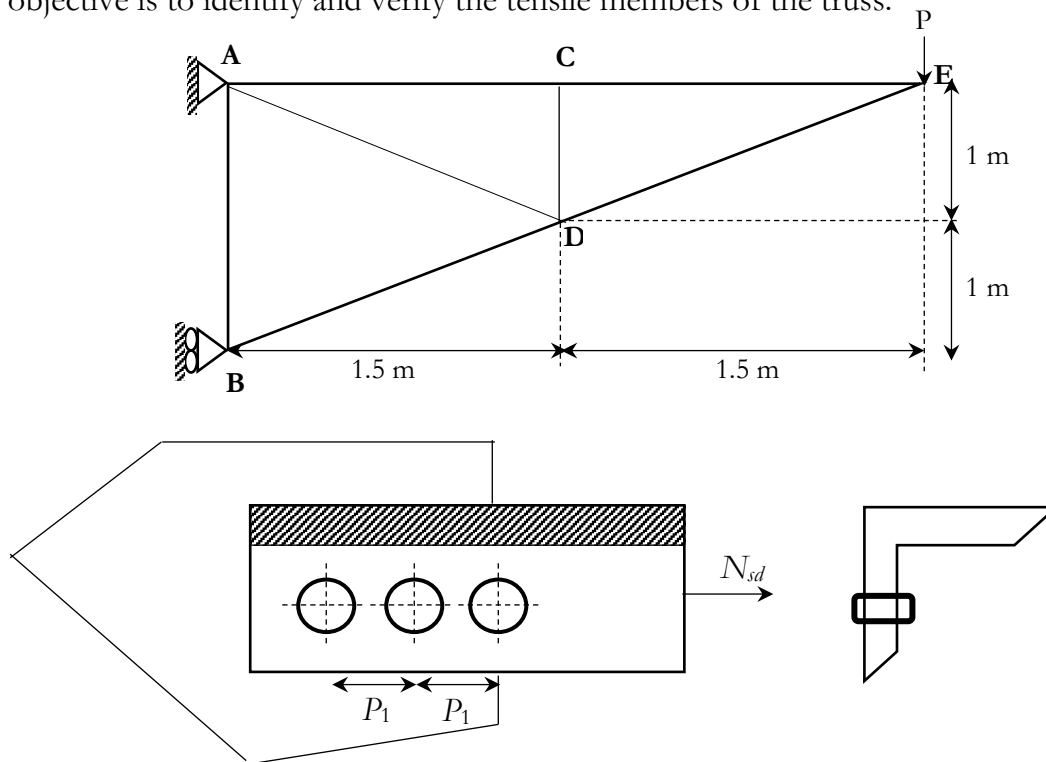


Figure 5.11. Truss system and connection detail.

Expected technical work:

- Determine the axial forces in the truss members using equilibrium of joints or sections.
- Identify the bars subjected to tension under the applied loading configuration.
- For each tension member, determine the gross and net resisting areas.
- Evaluate the tensile resistance controlled by gross-section yielding and net-section fracture.

- Compare the internal tensile force N_{sd} in each bar with the corresponding design resistance and state whether the verification is satisfied.

Exercise No. 04

Verify the resistance of the tension and compression members of the structural system.

The members of the system are made of double UAP80 sections connected only at the joints. Each member connection is made with four bolts of hole diameter $d_0 = 15$ mm, spaced at $P_1 = 55$ mm. The steel grade is $f_y = 235$ N/mm² and $f_u = 360$ N/mm², and the applied force is $F = 300$ kN (Figure 5.12).

The objective is to verify the members subjected to tension and compression by considering both the axial force distribution and the relevant resistance checks.

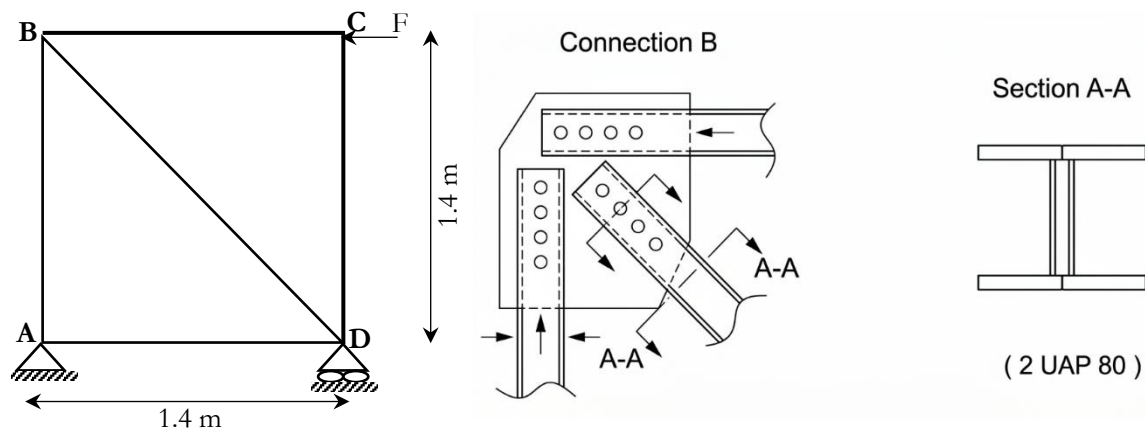


Figure 5.12. Truss system and connection detail.

Expected technical work:

- Determine the axial forces in all members of the system using static equilibrium.
- Classify the members according to the nature of the internal force: tension or compression.
- For tension members, evaluate the gross and net cross-sections and verify yielding and fracture resistance.
- For compression members, determine the effective length and assess the buckling resistance.
- Compare each design axial force with the corresponding design resistance and conclude on the adequacy of the system.

Chapter 6

Simple Compression (Simple Buckling)

Learning objectives:

- ✎ At the end of this chapter, the student should be able to explain the behavior of compressed steel members, distinguish cross-section resistance from member buckling resistance, calculate slenderness and reduced slenderness, select an appropriate buckling curve, and verify the design resistance of members subjected to simple compression.

6.1. Introduction:

A structural member is considered to be under simple compression when the internal force system is reduced essentially to an axial compressive normal force directed toward the interior of the member. In steel construction, such members are common in columns, truss diagonals and verticals, bracing systems, and numerous secondary structural components. Although axial compression may appear mechanically simple, the design of compression members is governed not only by material strength but also, and often predominantly, by stability considerations. For slender members, failure may occur by loss of stability at a load level much lower than the squash load of the full section. Consequently, the study of simple compression is inseparable from the study of buckling.

6.2. Field of application:

Simple compression is encountered in a wide range of steel structural systems. In practical design, compressed elements are not limited to isolated columns; they also appear in triangulated systems, braced frames, and stability subsystems whose primary function is to transmit axial forces while ensuring global equilibrium (Figure 6.1).

- **Building columns:** vertical members that transfer floor loads, roof loads, and other gravity actions to the foundations. Their design generally combines axial force with the possible influence of moments arising from imperfections or frame action.
- **Truss members in compression:** certain diagonals, struts, or top chords of trusses carry mainly compressive axial force. Their efficiency depends strongly on member slenderness, end restraint conditions, and out-of-plane bracing.
- **Stability systems:** bracing members in roofs, façades, industrial halls, and towers participate in the overall stability of the structure by resisting horizontal actions and redistributing internal forces.

The designer must therefore distinguish between a short member, for which yielding may govern, and a slender member, for which instability controls the resistance.

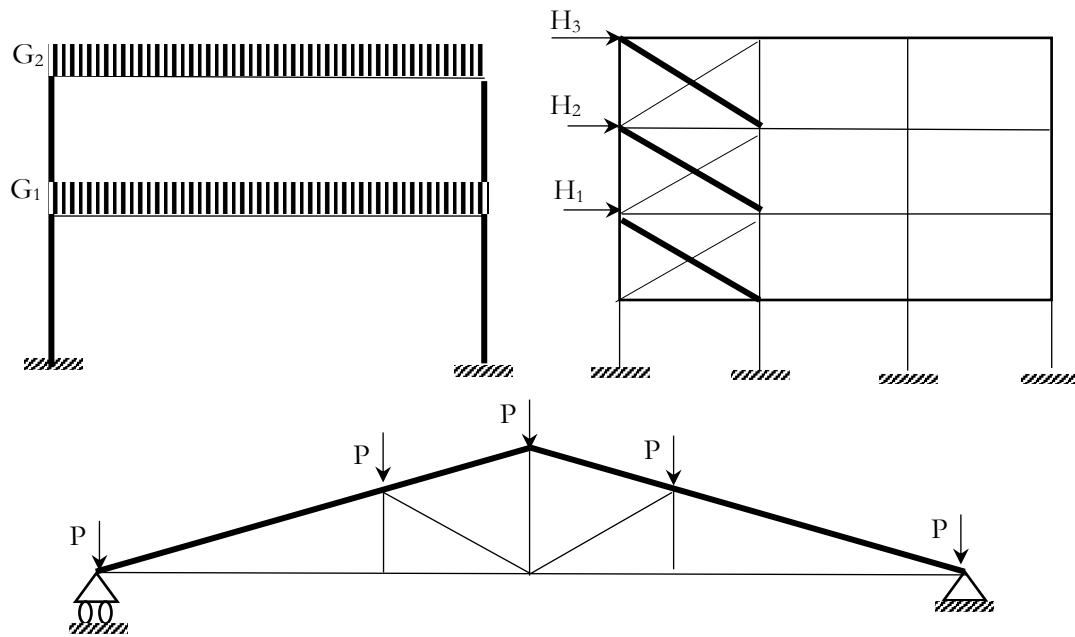


Figure 6.1. Field of application of simple compression [9].

6.3. Behavior and sizing of compressed members:

The mechanical response of a compressed steel member depends on its cross-sectional geometry, its length, the degree of restraint provided at its ends, the distribution of residual stresses, and inevitable geometric imperfections introduced during fabrication and erection. For this reason, the sizing of compression members cannot be based solely on the yield strength of steel.

In practice, three main failure modes must be considered when a steel member is subjected to axial compression:

- Complete section plasticization, which occurs when the cross-section reaches its compressive strength before instability develops. This mode is more common in short members with low slenderness.
- Localized buckling, which affects individual plate elements of the cross-section such as webs or flanges. It is especially relevant for thin-walled sections and influences the effective area available for resistance.
- Overall buckling, which corresponds to a global loss of equilibrium of the member about one of its principal axes. This is generally the governing mode for slender members.

Accordingly, member sizing requires the verification of the gross or effective cross-section, the identification of the relevant buckling axis, and the determination of the design buckling resistance.

6.4. Buckling curve:

Experimental results for compressed steel members are commonly represented through buckling curves, which relate the reduction factor of compressive resistance to the reduced slenderness of the member (Figure 6.2). These curves provide a practical way of incorporating the effects of imperfections, residual stresses, and non-ideal material behavior into design.

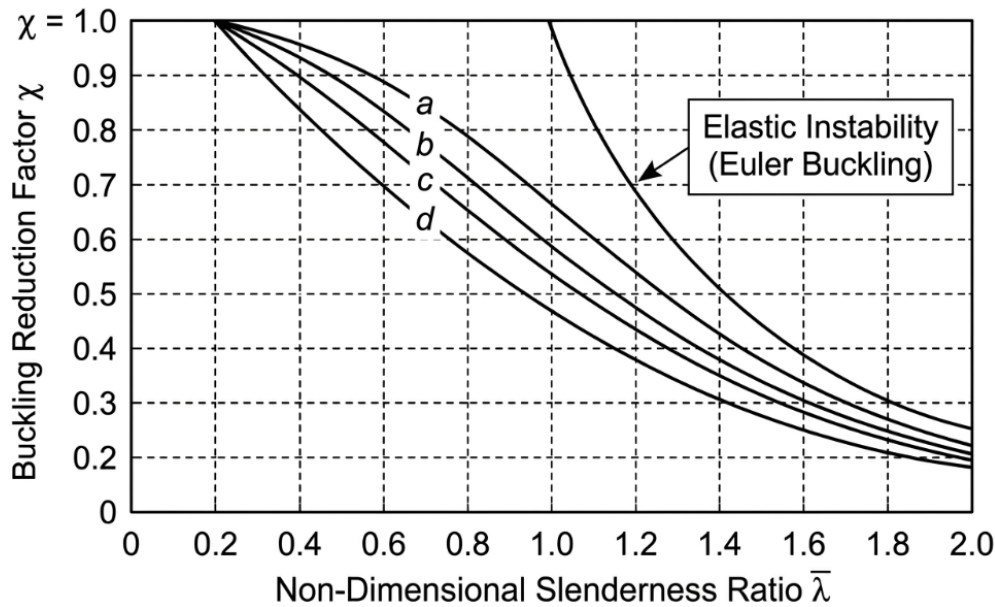


Figure 6.2. Buckling curve [4].

The fundamental parameter governing the phenomenon is the reduced slenderness, usually written as $\bar{\lambda}$, which compares the actual slenderness of the member with a reference elastic slenderness. In a simplified form, it may be expressed as:

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} \quad (6.1)$$

Where λ is the geometric slenderness is given by:

$$\lambda = \frac{l_{bc}}{i} = \frac{KL}{i} \quad (6.2)$$

where KL is the buckling length, i is the radius of gyration about the considered buckling axis, f_y is the yield strength of steel, and E is Young's modulus.

The radius of gyration i is defined from the section properties by:

$$i = \sqrt{\frac{I}{A}} \quad (6.3)$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 93.9 \varepsilon / \varepsilon = \sqrt{\frac{235}{f_y}} \quad (6.4)$$

A low value of $\bar{\lambda}$ indicates a stocky member with limited sensitivity to instability, whereas a high value indicates a slender member whose resistance is strongly reduced by buckling.

To account for different fabrication routes, section types, and imperfection sensitivities, practical design adopts four conventional buckling curves, denoted a, b, c, and d, which are distinguished by the imperfection factor α .

The buckling curves depend on the imperfection factor " α " where their values are presented in Table 6.1.

Table 6.1. Values of the imperfection coefficient α

Buckling curve	a	b	c	d
Imperfection factor α	0.21	0.34	0.49	0.76

The reduction factor χ associated with a given buckling curve is calculated analytically using the conventional expressions:

$$\phi = 0.5 \left[1 + \alpha (\bar{\lambda} - 0.2) + \bar{\lambda}^2 \right] \quad (6.5)$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{with} \quad \chi \leq 1.0 \quad (6.6)$$

For a given member, χ must be determined about each relevant principal axis, because the member may buckle in either plane. The lower value is retained for design.

$$\chi_{\min} = \min [\chi_y, \chi_z] \quad (6.7)$$

6.5. Sizing criteria:

The design axial compression force, usually denoted N_{sd} must not exceed the design resistance of the member. Two levels of verification are typically required: cross-sectional resistance and member buckling resistance.

The compression force N_{sd} in the element must satisfy:

$$N_{sd} \leq N_{Rd} \quad (6.8)$$

N_{Rd} depends on the reduced slenderness ratio $\bar{\lambda}$ and the profile class (Class 1, 2, 3, and 4).

➤ **Case 1:** $\bar{\lambda} \leq 0.2$

- **Class 1, 2, 3:**

$$N_{Rd} = N_{pl} = \frac{Af_y}{\gamma_{m0}} \quad (6.9)$$

A : Area of the gross section.

f_y : Yield strength (elastic limit).

γ_{m0} : Partial safety coefficient $\gamma_{m0} = 1.1$.

- **Class 4:**

$$N_{Rd} = \frac{A_{eff} f_y}{\gamma_{m2}} \quad (6.10)$$

A_{eff} : Effective section.

γ_{m2} : Partial safety coefficient $\gamma_{m2} = 1.25$.

➤ **Case 2:** $\bar{\lambda} > 0.2$

- **Class 1, 2, 3:**

$$N_{Rd} = \chi_{\min} \frac{Af_y}{\gamma_{m0}} \quad (6.11)$$

- **Class 4:**

$$N_{rd} = \chi_{\min} \frac{A_{eff} f_y}{\gamma_{m2}} \quad (6.12)$$

6.6. Determination of buckling length:

The buckling length is a decisive parameter because the elastic critical load varies inversely with the square of the effective member length. In design, this length is not always identical to the actual physical length of the member. Instead, it is an equivalent length that reflects end restraints, rotational continuity, and the global structural configuration (Figure 6.3).

For isolated members, the effective length is often written as:

$$l_{bc} = Kl_0 \quad (6.13)$$

where l_0 is the system length and k is the effective length factor determined from the support conditions and the degree of end restraint.

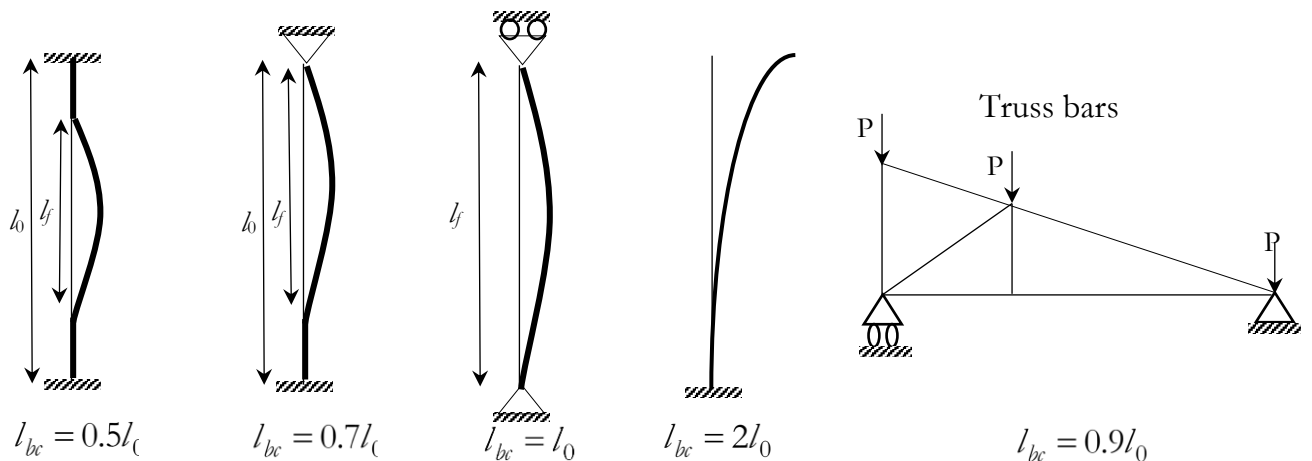


Figure 6.3. Buckling length [6].

A lower value of k indicates stronger rotational restraint and therefore a greater buckling resistance. For truss bars and bracing members, the effective length is often governed by the nodal configuration and the practical detailing of the joints.

6.7. Profile classes used in simple compression:

Cross-section classification remains essential in compression design because local plate slenderness influences the extent to which the section can mobilize yield stress before local buckling occurs. The class of a profile is determined by comparing the width-to-thickness ratio of each plate element to codified limits.

For simple compression, the web and flange elements are generally checked separately. In simplified course form, the following class limits (Table 6.2 and Figure 6.4) are commonly used:

Table 6.2. Profile classes in simple compression

Element	Class 1	Class 2	Class 3	Class 4	$\varepsilon = \sqrt{\frac{235}{f_y}}$
Web ratio « d / t_w »	33ε	38ε	42ε	$> 42 \varepsilon$	
Flange ratio « c / t_f »	10ε	11ε	15ε	$> 15 \varepsilon$	

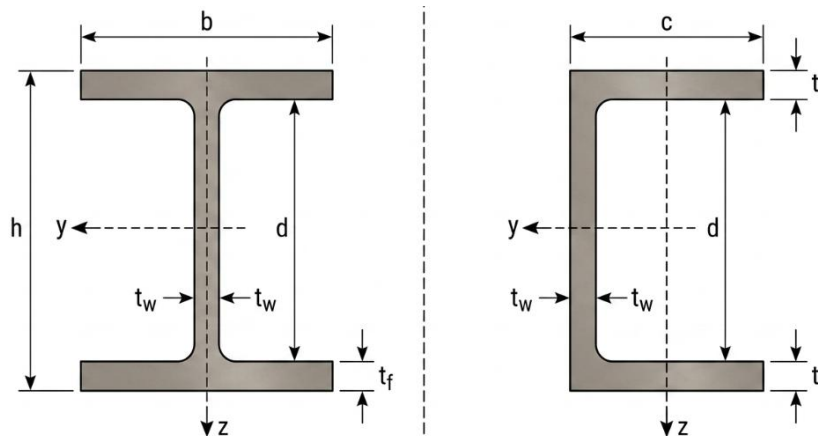


Figure 6.4. Profile dimensions [8].

Class 1 and Class 2 sections can generally develop plastic resistance, whereas Class 3 sections are limited to elastic resistance.

Class 4 sections are slender sections for which local buckling reduces the effective resisting area. From a design perspective, the interaction between section class and member slenderness is fundamental.

6.8. Conclusion:

The design of members subjected to simple compression combines strength verification with stability analysis. The engineer must identify whether the governing limit state is plastic crushing of the section, local buckling of plate elements, or global buckling of the member.

The use of reduced slenderness, imperfection-sensitive buckling curves, effective buckling length, and section classification provides a rational framework for translating the physical instability phenomenon into practical design checks. Compression-member

design is therefore a stability-driven process in which geometry, end restraint, cross-sectional proportions, and imperfections jointly determine the load-carrying capacity of the member.

6.9. Application Exercises:

This part presents a set of application exercises devoted to the verification of steel members subjected to axial compression and simple buckling. The proposed work emphasizes section classification, slenderness evaluation, buckling verification, and the determination of the design compression resistance of steel members encountered in practice.

Exercise No. 01

Determine the maximum design compression resistance N_{rd} of the steel column shown in the statement.

The member is a rolled IPE 140 section made of structural steel with a yield strength $f_y = 235 \text{ N/mm}^2$. The cross-section is classified as Class 1, and the member length is $L = 4.0 \text{ m}$ (Figure 6.5). The objective is to evaluate the axial load-carrying capacity of the member in simple compression by applying the relevant buckling verification procedure.

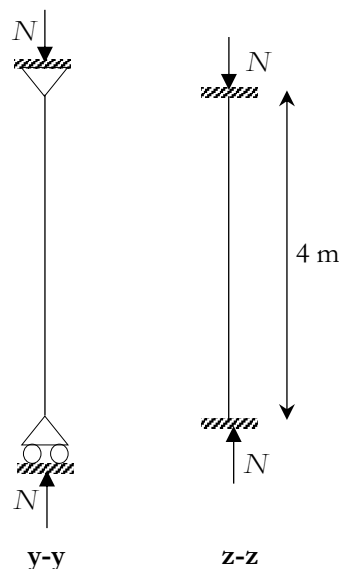


Figure 6.5. Column member under axial force

Expected technical work:

- Identify the principal buckling axes and establish which axis is likely to govern the verification.
- Determine the appropriate slenderness parameters using the section properties of IPE 140.
- Select the relevant buckling curve and evaluate the corresponding reduction coefficient.
- Calculate the design compression resistance N_{rd} .

Exercise No. 02

Determine the cross-section class of two compressed rolled profiles:

An IPE 600 section made of steel with $f_y = 355 \text{ N/mm}^2$ and an HEA 500 section made of steel with $f_y = 235 \text{ N/mm}^2$.

Using the HEA 500 profile, verify the resistance of the compressed column represented in the statement (Figure 6.6).

The exercise links local cross-section classification with the global buckling verification of a steel column.

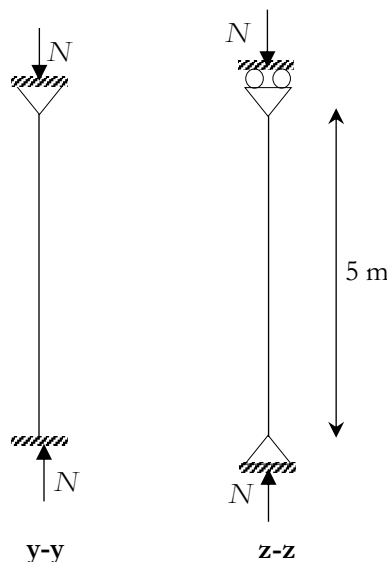


Figure 6.6. Column member under axial force.

Expected technical work:

- Classify the IPE 600 and HEA 500 profiles by checking the width-to-thickness limits of the relevant plate elements.

- For the HEA 500 column, establish the governing buckling axis and determine the associated effective slenderness.
- Apply the appropriate design relation for compression members and verify whether the profile satisfies the resistance condition.
- State clearly whether the verification is controlled by section resistance, global buckling, or cross-section class.

Exercise No. 03

Consider the truss system defined by the nodes A, B, C, D, and E, with the panel lengths indicated in the original statement (1.0 m, 1.0 m, 1.5 m, and 1.5 m). The truss members to be checked are made of an equal-angle section L 60 × 60 × 6.

The connection is formed by three bolts of hole diameter $d_0 = 16$ mm, with a spacing $P_1 = 30$ mm. A load $P = 20$ kN is applied as indicated in the problem (Figure 6.7).

The steel grade is defined by $f_y = 235$ N/mm² and $f_u = 360$ N/mm².

The objective is to verify the resistance of the compressed bars of the truss while accounting for both the angle section and its bolted connection.

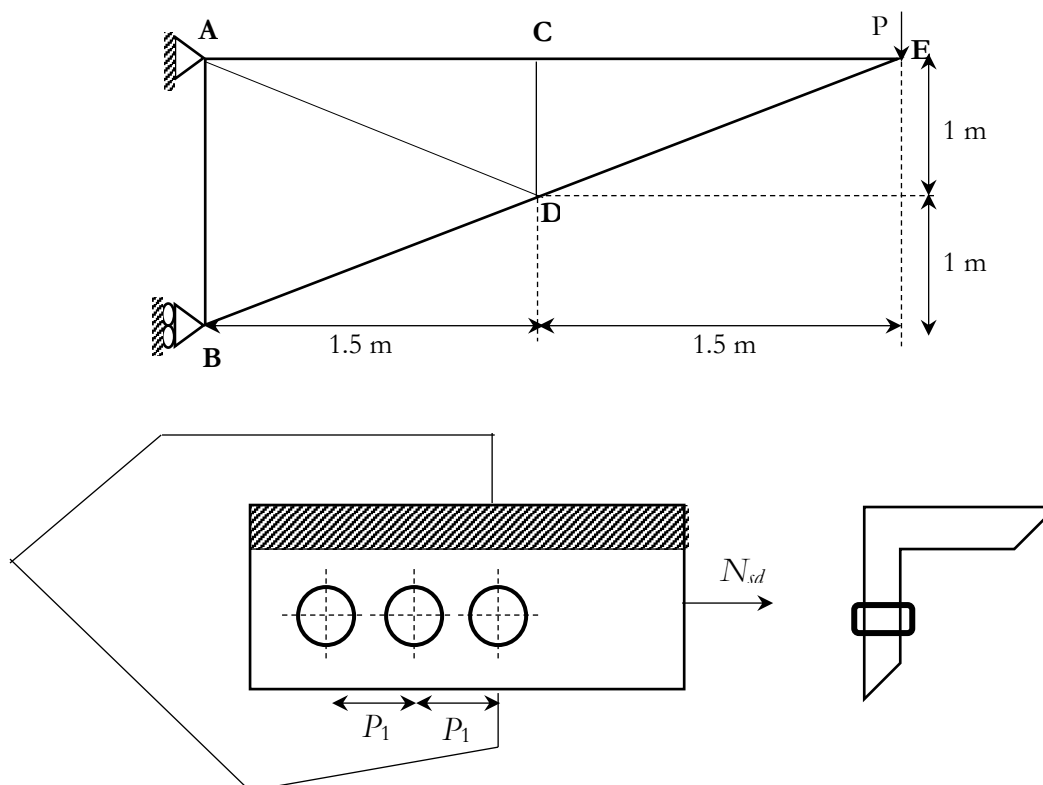


Figure 6.7. Truss system and connection of three bolts.

Expected technical work:

- Identify the truss bars subjected to compression under the applied loading configuration.
- Determine the gross and, when relevant, the net resisting section associated with the bolted angle member.
- Evaluate the susceptibility of the compressed angle member to buckling by using an appropriate effective length and section properties.
- Conclude on the adequacy of the compressed members with respect to the design axial force N_{sd} .

Exercise No. 04

Determine the maximum design compression resistance N_{rd} of the circular hollow steel column presented in the statement. The member has an external diameter $D = 115$ mm and an internal diameter $d = 105.5$ mm.

It is made of structural steel with $f_y = 235$ N/mm², and the cross-section is classified as Class 2. The column length is $L = 4.0$ m (Figure 6.8).

This exercise compares the compression behavior of a hollow tubular member with that of the rolled open sections studied in the previous exercises.

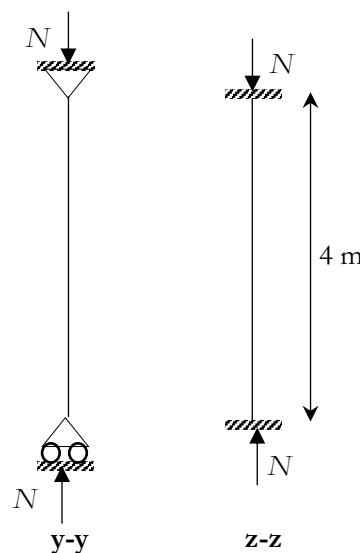


Figure 6.8. Column member under axial force.

Expected technical work:

- Compute the geometric properties of the circular hollow section required for compression and buckling verification.
- Determine the slenderness of the member and assess the relevant reduction factor.
- Calculate the design compression resistance N_{rd} of the tubular column.
- Briefly discuss the structural advantages of hollow sections in compression members with respect to stiffness and torsional efficiency.

Chapter 7

Simple Bending

Learning objectives:

- ✎ After studying this chapter, the student should be able to identify the internal actions acting on a beam in simple bending, distinguish between shear and flexural design criteria, select the appropriate resistance expression according to the section class, account for the influence of high shear on bending capacity, and apply interaction formulae for biaxial bending.

7.1. Introduction:

This chapter presents the design principles for steel members subjected primarily to simple bending. The aim is to provide a clear engineering interpretation of the design checks commonly used for beams and flexural members, while keeping the same scope as the original lecture notes: shear resistance, bending resistance according to the class of the cross-section, reduction of bending resistance under high shear, and verification under biaxial bending. The text is written in an academic style suitable for lecture handouts, course notes, or a technical design chapter.

7.2. Mechanical meaning of simple bending:

A steel member is said to be subjected to simple bending when the dominant internal action is a bending moment acting about one principal axis of the cross-section (Figure 7.1). In practical steel structures, simple bending is most commonly encountered in beams, secondary joists, purlins, floor girders, and frame members carrying transverse loads. Although the bending moment is the principal action, shear force usually acts simultaneously and must also be verified.

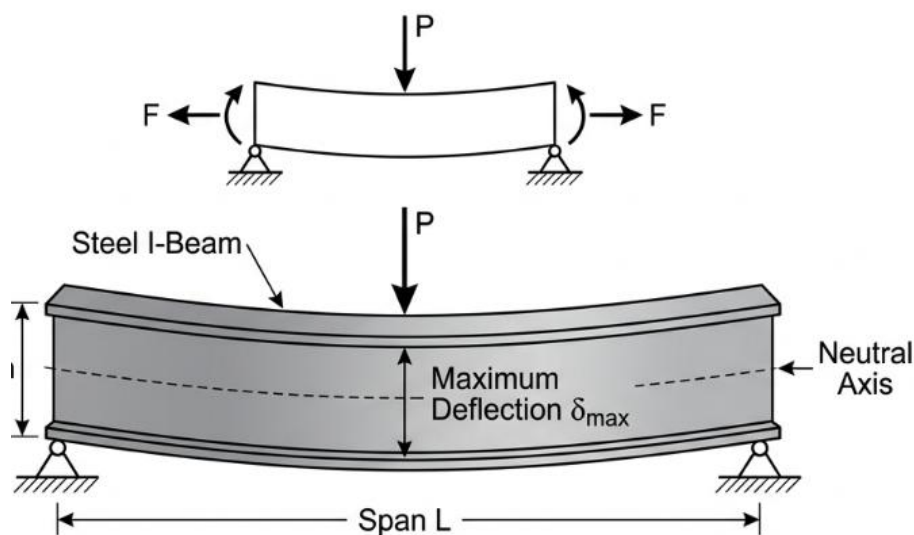


Figure 7.1. A steel member subjected to simple bending [1].

The bending moment produces a linear distribution of normal stresses over the depth of the section when the behavior remains elastic. Fibers on one side of the neutral axis are compressed, whereas fibers on the opposite side are tensioned. As the bending

moment increases, yielding may begin at the extreme fibers and progressively spread through the cross-section. The design resistance therefore depends on the capacity of the section to develop either an elastic stress distribution, a plastic stress distribution, or a reduced effective section in the case of local buckling.

In engineering design, the verification is not limited to a comparison between applied moment and section capacity. The designer must also examine the effect of shear, the classification of the cross-section, the possible interaction between moment components, and the suitability of the member for serviceability requirements such as deflection and vibration. This chapter focuses on ultimate limit state resistance of the cross-section, which is the direct scope of the original notes.

Note. The notation V_{sd} and M_{sd} refers to design effects obtained from factored load combinations. The notation R_d indicates a design resistance after the introduction of partial safety factors.

7.3. Design criteria in simple bending:

For a beam or flexural member, two fundamental resistance checks are required at the cross-section level: a shear-force verification and a bending-moment verification. Both checks must be satisfied for the member to be acceptable at the ultimate limit state.

7.3.1. Shear resistance criterion:

The design shear force acting on the member must not exceed the plastic shear resistance of the cross-section. This condition is expressed as:

$$V_{sd} \leq 50\% V_{pl,Rd} \quad (7.1)$$

The plastic shear resistance is generally evaluated from the shear area of the section and the yield strength of steel.

$$V_{pl,Rd} = \frac{A_{vz} f_y}{\sqrt{3} \gamma_{m0}} \quad (7.2)$$

A_{vz} : shear area of the section, can be found as follows :

- **I and H Profiles:** For rolled I- and H-sections, the web carries most of the vertical shear force, which explains why the shear area is strongly related to the web geometry.

$$A_{v\chi} = A - bt_f + (t_w + 2r)t_f \quad (7.3)$$

- **U Profiles:** For channel sections, the shear area is also linked to the web, but the asymmetry of the section should be kept in mind when interpreting the distribution of internal stresses.

$$A_{v\chi} = A - bt_f + (t_w + r)t_f \quad (7.4)$$

In design practice, the shear check is usually performed before the final bending verification because high shear may reduce the effective bending resistance. The shear force is especially important near supports, concentrated loads, beam-column joints, and locations where web openings or notches are present.

7.3.2. Bending resistance criterion:

The design bending moment must be less than or equal to the design bending resistance of the cross-section:

$$M_{sd} \leq M_{c,Rd} \quad (7.5)$$

The value of the design bending resistance depends on the class of the cross-section. Cross-section classification is essential because it indicates whether the section can reach its plastic resistance, only its elastic resistance, or a reduced resistance due to local buckling of slender plate elements.

- **Class 1 and 2:** In class 1 the section can form a plastic hinge with sufficient rotation capacity and for class 2 the section can reach plastic resistance but has limited rotation capacity.
- **Class 3:** In class 3 the extreme fiber can reach yield, but local buckling prevents full plastic redistribution.

- **Class 4:** In class 4, slender elements buckle locally before the yield stress is fully reached; effective properties are required.

7.4. Role of cross-section classification:

The classification of a steel section is a central concept in modern steel design. It connects the geometry of plate elements, such as flanges and webs, with the mechanical response of the whole cross-section. Stocky plates can develop large plastic stress blocks, whereas slender plates may buckle locally and lose part of their compressive effectiveness before the nominal yield strength is fully mobilised.

For Class 1 and Class 2 sections, the plastic section modulus W_{pl} is used. This reflects the ability of the section to redistribute stresses beyond first yield and to develop a rectangular plastic stress distribution. For Class 3 sections, the elastic section modulus W_{el} is used because the design is limited by the first attainment of yield at the most stressed fiber. For Class 4 sections, the effective section modulus W_{eff} is used to account for local buckling and the reduction of the effective compressed width.

Note. The use of W_{pl} , W_{el} , or W_{eff} is not a matter of convenience; it is a direct consequence of the cross-section slenderness and the expected stability behavior of its plate elements.

7.4.1. Plastic bending resistance for Class 1 and Class 2 sections:

$$M_{c,Rd} = M_{pl} = \frac{W_{pl} f_y}{\gamma_{m0}} \quad (7.6)$$

W_{pl} : The plastic section modulus.

This expression assumes that the cross-section is sufficiently compact to develop a fully plastic stress distribution. It is appropriate for beams where the compressed flange and the compressed part of the web do not buckle prematurely. In this case, the section may exploit the reserve capacity between first yield and full plasticity.

7.4.2. Elastic bending resistance for Class 3 sections:

$$M_{c,Rd} = M_{el} = \frac{W_{el} f_y}{\gamma_{m0}} \quad (7.7)$$

For Class 3 sections, the section is not allowed to rely on plastic redistribution. The resistance is based on the elastic section modulus W_{el} , which corresponds to the onset of yield at the extreme fiber. This provides a safer representation of the behavior when local buckling may prevent development of a fully plastic stress block.

7.4.3. Effective bending resistance for Class 4 sections:

$$M_{c,Rd} = \frac{W_{eff} f_y}{\gamma_{m0}} \quad (7.8)$$

For Class 4 sections, slender compressed plate elements are susceptible to local buckling before the full cross-section becomes effective. Consequently, the gross section properties are replaced by effective properties. The effective section modulus W_{eff} is calculated after reducing the widths of slender elements that are affected by local buckling.

7.5. Influence of shear on bending resistance:

When shear force is moderate, its influence on bending resistance is usually small and may be neglected in cross-section resistance checks. However, when the design shear force exceeds one half of the plastic shear resistance, the web is significantly mobilised in shear, and the bending resistance must be reduced. This interaction reflects the fact that the same material cannot simultaneously develop full shear yielding and full bending resistance without loss of capacity.

If $V_{sd} > 50\% V_{pl,Rd}$, reduce $M_{c,Rd}$ to $M_{v,Rd}$

For equal-flange I- or H-sections bent about the major axis, the reduction is commonly applied through a shear-dependent reduction factor associated with the shear area A_v . The original lecture note indicates this case separately because it is frequent in beam design and has a convenient engineering formulation. The reduction primarily affects the

contribution of the web to bending resistance, whereas the flanges remain the main contributors to the flexural capacity about the strong axis.

$$M_{v,rd} = \left[W_{pl} - \frac{\rho A_v^2}{4t_w} \right] \frac{f_y}{\gamma_{m0}} \quad (7.9)$$

$$\rho = \left[\frac{2V_{sd}}{V_{pl,Rd}} - 1 \right] \quad (7.10)$$

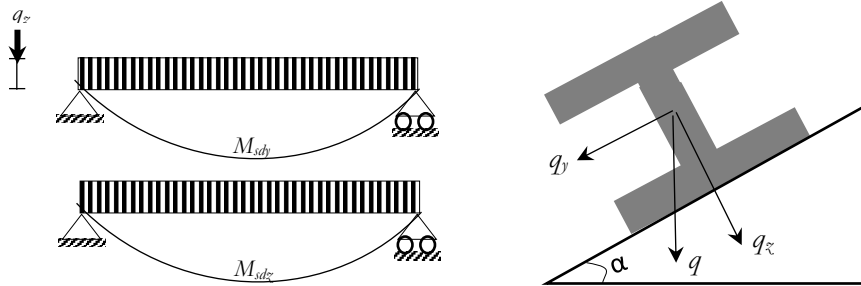
The coefficient ρ increases from zero when $V_{sd} = 50\% V_{pl,Rd}$ to one when $V_{sd} = V_{pl,Rd}$. Thus, it represents the increasing severity of shear-bending interaction. In practical design, the reduced moment resistance $M_{v,Rd}$ must be used instead of the unreduced value $M_{e,Rd}$ whenever the high-shear condition is met.

For other cases:

$$M_{v,rd} = \left[W_{pl}(1 - \rho) \right] \frac{f_y}{\gamma_{m0}} \quad (7.11)$$

7.6. Biaxial bending:

Biaxial bending occurs when a cross-section is subjected to bending moments about both principal axes (see Figure 7.2). This situation may result from eccentric loading, inclined transverse loads, unsymmetrical support conditions, spatial frames, secondary effects, or load transfer through connections that do not pass through the shear center. In biaxial bending, the stress distribution is the superposition of the effects produced by each moment component.



$$M_{sdy} = q_z \frac{l^2}{8} \quad M_{sdz} = q_y \frac{l^2}{8} / \quad q_z = q \cos \alpha, \quad q_y = q \sin \alpha$$

Figure 7.2. Biaxial bending of a member subjected to bending moments about both principal axes.

The resistance criterion is expressed as an interaction equation rather than by checking each moment independently. This is necessary because the same cross-section must resist both bending components simultaneously. The general design philosophy is to ensure that the combined utilization remains less than or equal to unity.

7.6.1. Interaction criterion for Class 1 and Class 2 sections:

For Class 1 and Class 2 cross-sections, the interaction can be based on plastic bending resistances about the two axes. The general form is:

$$\frac{M_{sdy}}{\frac{W_{ply}f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{plz}f_y}{\gamma_{m0}}} \leq 1 \quad (7.12)$$

The exponents alpha and beta depend on the type of cross-section and the adopted design rule. They account for the shape of the interaction surface between major-axis and minor-axis plastic bending resistance. A conservative simplified form may use linear interaction when detailed parameters are not available.

7.6.2. Interaction criterion for Class 3 sections:

For Class 3 sections, the elastic limit controls the verification. The combined stress at the most unfavorable fiber should not exceed the design yield stress. In practice, this leads to an interaction based on elastic bending resistances:

$$\frac{M_{sdy}}{\frac{W_{ply}f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{plz}f_y}{\gamma_{m0}}} \leq 1 \quad (7.13)$$

This expression reflects the elastic superposition of stresses. Since plastic redistribution is not relied upon, the criterion is generally more restrictive than for compact sections capable of developing plastic resistance.

7.6.3. Interaction criterion for Class 4 sections:

For Class 4 cross-sections, effective properties must be used. The interaction should be based on effective section moduli about the relevant axes, and the shift of the neutral axis may have to be considered when calculating the effective stress distribution. The verification is therefore more sensitive to geometric details than in Class 1, 2, or 3 sections.

$$\frac{M_{sdy}}{\frac{W_{effy} f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{effz} f_y}{\gamma_{m0}}} \leq 1 \quad (7.14)$$

Note. For Class 4 sections, the design resistance may vary depending on which plate elements are in compression. The sign and distribution of bending moments can therefore influence the effective section properties.

7.7. Conclusion:

Simple bending design in steel structures requires a rational sequence of verifications. The applied shear force is compared with the plastic shear resistance, while the applied bending moment is compared with the bending resistance of the cross-section. The bending resistance depends directly on the class of the section: plastic modulus for Class 1 and 2 sections, elastic modulus for Class 3 sections, and effective modulus for Class 4 sections. When shear is high, the bending resistance must be reduced to reflect shear-moment interaction. If bending takes place about both principal axes, an interaction criterion is required to verify the combined effect. These principles form the basis for safe and consistent design of steel beams and flexural members.

7.8. Application Exercises:

This part presents a set of application exercises devoted to the verification and preliminary design of steel beams subjected to simple bending and, where applicable, oblique or biaxial bending. The proposed work emphasizes the identification of the bending axis, load decomposition, evaluation of design bending moments, selection of the relevant IPE section properties, and verification of the design bending resistance of Class 1 rolled steel sections. Unless additional instability data are provided in the original statement, the verification is focused on the cross-section bending resistance and associated shear checks.

Exercise No. 01

Determine whether the simply supported steel beam shown in the original statement satisfies the bending resistance requirement.

The member is a rolled IPE 400 beam made of structural steel with $f_y = 235 \text{ N/mm}^2$. The beam is subjected to a uniformly distributed load $q = 20 \text{ kN/m}$ over a span $L = 8.0 \text{ m}$. The cross-section is classified as Class 1 (Figure 7.3).

The objective is to evaluate the design bending action produced by the distributed load and compare it with the design bending resistance of the IPE 400 cross-section.

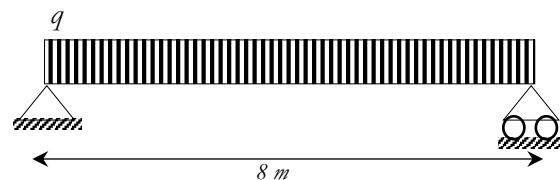


Figure 7.3. Steel beam subjected to a uniformly distributed load.

Expected technical work:

- Idealize the beam according to the support and loading conditions shown in the statement.
- Draw or establish the shear-force and bending-moment diagrams for the uniformly loaded span.
- Determine the maximum design bending moment M_{sd} and the maximum design shear force V_{sd} .

- Extract the relevant IPE 400 section properties from steel tables, mainly $W_{pl,y}$ and the shear area $A_{v,z}$ when a shear check is required.
- For a Class 1 section, calculate the plastic design bending resistance $M_{pl,y,Rd}$.
- Check whether the shear force is low enough to neglect bending-shear interaction; if not, use the reduced bending resistance required by the adopted code.
- Verify the resistance condition $M_{sd} \leq M_{y,Rd}$ and state whether the IPE 400 beam is adequate.

Exercise No. 02

Design a steel beam within the IPE series and verify its resistance. The beam is subjected to an oblique load q , producing bending effects about both principal axes.

The original statement indicates a total span composed of two 4.0 m segments, steel grade $f_y = 235 \text{ N/mm}^2$, and the bending components M_{sdy} and M_{sdz} associated with the load components q_y and q_z (Figure 7.4).

The objective is to select a suitable rolled IPE profile by considering bending about the major and minor principal axes and then verify the chosen cross-section under combined bending.

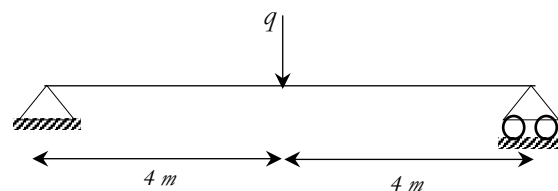


Figure 7.4. Steel beam subjected to an oblique load.

Expected technical work:

- Identify the beam span, support conditions, load direction and the principal axes y-y and z-z from the statement figure.
- Resolve the oblique load q into its components q_y and q_z according to the adopted axis convention and the angle α .
- Calculate the design bending moments about both axes: M_{sdy} due to the load component acting in the z direction, and M_{sdz} due to the load component acting in the y direction.

- Perform a preliminary design by estimating the required section moduli $W_{ply,req}$ and $W_{plz,req}$ from the calculated bending moments.
- Select the first IPE section that satisfies the required section modulus criteria, then check whether the section class permits the use of plastic resistance.
- Calculate the design bending resistances $M_{ply,Rd}$ and $M_{plz,Rd}$ for the selected IPE section.
- Check the combined bending condition.
- Check shear resistance if the shear forces are significant and confirm whether bending-shear interaction affects the result.
- Conclude whether the selected IPE profile is sufficient; if it is not sufficient, choose the next larger IPE profile and repeat the verification.

Exercise No. 03

Verify the resistance of a rolled IPE 270 steel beam subjected to an inclined uniformly distributed load. The span is $L = 10.0$ m, the steel yield strength is $f_y = 235$ N/mm², the load intensity is $q = 50$ kN/m, and the load is inclined at $\alpha = 30^\circ$. The cross-section is classified as Class 1 (Figure 7.5).

The objective is to verify the IPE 270 section under oblique bending by decomposing the applied load into components acting along the principal directions and checking the corresponding bending resistances.

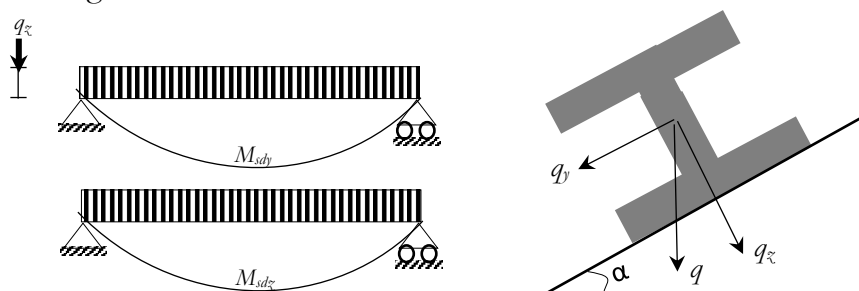


Figure 7.5. Steel beam subjected to an inclined uniformly distributed load.

Expected technical work:

- Define the principal axes of the IPE 270 section and establish the direction of the inclined load q .

- Resolve the load into its components q_y and q_z using $\alpha = 30^\circ$ and the course sign convention.
- Determine the maximum design bending moments $M_{sd,y}$ and $M_{sd,z}$ produced by the corresponding load components over the 10.0 m span.
- Extract from steel tables the relevant properties of IPE 270: W_{ply} , W_{plz} , and shear areas if needed.
- For the Class 1 cross-section, calculate the design plastic bending resistances $M_{ply,Rd}$ and $M_{plz,Rd}$.
- Verify the beam under combined bending using the interaction rule specified in class.
- Verify shear resistance and assess whether bending-shear interaction is relevant.
- State whether the IPE 270 beam is adequate for the applied inclined load; if not, indicate that a larger section or additional lateral support is required.

Chapter 8

Combined Bending (buckling with bending)

Learning objectives:

- ↳ At the end of this chapter, the student should be able to identify members subjected to simultaneous axial force and bending, understand second-order effects and the interaction between internal forces, distinguish the verification formats used for different section classes, and apply combined-resistance checks for beam-columns in steel structures.

8.1. Introduction:

A structural member is said to be subjected to combined bending, or beam-column action, when it resists at the same time an axial compressive force and a bending moment (Figure 8.1). This loading condition is particularly important in steel structures because axial compression reduces the ability of the member to resist lateral displacement, whereas bending generates a non-uniform stress distribution over the cross-section. The result is a coupled structural response in which resistance, stiffness, and stability can no longer be assessed independently.

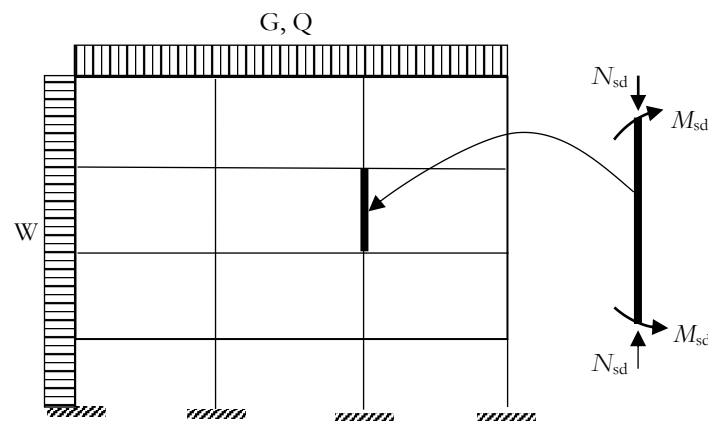


Figure 8.1. Columns subjected to combined bending (buckling with bending) [7].

The verification of members under buckling with bending therefore extends the simple compression problem studied previously. In pure compression, the designer checks the resistance of the section and the susceptibility of the member to global buckling. Under combined bending, however, the situation becomes more complex because the design must take into account the interaction between axial force, bending moment, member slenderness, cross-section class, and the real distribution of moments along the member length.

In practical steel construction, members subjected to combined axial compression and bending are mainly columns, portal-frame members, and compression members carrying eccentric loading. Such elements may be loaded simultaneously by gravity actions, wind, secondary bending, or frame effects arising from continuity. For this reason, the verification of beam-columns is one of the central topics in steel design, since it directly influences safety, service performance, and the overall economy of the structure.

8.2. Verification of members under buckling with bending:

Members subjected to combined axial compression and bending must satisfy interaction criteria that depend on the class of the cross-section and on the distribution of internal forces. In design practice, the verification is not performed by checking axial resistance and bending resistance separately. Instead, code-based interaction relations are used to ensure that the combined demand produced by normal force and bending moment remains lower than the available member resistance.

The classification of the section plays a decisive role in this verification. Compact sections may develop plastic resistance, semi-compact sections are generally verified on the basis of elastic resistance, and slender sections must be treated through effective section properties because of local buckling effects. Consequently, the resistance format adopted for Class 1 and 2 sections is not identical to that used for Class 3 or Class 4 sections.

8.2.1. Combined bending with an axial compressive force:

This first case concerns members subjected to axial compression combined with bending about a single principal axis. The stress field in the cross-section results from the superposition of a uniform compressive stress due to the design axial force N_{sd} and a linear stress variation due to the design bending moment. The most compressed fibers are therefore the most critical zone for design, especially when member slenderness amplifies second-order effects.

- **Class 1 and 2:**

The verification is expressed by an interaction criterion combining the ratio of the design axial force to the available compressive resistance with the ratio of the design bending moment to the available bending resistance.

$$\frac{N_{sd}}{\chi_{\min} \frac{Af_y}{\gamma_{m0}}} + \frac{k_y M_{sdy}}{\frac{W_{ply} f_y}{\gamma_{m0}}} \leq 1 \quad (8.1)$$

Since these sections possess sufficient plastic deformation capacity, the design model may rely on plastic resistance concepts and on a redistribution of stresses within the section.

In this interaction expression, the coefficient k_y acts as a modification factor accounting for the influence of axial compression on flexural resistance. Its role is to reduce the admissible bending moment when the level of compression increases.

$$k_y = 1 - \frac{\mu_y N_{sd}}{\chi_y A f_y} \leq 1.50, \quad \mu_y = \bar{\lambda}_y (2\beta_{My} - 4) + \frac{W_{ply} - W_{ely}}{W_{ely}} \leq 0.9 \quad (8.2)$$

The factor β_{My} is introduced to account for the actual distribution of bending moments along the member length, because a non-uniform bending diagram does not affect stability in the same way as a uniform moment diagram.

- k_y : modification factor.
- β_{My} : uniform moment factor representing the influence of the moment diagram shape.
- N_{sd} : design axial compressive force in the member.
- M_{sd} : design bending moment acting about the relevant principal axis.

- **Class 3:**

For Class 3 cross-sections, the general verification philosophy remains the same, but the bending resistance is evaluated using elastic section properties. These sections do not possess sufficient ductility to justify a fully plastic stress distribution in the compressed zone. The design approach is therefore more restrictive than for Class 1 and 2 sections, even if the member geometry and loading appear similar.

From an engineering point of view, this means that Class 3 sections are still suitable for many structural applications, but they must be checked with greater attention to elastic stress limits. The interaction criterion continues to combine axial force and bending, yet the available moment resistance is lower than that of an equivalent compact section.

$$\frac{N_{sd}}{\chi_{\min} \frac{A f_y}{\gamma_{m0}}} + \frac{k_y M_{sd}}{\frac{W_{ely} f_y}{\gamma_{m0}}} \leq 1 \quad (8.3)$$

$$k_y = 1 - \frac{\mu_y N_{sd}}{\chi_y A f_y} \leq 1.50 \quad (8.4)$$

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) \leq 0.9 \quad (8.5)$$

• **Class 4:**

In this class, the verification must be performed on the basis of effective section properties. This is because local buckling may occur in the slender plate elements of the web or flange before the gross section can fully develop its nominal resistance. As a result, the effective area and the effective section modulus are used instead of the gross properties.

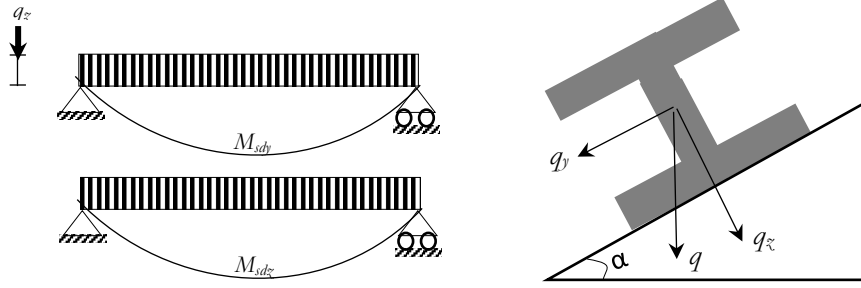
$$\frac{N_{sd}}{\chi_{\min} \frac{A_{eff} f_y}{\gamma_{m0}}} + \frac{k_y (M_{sdy} + N_{sd} \times e_{NZ})}{\frac{W_{effy} f_y}{\gamma_{m0}}} \leq 1 \quad (8.6)$$

The effective section modulus W_{eff} represents the reduced flexural capacity of the section after allowance for local instability. In addition, the term $(N_{sd} \times e_{NZ})$ accounts for the additional moment caused by the offset between the neutral axis of the gross section and that of the effective section. This term is particularly important for slender sections, because local buckling modifies the distribution of stress and shifts the effective centroidal properties used in design.

The use of effective properties provides a rational representation of local plate instability. It does not change the global scope of the design problem, but it makes the verification more realistic for thin-walled members and for sections in which the compressed parts are especially sensitive to local buckling

8.2.2. Combined deviated bending with an axial compressive force :

The second case corresponds to biaxial or deviated bending, where the member is subjected simultaneously to an axial compressive force and to bending moments about both principal axes. This situation is common in spatial frames, corner columns, industrial structures, and members exposed to accidental eccentricities or unsymmetrical load transfer (Figure 8.2).



$$M_{sdy} = q_z \frac{l^2}{8} \quad M_{sdz} = q_y \frac{l^2}{8} \quad q_z = q \cos \alpha, \quad q_y = q \sin \alpha$$

Figure 8.2. Biaxial (deviated bending).

The resistance check is more severe in this configuration because the cross-section mobilizes axial resistance and flexural resistance about two orthogonal directions at the same time. In practice, the design is based on interaction equations in which the contribution of the y-axis bending and the z-axis bending are both taken into account.

- **Class 1 and 2:**

For these class sections, the interaction criterion combines the design axial force with the design bending moments about the two principal axes. Because these sections can develop plastic resistance, the design equations allow the beneficial effect of plastic stress redistribution to be considered, provided the code conditions for compact behavior are satisfied.

The important point is that biaxial resistance cannot be checked by two independent uniaxial verifications. The resistance consumed about one axis is no longer fully available about the other axis. The design must therefore be treated as a coupled problem in which all internal actions contribute simultaneously to the safety assessment.

$$\frac{N_{sd}}{\chi_{\min} \frac{A f_y}{\gamma_{m0}}} + \frac{k_y M_{sdy}}{\frac{W_{ply} f_y}{\gamma_{m0}}} + \frac{k_z M_{sdz}}{\frac{W_{plz} f_y}{\gamma_{m0}}} \leq 1 \quad (8.7)$$

$$\text{Where: } k_y = 1 - \frac{\mu_y N_{sd}}{\chi_y A f_y} \leq 1.50, \quad k_z = 1 - \frac{\mu_z N_{sd}}{\chi_z A f_y} \leq 1.50 \quad (8.8)$$

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) + \frac{W_{ply} - W_{ely}}{W_{ely}} \leq 0.9, \quad \mu_z = \bar{\lambda}_z (2\beta_{Mz} - 4) + \frac{W_{plz} - W_{elz}}{W_{elz}} \leq 0.9 \quad (8.9)$$

- **Class 3:**

For Class 3 sections, the same coupled verification format is adopted, but the bending resistances about the principal axes are based on elastic section properties. This approach reflects the lower deformation capacity of the section and the impossibility of mobilizing a fully plastic stress block.

In many practical cases, the weaker principal axis governs the design, especially for open rolled sections with significantly different moments of inertia about the two axes. It is therefore essential to identify the critical axis and to interpret the interaction equations with regard to the real geometry of the section.

$$\frac{N_{sd}}{\chi_{\min} \frac{Af_y}{\gamma_{m0}}} + \frac{k_y M_{sdy}}{\frac{W_{ely} f_y}{\gamma_{m0}}} + \frac{k_z M_{sdz}}{\frac{W_{elz} f_y}{\gamma_{m0}}} \leq 1 \quad (8.10)$$

$$\text{Where: } k_y = 1 - \frac{\mu_y N_{sd}}{\chi_y Af_y} \leq 1.50, \quad k_z = 1 - \frac{\mu_z N_{sd}}{\chi_z Af_y} \leq 1.50 \quad (8.11)$$

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) \leq 0.9, \quad \mu_z = \bar{\lambda}_z (2\beta_{Mz} - 4) \leq 0.9 \quad (8.12)$$

- **Class 4:**

For Class 4 sections, the interaction check must again be based on effective section properties. Local buckling reduces the effective width of the compressed elements and therefore reduces both the available axial resistance and the flexural resistance about one or both principal axes.

The designer must consequently consider two instability mechanisms together: first, the global reduction in member resistance due to axial force combined with biaxial bending; and second, the local reduction in section efficiency caused by slender plate elements. This makes Class 4 members the most demanding case in combined bending design.

$$\frac{N_{sd}}{\chi_{\min} \frac{A_{eff} f_y}{\gamma_{m0}}} + \frac{k_y (M_{sdy} + N_{sd} \times e_{NZ})}{\frac{W_{effy} f_y}{\gamma_{m0}}} + \frac{k_z (M_{sdz} + N_{sd} \times e_{NZ})}{\frac{W_{effz} f_y}{\gamma_{m0}}} \leq 1 \quad (8.13)$$

8.3. Uniform moment factor β_{My} :

The factor β_{My} is introduced to represent the influence of the moment diagram shape along the member. Two members may have the same maximum bending moment, yet not the same stability behavior, if one is subjected to uniform end moments while the other is loaded by transverse actions producing a non-uniform moment distribution. For this reason, the factor β_{My} is essential in a realistic beam-column verification.

8.3.1. Case of moments at the extremities (end moments):

When the bending action is produced only by end moments, the value of β_{My} depends mainly on the ratio and sign of the moments applied at the two ends (Figure 8.3). This distinction separates single-curvature bending from reverse-curvature bending. A member bent in reverse curvature is generally less critical from the viewpoint of instability than a member bent entirely in one direction.

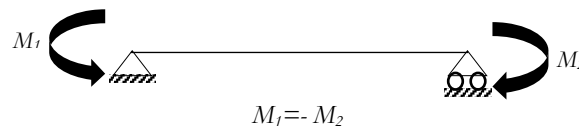


Figure 8.3. Moments at the extremities.

$$M_1 = \psi M_2 \Rightarrow \psi = \frac{M_1}{M_2}, \quad \text{with : } -1 \leq \psi \leq 1 \quad (8.14)$$

$$\beta_{M_\psi} = 1.8\psi - 1.7$$

8.3.2. Case of transverse loads:

When the member is subjected to transverse loads, the bending moment varies continuously along the span (Figure 8.4). The corresponding value of β_{My} reflects this non-uniform shape of the bending moment diagram and allows the designer to avoid using an overly simplified uniform-moment assumption. This leads to a more accurate estimation of the severity of the instability problem.

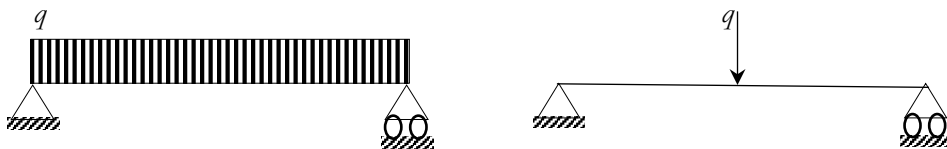


Figure 8.4. Transverse loads.

$$\beta_{M_Q} = 1.3 \quad (8.15)$$

8.3.3. Case of transverse loads combined with end moments:

In real structures, transverse loads and end moments frequently act together (Figure 8.5). The final bending moment diagram is then obtained by superposition of the moment caused by the transverse load and that caused by the end moments. The design value of β_{My} must therefore be determined from the combined distribution, because it is this real distribution that governs the interaction between bending and axial compression.

In this context, $M_{\max}$ denotes the maximum bending moment produced by the transverse load Q acting alone. It is used to characterize the contribution of the distributed loading when evaluating the actual curvature form of the member and the corresponding instability sensitivity.

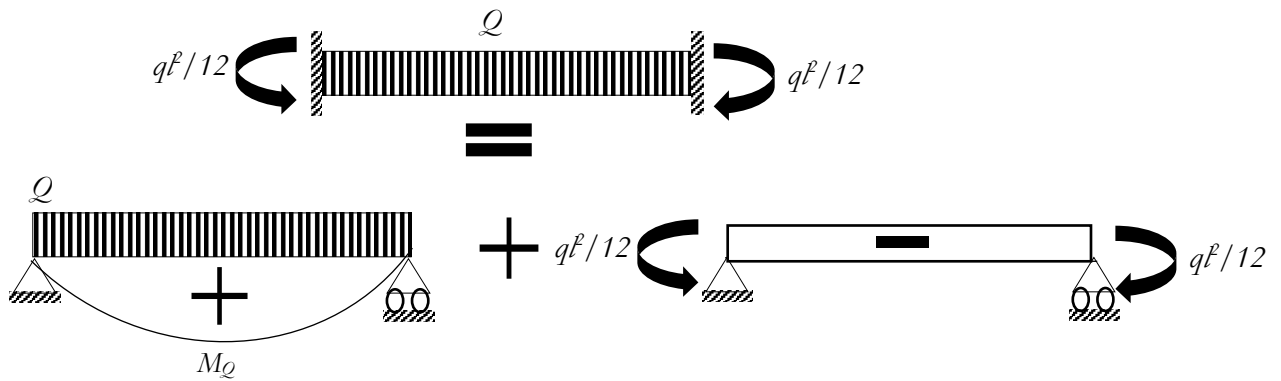


Figure 8.5. Transverse loads combined with end moments.

$$\beta_M = \beta_{M_\psi} + \frac{M_Q}{\Delta M} (\beta_{M_Q} - \beta_{M_\psi}) \quad (8.16)$$

$M_Q = |M_{\max}|$ is the maximum bending moment caused by the transverse load Q alone.

$$\Delta M = |M_{\max}^-| + |M_{\max}^+|$$

8.4. Profile classes used in bending:

The profile classes used in bending are established from the width-to-thickness ratios of the compressed web and flange elements. These limits are essential because they indicate whether the section can develop plastic resistance, whether it must be limited to elastic resistance, or whether it should be treated as a slender section with effective properties (see Table 8.1 and Figure 8.6).

In design practice, the classification of the section is a preliminary stage before applying the interaction criteria presented above. An incorrect class assignment may lead

either to an unsafe overestimation of resistance or to an unnecessarily conservative member design.

Table 8.1. Profile classes in bending

	Class 1	Class 2	Class 3	Class 4	$\varepsilon = \sqrt{\frac{235}{f_y}}$
Web « d / t_w »	72ε	38ε	42ε	$> 42 \varepsilon$	
Flange « c / t_f »	10ε	83ε	124ε	$> 124 \varepsilon$	

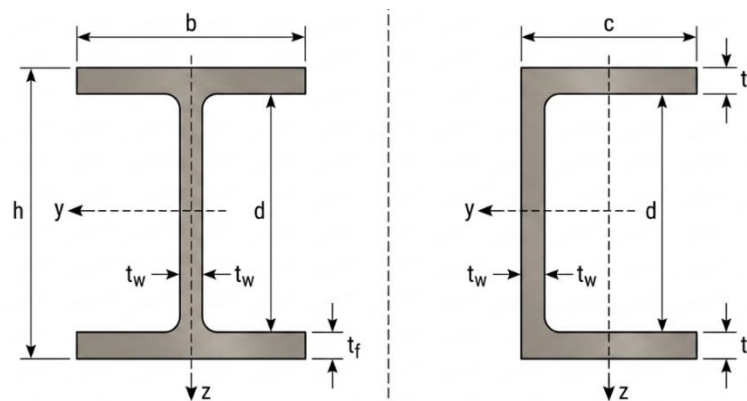


Figure 8.6. Cross-section class limits in bending [8].

Overall, the design of beam-columns requires a unified treatment of axial compression, bending, cross-section classification, and moment distribution. The concepts presented in this chapter provide the basis for understanding the behavior of steel members under combined internal actions and for applying the appropriate code-based resistance checks in engineering practice.

8.5. Conclusion:

Combined bending shows that steel-member design cannot always be separated into isolated checks for axial force and bending moment. When both actions act simultaneously, the resistance depends on section class, member slenderness, buckling mode, moment distribution, and the interaction between forces. A reliable design therefore requires a global interpretation of member behavior and the use of appropriate interaction criteria.

8.6. Application Exercises:

This part presents a set of application exercises devoted to the verification of steel members subjected to combined bending. The proposed work emphasizes the identification of the governing internal forces, the distinction between simple bending and deviated bending, the evaluation of cross-sectional resistance according to the relevant section class, and the application of appropriate interaction criteria for members subjected to compression combined with bending moments about one or both principal axes.

Exercise No. 01

Consider a steel column made of an HEB 200 section and subjected to a compressive force together with transverse loading, as represented in Figure 8.7.

$f_y = 235 \text{ N/mm}^2$, Axial force $N = 60 \text{ kN}$, Transverse loading $q = 15 \text{ kN}$

Cross-section class 1.

Verify the strength of the column by determining the design effects induced by the loading system and by checking the appropriate combined bending interaction criterion for a Class 1 section.

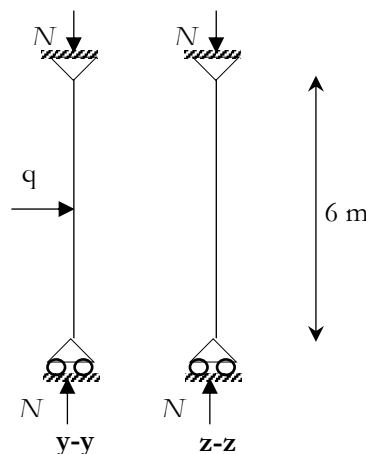


Figure 8.7. Column member under a compressive force with transverse loading.

Expected technical work:

- Identify the type of combined loading acting on the HEB 200 member and determine the principal bending axis involved.

- Evaluate the design bending moment generated by the transverse loading according to the support and loading conditions represented in the Figure 8.7.
- Determine the design plastic compression resistance of the Class 1 cross-section using the relevant section properties.
- Calculate the design plastic bending resistance about the governing principal axis.
- Apply the appropriate interaction criterion for a Class 1 section subjected to axial compression and bending about one principal axis.
- Conclude whether the HEB 200 member satisfies the required resistance condition under the combined action of compression and bending.

Exercise No. 02

A second column is formed by a HEB 300 section and is subjected to axial compression $N = 100$ kN together with a transverse loading $q = 25$ kN that generates bending effects requiring a combined check (Figure 8.8).

$$f_y = 235 \text{ N/mm}^2,$$

Verify the strength of the member using the interaction criterion corresponding to Class 2 sections.

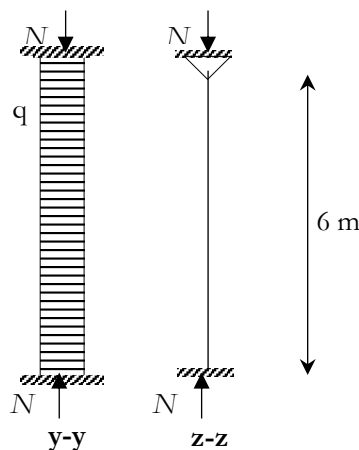


Figure 8.8. Column member under a compressive force with distributed loading.

Expected technical work:

- Identify the nature of the loading acting on the HEB 300 member and distinguish between axial compression and bending effects.

- Determine the design bending moment produced by the transverse loading and identify the axis about which bending is developed.
- Evaluate the plastic compression resistance of the Class 2 cross-section.
- Calculate the plastic bending resistance using the appropriate plastic section modulus of the HEB 300 profile.
- Apply the interaction criterion corresponding to a Class 2 section subjected to combined axial compression and bending.
- Interpret the result of the interaction check and state whether the member satisfies the resistance requirement.

Exercise No. 03

Consider a member made of an IPE 220 section. The element is subjected to both axial compression $N = 40$ kN and distributed loading $q = 15$ kN/m, and the statement additionally specifies an angle $\alpha = 5^\circ$, which introduces a directional effect that must be considered when interpreting the bending action (Figure 8.9).

$$f_y = 235 \text{ N/mm}^2,$$

Verify the resistance of the element under combined bending and compression using the interaction criterion corresponding to Class 1 sections.

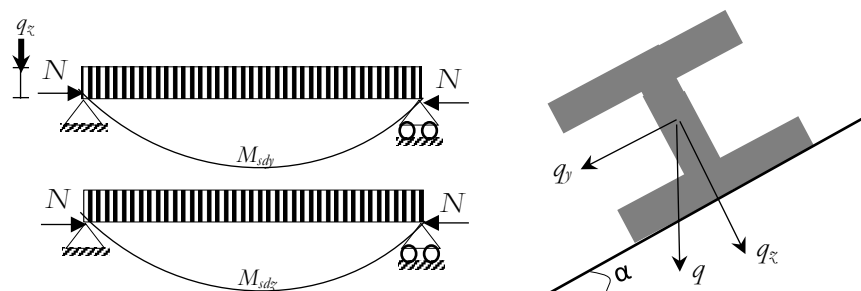


Figure 8.9. Beam subjected to axial compression and distributed loading about both principal axes.

Expected technical work:

- Interpret the inclined loading system and explain the role of the angle α in producing bending about both principal axes.
- Resolve the applied distributed load into its components along the principal directions of the cross-section.

- Determine the design bending moments M_{sdy} and M_{sdz} generated by the corresponding load components.
- Evaluate the plastic compression resistance of the Class 1 IPE 220 cross-section.
- Calculate the plastic bending resistances about both principal axes using W_{ply} and W_{plz} .
- Apply the appropriate interaction criterion for a Class 1 section subjected to axial compression and biaxial bending.
- Conclude on the adequacy of the IPE 220 member by identifying whether the combined compression and deviated bending effects remain within the available design resistance.

Chapter 9

Lateral-Torsional Buckling

Learning objectives:

- ✦ At the end of this chapter, the student should be able to describe lateral-torsional buckling of laterally unrestrained beams, interpret the role of the elastic critical moment, evaluate the reduced slenderness and reduction factor, and apply stability checks for members subjected to simple or combined bending.

9.1. Introduction:

This chapter presents the fundamental concepts used to assess the lateral-torsional stability of steel members subjected to bending, with particular emphasis on the elastic critical moment, the reduction factor associated with instability, and the verification of members in simple bending, biaxial bending, and combined bending-compression.

9.2. Description of the phenomenon:

Lateral-torsional buckling (LTB) is a global instability mode that affects laterally unrestrained beams when the bending resistance of the cross-section is coupled with insufficient torsional and lateral stiffness. In practical steel design, the phenomenon is especially important for open thin-walled sections, such as I- and H-sections, because these profiles possess high flexural stiffness about their major axis but comparatively limited torsional rigidity.

When a beam is subjected to a bending moment M_{sdy} about its major axis, one flange is placed in compression while the opposite flange is placed in tension. The compressed flange tends to move laterally away from the original plane of bending. Since the two flanges are connected by the web and are separated by a finite distance, this lateral displacement cannot develop independently; it is accompanied by a twist of the member about its longitudinal axis. The instability therefore results from the interaction between lateral translation and torsional rotation (Figure 9.1).

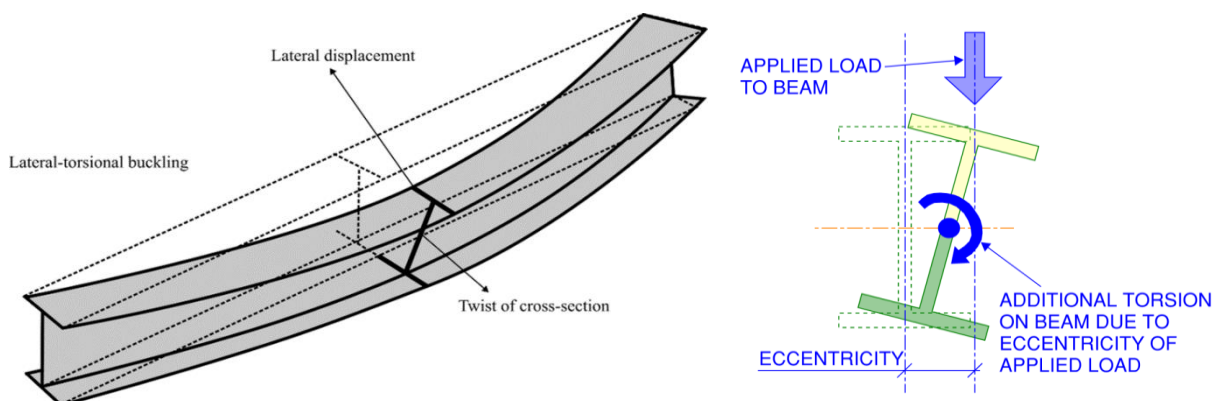


Figure 9.1. Description of LTB [1].

From a mechanical point of view, LTB may develop before the steel reaches its yield strength. For that reason, the design of an unrestrained beam cannot be based solely on cross-section strength. Stability must also be checked, particularly when the member has a long-unbraced length, when the load is applied above the shear center, or when the cross-section is open and slender.

The sensitivity of a beam to LTB is governed mainly by the following parameters: the unbraced length of the member, the torsional constant I_t , the warping constant I_w , the minor-axis second moment of area I_y , the loading arrangement, the end-restraint conditions, and the shape of the bending-moment diagram.

Consequently, two beams with the same steel grade and the same plastic section modulus may exhibit very different stability behavior if their restraint or loading conditions differ.

Remark: Lateral-torsional buckling primarily concerns beams bent about their major (strong) axis, while the resulting instability displacement occurs laterally, generally in the minor-axis direction, together with twist.

9.3. Elastic critical moment M_{cr} :

The elastic critical moment M_{cr} is the theoretical moment at which a perfectly straight, perfectly elastic member with ideal support conditions loses lateral-torsional stability. It plays the same conceptual role in beam stability that Euler's critical load plays in column buckling.

In design, M_{cr} is not used directly as the design resistance; rather, it serves as the reference quantity from which the reduced slenderness and the instability reduction factor are derived.

For a member with a constant and doubly symmetric cross-section, the general expression of the elastic critical moment may be written in the following form:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_z} + \frac{(KL)^2 GI_t}{\pi^2 EI_z} + (C_2 Z_g - C_3 Z_j)^2 \right]^{0.5} - (C_2 Z_g - C_3 Z_j) \right\} \quad (9.1)$$

In this expression, the coefficients C_1 , C_2 , and C_3 depend on the shape of the bending-moment diagram and on the boundary conditions of the member. Their role is to account

for the beneficial or unfavourable effect of non-uniform bending and end restraint on the onset of instability.

K : coefficient related to the in-plane rotational restraint at the beam ends. In many practical cases: $K = 0.5$ for fully fixed ends, $K = 0.7$ for one fixed end and one simply supported end, and $K = 1.0$ for simple supports (see Figure 9.2).

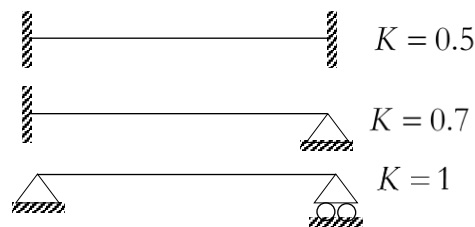


Figure 9.2. Coefficient K .

K_w : coefficient associated with warping restraint at the ends; in many ordinary applications it is taken as $K_w = 1.0$.

L : lateral-torsional buckling length, that is, the distance between adjacent effective lateral restraints.

I_w : warping constant of the cross-section, which reflects the resistance of the member to restrained torsional deformation.

I_t : Saint-Venant torsional constant, which characterises the pure torsional stiffness of the section.

E : modulus of elasticity of steel, commonly taken as $E = 210,000 \text{ N/mm}^2$.

ν : Poisson's ratio of steel, generally $\nu = 0.30$.

G : shear modulus of steel, $G = E / [2(1 + \nu)]$, approximately $80,770 \text{ N/mm}^2$.

$I_{\tilde{x}}$: minor-axis second moment of area of the section, which governs the lateral flexural stiffness.

Z_g : distance between the shear center and the point of load application; when the load is applied above the shear center, the member generally becomes less stable (Figure 9.3).

$$Z_g = Z_a - Z_s \quad (9.2)$$

Z_a and Z_s : coordinates of the load application point and the shear center, respectively.

$$\begin{aligned}
Z_j &= 0.8(2\beta_f - 1)\frac{h_s}{2} \rightarrow \beta_f > 0.5 \\
Z_j &= (2\beta_f - 1)\frac{h_s}{2} \rightarrow \beta_f < 0.5 \quad , \quad \beta_f = \frac{I_{fc}}{I_{fc} + I_{ft}} \\
Z_j &= 0 \rightarrow \beta_f = 0.5
\end{aligned} \tag{9.3}$$

I_{fc} and I_{ft} : second moments of area of the compressed and tension flanges, used in formulations that distinguish the contribution of each flange.

h_s : distance between the centroids of the two flanges.

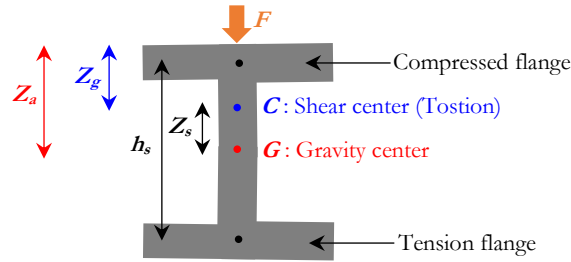


Figure 9.3. Representation of LTB distances [6].

For the common case of a doubly symmetric section loaded through its shear center, the preceding relationship becomes simpler because $Z_g = 0$ and $Z_j = 0$. The expression may then be written as:

$$M_{cr} = C_1 \frac{\pi^2 EI_Z}{(KL)^2} \left\{ \left[\left(\frac{K}{K_w} \right)^2 \frac{I_w}{I_Z} + \frac{(KL)^2 GI_t}{\pi^2 EI_Z} \right]^{0.5} \right\} \tag{9.4}$$

This formula clearly shows that the elastic critical moment increases with flexural stiffness about the minor axis, torsional stiffness, and warping stiffness, while it decreases rapidly as the unbraced length L increases. It also shows why open sections are generally more vulnerable than closed sections: their torsional stiffness is comparatively low, which reduces M_{cr} and therefore lowers the stability reserve of the beam.

9.4. Verification of the lateral-torsional buckling strength of the beam:

9.4.1. Member in simple bending:

For a beam subjected to simple bending about its major axis and not continuously restrained laterally (Figure 9.4), the verification against LTB is expressed by comparing the

applied design bending moment with the design buckling resistance moment of the member:

$$M_{sdy} \leq M_{brdy} \quad (9.5)$$

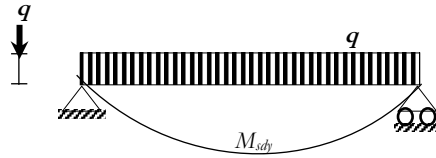


Figure 9.4. Beam subjected to simple bending.

9.4.1.1. Lateral-torsional buckling resistance moment:

The design buckling resistance moment of the member may be written in the following form:

$$M_{brdy} = \chi_{LT} \beta_W \left(\frac{W_{ply} f_y}{\gamma_{m0}} \right) \quad (9.6)$$

In this expression, χ_{LT} is the reduction factor for lateral-torsional buckling, β_W is a coefficient related to the class of the cross-section, W_{ply} denotes the appropriate section modulus for bending about the major axis, f_y is the yield strength, and γ_{m0} is the partial safety factor associated with member stability.

The factor β_W depends on the class of the cross-section, because the available bending resistance differs according to whether the section may develop plastic resistance, only elastic resistance, or an effective resistance reduced by local buckling. The usual values are summarised in Table 9.1.

Table 9.1. Usual values of the factor β_W [4]

Section class	β_W
Class 1 and 2	$\beta_W = 1$
Class 3	$\beta_W = \frac{W_{ely}}{W_{ply}}$

Section class	β_w
Class 4	$\beta_w = \frac{W_{effy}}{W_{ply}}$

9.4.1.2. Reduced slenderness:

The reduced lateral-torsional slenderness $\overline{\lambda}_{LT}$ is defined by comparing the reference bending resistance of the cross-section with the elastic critical moment. It may be written as:

$$\overline{\lambda}_{LT} = \sqrt{\beta_w \frac{W_{ply} f_y}{M_{cr}}} \quad (9.7)$$

A low value of $\overline{\lambda}_{LT}$ indicates a stocky member with a limited sensitivity to instability, whereas a high value indicates a slender member for which lateral-torsional buckling is likely to govern the design. In practical terms, the reduced slenderness provides the link between the elastic instability analysis and the design resistance check used in member verification.

Design note:

If the member is adequately laterally restrained along its length ($\overline{\lambda}_{LT} \leq 0.4$), the risk of lateral-torsional buckling may be neglected and the verification can be limited to cross-section resistance.

If the member is unrestrained ($\overline{\lambda}_{LT} > 0.4$), the risk of lateral-torsional buckling is considered and the reduction factor χ_{LT} must be evaluated.

9.4.1.3. Reduction factor:

The reduction factor χ_{LT} for lateral-torsional buckling is obtained from the standard member-buckling format used in steel design. It is expressed as:

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \quad (9.8)$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\overline{\lambda_{LT}} - 0.2) + \overline{\lambda_{LT}}^2 \right] \quad (9.9)$$

The imperfection factor α_{LT} reflects the sensitivity of the member to residual stresses, initial crookedness, and fabrication effects. It depends on the type of cross-section and fabrication process, as shown in the table 9.2.

Table 9.2. The imperfection factor α_{LT} [4]

Profile type	Rolled sections	Welded sections
α_{LT}	0.21	0.49
Buckling curve	Curve a	Curve c

9.4.2. Member in deviated (biaxial) bending:

When the beam is subjected simultaneously to bending about both principal axes, the verification becomes more complex because the major-axis bending component is influenced by LTB while the minor-axis component remains primarily a cross-section and flexural-resistance problem (Figure 9.5). The design is therefore carried out using interaction equations that combine the two bending components and account for the class of the cross-section.

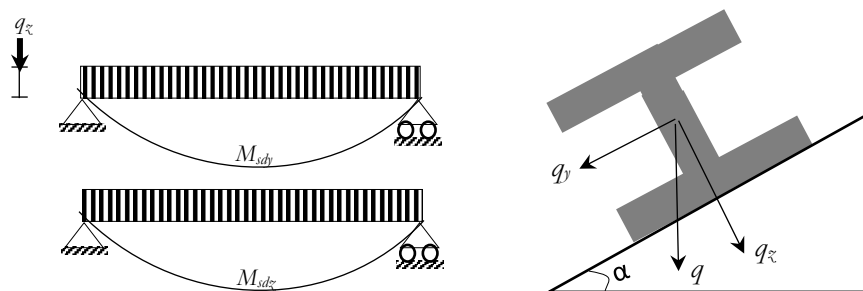


Figure 9.5. Beam subjected to simple bending about both principal axes.

$$M_{sdy} = q_z \frac{l^2}{8} \quad M_{sdz} = q_y \frac{l^2}{8} / \quad q_z = q \cos \alpha, \quad q_y = q \sin \alpha$$

- **Class 1 and 2 :**

The interaction check is usually based on a plastic resistance format:

$$\frac{M_{sdy}}{\chi_{LT} \frac{W_{ply} f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{plz} f_y}{\gamma_{m0}}} \leq 1 \quad (9.10)$$

- **Class 3 :**

The verification is generally performed in an elastic format:

$$\frac{M_{sdy}}{\chi_{LT} \frac{W_{ely} f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{elz} f_y}{\gamma_{m0}}} \leq 1 \quad (9.11)$$

- **Class 4 :**

The effective properties of the section must be used, since local plate slenderness modifies both the section modulus and the location of the neutral axis:

$$\frac{M_{sdy}}{\chi_{LT} \frac{W_{effy} f_y}{\gamma_{m0}}} + \frac{M_{sdz}}{\frac{W_{effz} f_y}{\gamma_{m0}}} \leq 1 \quad (9.12)$$

In engineering terms, biaxial bending requires the designer to ensure that instability about the major axis does not compromise the combined resistance of the member. The major-axis moment is therefore reduced by the factor associated with lateral-torsional buckling, while the minor-axis moment is checked using the corresponding bending resistance of the cross-section.

9.5. Verification of members subjected to bending and axial compression:

In steel structures, many beam-columns are subjected simultaneously to axial compression and bending (Figure 9.6). In such members, the compressive force amplifies instability effects and reduces the bending resistance available at the cross-section level. The design check must therefore account for the interaction between compression, flexural buckling, and lateral-torsional buckling.

The verification is generally performed through interaction equations in which the design axial force N_{sd} and the design bending moments are compared with the corresponding member resistances. These expressions depend on the cross-section class

and on the verification axis considered. In all cases, the engineer must verify that the interaction between axial force and bending remains within the permitted design domain.

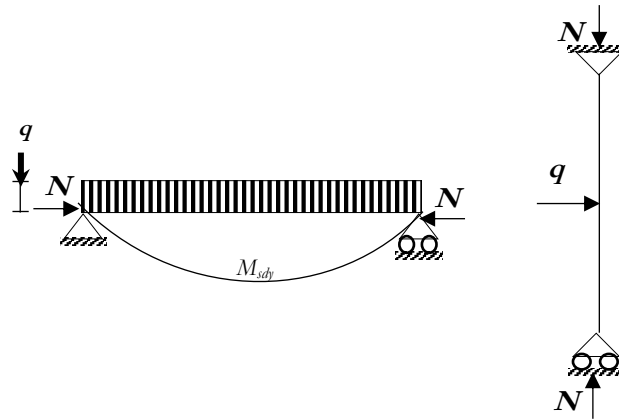


Figure 9.6. Beam-column subjected to axial compression and bending.

9.5.1. Combined bending with axial compression:

- **Class 1 and 2 :**

The verification may be based on plastic interaction criteria, because these sections are capable of developing significant plastic redistribution before local buckling occurs. The combined resistance therefore depends on both the reduction in bending capacity caused by compression and the instability reduction associated with lateral-torsional buckling:

$$\frac{N_{sd}}{\chi_Z \frac{Af_y}{\gamma_{m0}}} + \frac{k_{LT} M_{sdy}}{\chi_{LT} \frac{W_{ply} f_y}{\gamma_{m0}}} \leq 1 \quad (9.13)$$

- **Class 3 :**

The interaction is commonly evaluated using an elastic resistance format. In this case, the stress distribution remains essentially elastic at the design limit state, and the instability verification must ensure that the member does not reach excessive stress or lose stability prematurely:

$$\frac{N_{sd}}{\chi_Z \frac{Af_y}{\gamma_{m0}}} + \frac{k_{LT} M_{sdy}}{\chi_{LT} \frac{W_{ely} f_y}{\gamma_{m0}}} \leq 1 \quad (9.14)$$

- **Class 4 :**

The effective cross-sectional properties must be adopted. This is particularly important because local buckling modifies the effective width of compressed plates, changes the position of the effective neutral axis, and may introduce an additional moment associated with the eccentricity between the gross and effective sections:

$$\frac{N_{sd}}{\chi_Z \frac{A_{eff} f_y}{\gamma_{m2}}} + \frac{k_{LT} M_{sdy} + (N_{sd} \times e_Z)}{\chi_{LT} \frac{W_{eff} f_y}{\gamma_{m0}}} \leq 1 \quad (9.15)$$

The modification factor k_{LT} and the uniform moment factor β_{MLT} are introduced in many design formulations to account for the shape of the bending-moment diagram and the coupling between axial compression and lateral-torsional buckling.

Where k_{LT} represents a modification factor defined as follows:

$$k_{LT} = 1 - \frac{\mu_{LT} N_{sd}}{\chi_Z A f_y} \leq 1, \quad \mu_{LT} = (0.15 \bar{\lambda}_Z \beta_{MLT}) - 0.15 \leq 0.9 \quad (9.16)$$

β_{MLT} : It is a factor of uniform moment taking into account the non-uniformity of the bending moment.

9.5.2. Combined deviated bending with axial compression:

The most general case corresponds to members subjected to axial compression together with bending about both principal axes (Figure 9.7). Such beam-columns are frequent in frame structures, industrial buildings, portal frames, and multi-storey steel skeletons, where imperfections, load eccentricities, wind action, and joint continuity generate simultaneous biaxial bending.

In this situation, the verification must consider the combined effect of N_{sd} , M_{sdy} , and M_{sdz} .

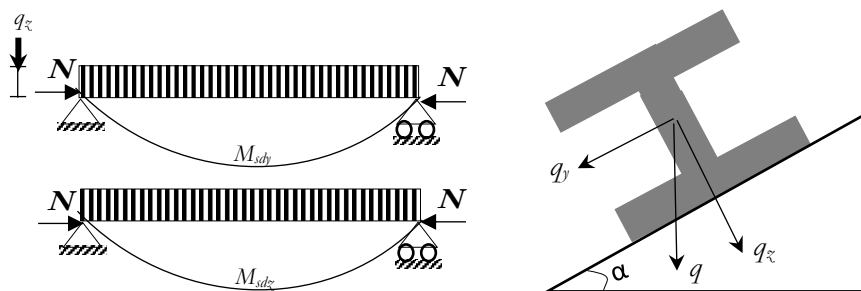


Figure 9.7. Beam-column subjected to axial compression and bending about both principal axes.

The section class remains decisive: plastic interaction is adopted for Class 1 and Class 2 sections, elastic interaction is adopted for Class 3 sections, and effective-section methods are used for Class 4 sections. In addition, second-order effects and global frame imperfections may become significant and should be considered whenever required by the design standard.

- **Class 1 and 2 :**

The resistance criterion is expressed as:

$$\frac{N_{sd}}{\chi_Z \frac{A f_y}{\gamma_{m0}}} + \frac{k_{LT} M_{sdy}}{\chi_{LT} \frac{W_{ply} f_y}{\gamma_{m0}}} + \frac{k_{\tilde{\chi}} M_{sd\tilde{\chi}}}{\frac{W_{pl\tilde{\chi}} f_y}{\gamma_{m0}}} \leq 1 \quad (9.17)$$

$$\text{Where : } k_{LT} = 1 - \frac{\mu_{LT} N_{sd}}{\chi_Z A f_y} \leq 1, \quad k_{\tilde{\chi}} = 1 - \frac{\mu_{\tilde{\chi}} N_{sd}}{\chi_{\tilde{\chi}} A f_y} \leq 1.50 \quad (9.18)$$

$$\mu_{LT} = (0.15 \bar{\lambda}_Z \beta_{MLT}) - 0.15 \leq 0.9, \quad \mu_{\tilde{\chi}} = \bar{\lambda}_{\tilde{\chi}} (2\beta_{M\tilde{\chi}} - 4) + \frac{W_{pl\tilde{\chi}} - W_{el\tilde{\chi}}}{W_{el\tilde{\chi}}} \leq 0.9 \quad (9.19)$$

- **Class 3 :**

The resistance criterion is written as:

$$\frac{N_{sd}}{\chi_Z \frac{A f_y}{\gamma_{m0}}} + \frac{k_{LT} M_{sdy}}{\chi_{LT} \frac{W_{ely} f_y}{\gamma_{m0}}} + \frac{k_{\tilde{\chi}} M_{sd\tilde{\chi}}}{\frac{W_{el\tilde{\chi}} f_y}{\gamma_{m0}}} \leq 1 \quad (9.20)$$

$$\mu_{\tilde{\chi}} = \bar{\lambda}_{\tilde{\chi}} (2\beta_{M\tilde{\chi}} - 4) \leq 0.9 \quad (9.21)$$

- **Class 4 :**

The resistance criterion is expressed as:

$$\frac{N_{sd}}{\chi_Z \frac{A_{eff} f_y}{\gamma_{m2}}} + \frac{k_{LT} (M_{sdy} + N_{sd} \times e_{NZ})}{\chi_{LT} \frac{W_{effy} f_y}{\gamma_{m0}}} + \frac{k_{\tilde{\chi}} (M_{sd\tilde{\chi}} + N_{sd} \times e_{NZ})}{\frac{W_{eff\tilde{\chi}} f_y}{\gamma_{m0}}} \leq 1 \quad (9.22)$$

From an engineering design perspective, the most important practical measures for improving stability are to reduce the unbraced length, provide adequate lateral restraints to the compression flange, increase torsional rigidity, choose a section less sensitive to warping and twist, and control the position of the applied loads relative to the shear center. These measures increase M_{cr} , reduce the reduced slenderness, and therefore improve the design resistance of the member.

9.6. Conclusion:

This chapter has highlighted that beams subjected to bending may fail by lateral-torsional instability before the full cross-section resistance is reached. The phenomenon is controlled by unbraced length, torsional and warping stiffness, end restraints, loading position, and moment distribution. Effective design therefore requires adequate lateral restraint, suitable section selection, and careful application of the reduction factors used in steel design.

9.7. Application Exercises:

This part presents a set of application exercises devoted to the verification of steel members subjected to lateral-torsional buckling in simple bending and combined bending cases. The proposed work emphasizes the identification of the governing bending effects, the evaluation of flexural resistance about the major axis, the influence of lateral-torsional instability on laterally unrestrained members, and the application of appropriate verification procedures for beams and beam-column elements encountered in practice.

Exercise No. 01 - Simple bending

Determine the lateral-torsional buckling resistance of the IPE 400 beam shown in the statement. The member is made of steel with $f_y=235 \text{ N/mm}^2$, subjected to a uniformly distributed load $q=20 \text{ kN/m}$, and has a span of $L = 8\text{m}$. The objective is to verify whether the beam remains stable under simple bending when lateral restraints are not sufficient to prevent out-of-plane instability (Figure 9.8).

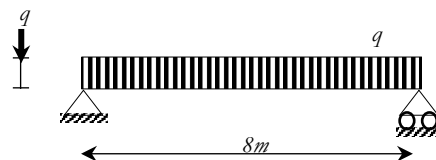


Figure 9.8. Simply supported beam under uniformly distributed loading.

Expected technical work:

- Identify the structural behavior of the member and confirm that the beam is mainly subjected to simple bending about the major axis.

- Determine the maximum design bending moment produced by the uniformly distributed load.
- Adopt the relevant geometrical properties of the IPE 400 section from steel profile tables.
- Evaluate the elastic critical moment M_{cr} for the laterally unrestrained beam.
- Determine the non-dimensional slenderness for lateral-torsional buckling.
- Select the appropriate lateral-torsional buckling curve and calculate the reduction factor.
- Calculate the design buckling resistance moment of the beam.
- Compare the applied design bending moment with the available lateral-torsional buckling resistance.
- Conclude whether the IPE 400 beam satisfies the stability condition with respect to lateral-torsional buckling.

Exercise No. 02 - Combined bending

Verify the stability of the HEB 230 member subjected to axial compression and transverse loading. The steel grade is defined by $f_y=235 \text{ N/mm}^2$, the axial force is $N=60 \text{ kN}$, the transverse load is $q=15 \text{ kN}$, and the section is classified as Class 1.

The objective is to study the member as a beam-column and to verify its stability under combined compression and bending (Figure 9.9).

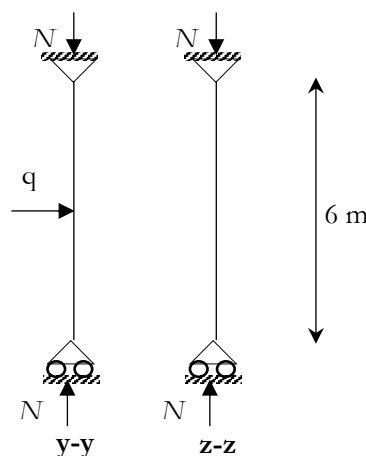
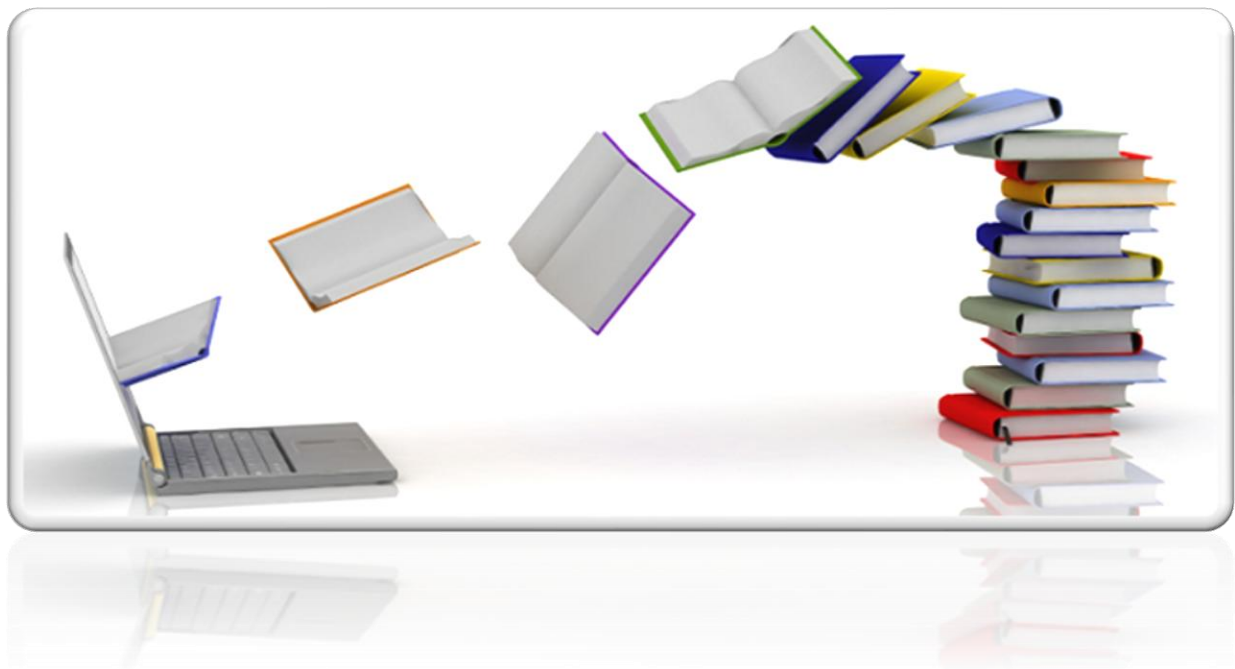


Figure 9.9. Column member under axial force and transverse loading.

Expected technical work:

- Identify the different actions applied to the member and distinguish between axial compression and bending effects.
- Determine the bending moment generated by the transverse load according to the support and loading conditions shown in the Figure 9.9.
- Identify the principal bending axis involved in the verification.
- Use the section class to justify the resistance model adopted for the cross-section.
- Adopt the relevant geometrical properties of the HEB section from steel profile tables.
- Discuss the possible influence of lateral-torsional buckling on the member, especially if the compressed flange is not laterally restrained.
- Evaluate the design bending resistance, taking into account lateral-torsional buckling when required.
- Assess the effect of the axial compressive force on the available resistance of the member.
- Apply the appropriate interaction verification between axial compression and bending resistance.
- Conclude whether the HEB member satisfies the stability and resistance requirements under the combined action of compression and bending.

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This handout presents the fundamental principles required for the analysis and design of steel structural members in civil engineering. It focuses on the mechanical behavior of structural steel, the identification of common steel products and sections, and the verification of members subjected to tension, compression, buckling, simple bending, combined bending, and lateral-torsional buckling.

Each chapter introduces essential theoretical concepts and practical design procedures, enabling students to understand the behavior of steel members under different loading and stability conditions. Worked applications and verification methods are provided to help students consolidate their knowledge, apply basic metal construction rules, and develop practical skills for safe and rational steel structural design.

