APPENDIX

I. Operations On Type-2 Sets

Consider two fuzzy sets of type-2,$\widetilde{A}$ and $\widetilde{B}$, in a universe X . Let $\mu_{\widetilde{A}}$ and $\mu_{\widetilde{B}}$ be the membership grades (fuzzy sets in $J_x \in [0,1]$) of these two sets, represented, for each x , as $\mu_{\widetilde{A}} = \int_u f_x(u)/u$ and $\mu_{\widetilde{B}} = \int_w g_x(w)/w$, respectively, where $u, w \in J_x$ indicate the primary memberships of x and $f_x(u), g_x(w) \in [0,1]$indicate the secondary memberships (grades) of x. Using Zadeh's Extension Principle [15,54,55], the membership grades for union, intersection and complement of type-2 fuzzy sets,$\widetilde{A}$ and $\widetilde{B}$ have been defined as follows [56]:

- Union

$$\tilde{A} \cup \tilde{B} \Leftrightarrow \mu_{\tilde{A}\cup\tilde{B}}(x) = \mu_{\tilde{A}} \sqcup \mu_{\tilde{B}} = \int_u l \int_w \left(f_x(u) \star g_x(w)\right)/(u \vee w) \qquad (1)$$

- Intersection

$$\tilde{A} \cap \tilde{B} \Leftrightarrow \mu_{\tilde{A}\cup\tilde{B}}(x) = \mu_{\tilde{A}} \sqcap \mu_{\tilde{B}} = \int_u l \int_w \left(f_x(u) \star g_x(w)\right)/(u \star w) \qquad (2)$$

- Complement

$$\bar{\tilde{A}} \Leftrightarrow \mu_{\bar{\tilde{A}}} = \neg\mu_{\tilde{A}} = \int_u f_x(u)/(u-1) \qquad (3)$$

Where $\vee$ represents the max t-conorm and $\star$ represents a t-norm. The integrals indicate logical union. In the sequel, we adhere to these definitions, and, as in [56], we refer to the operations $\sqcup, \sqcap$ and $\neg$ as join, meet and negation, respectively.[57]

II. Membership Functions Creation

Triangular and trapezoidal membership functions retained in this work are defined using 4 linear functions, completely described by 5 points (a, b, c, d and e) as defined in Equations 1, 2 and 3, and illustrated in Figure 1. Triangular MFs are a particular case of trapezoidal MFs where b=c as illustrated in Figure 2.

$$Trapezzoid(x, a, b, c, d, e) = max\big(0, min(y_1, y_2, e)\big) \qquad (1)$$

$$y_1(x, a, b, c) = e\frac{x-a}{b-a} \qquad (2)$$

$$y_2(x, c, d, e) = e\frac{d-x}{d-c} \qquad (3)$$

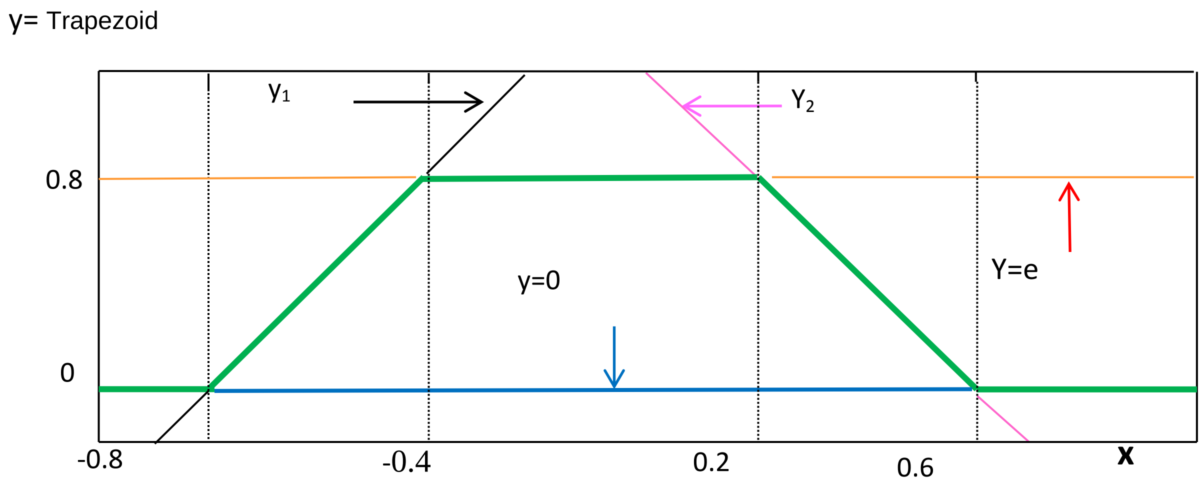


Fig 1 Trapezoidal MF example

a = -0:8; b = -0:4; c = 0:2; d = 0:6; e = 0:8

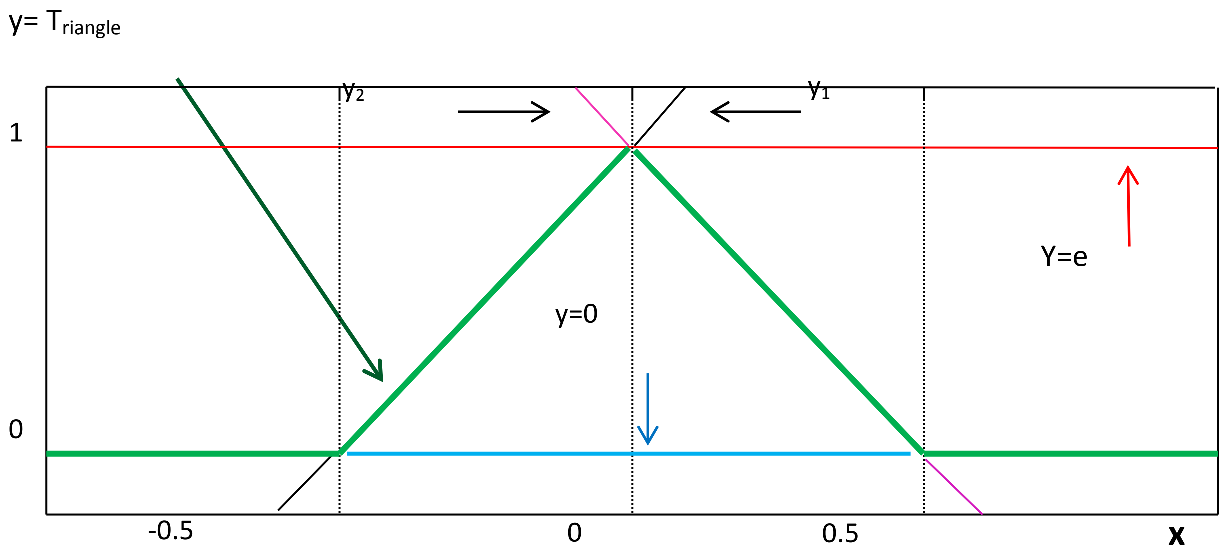
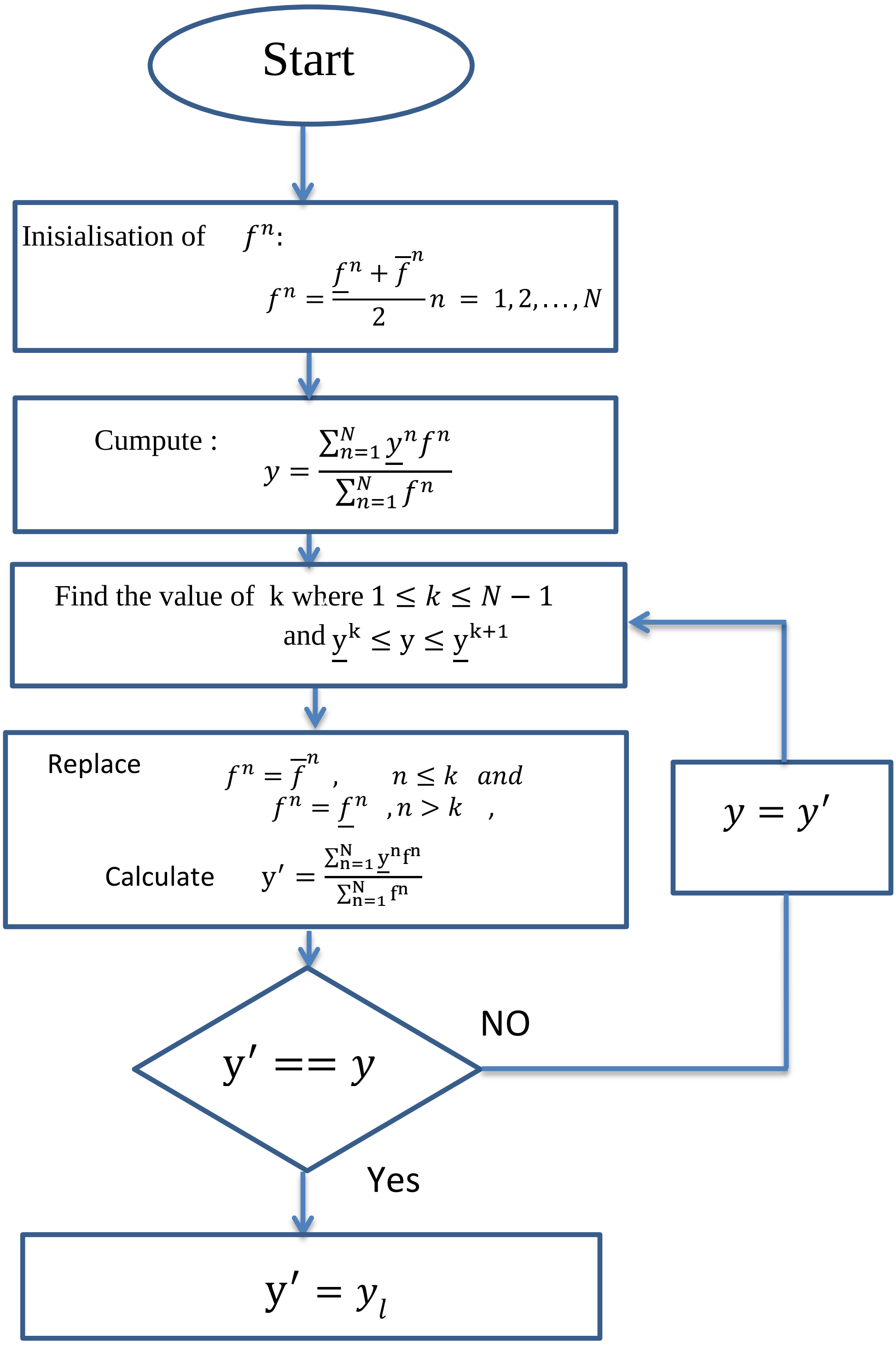


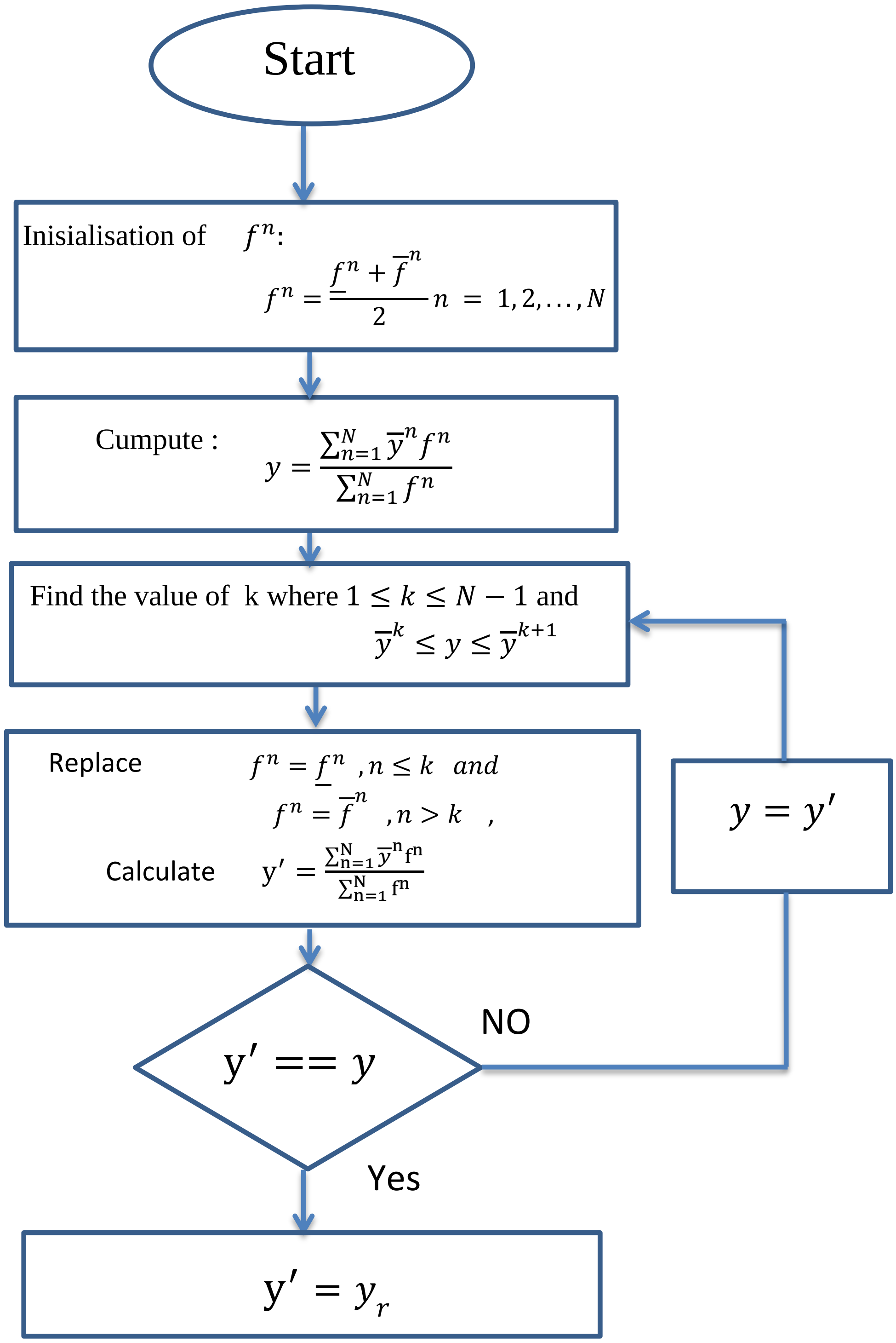
Fig 2 Triangular MF example

a = -0:5; b = c = 0; d = 0:5; e = 1

KM Algorithm for Computing y_L :



KM Algorithm for Computing y_r :



FUZZY PID STRUCTURES

we present some different FUZZY PID structures.**[24][21]** :
where e is error , Δe is change of error and Δ^2e rate of change of error**.**

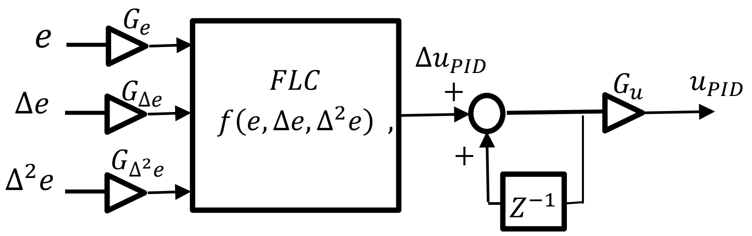


Fig 3 Three-input fuzzy PID (coupled rules)

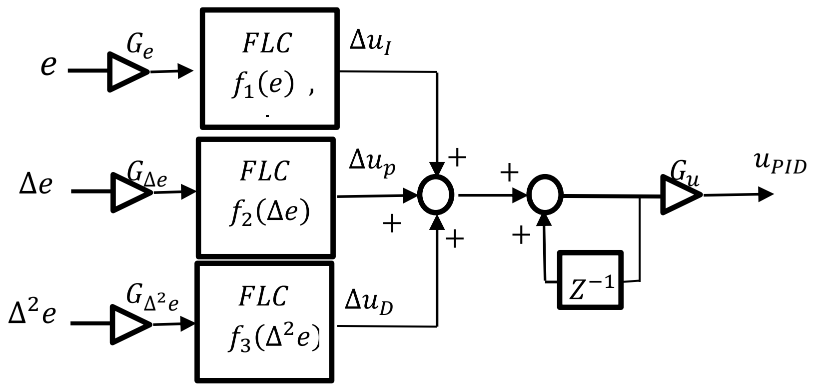


Fig 4 Three-input fuzzy PID (decoupled rules)

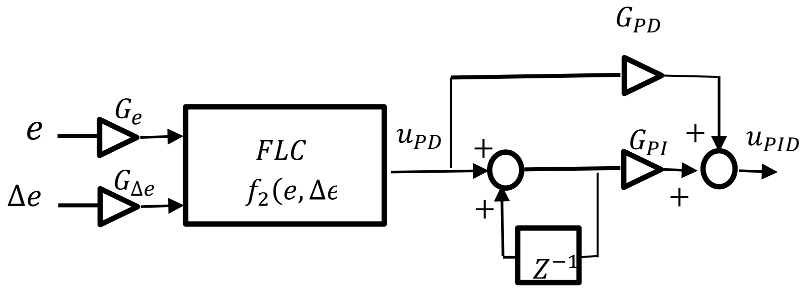


Fig 5 Two-input fuzzy PID (coupled rules)

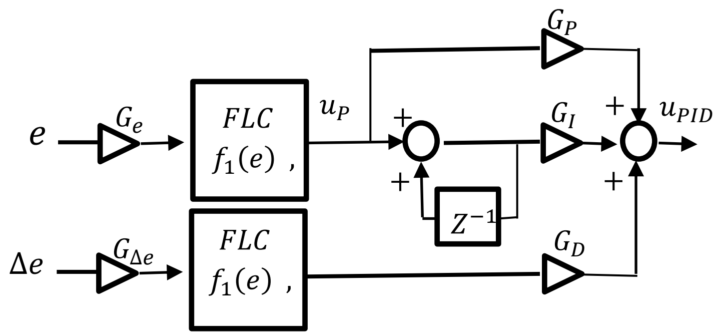


Fig 6 Two-input fuzzy PID (decoupled rules)

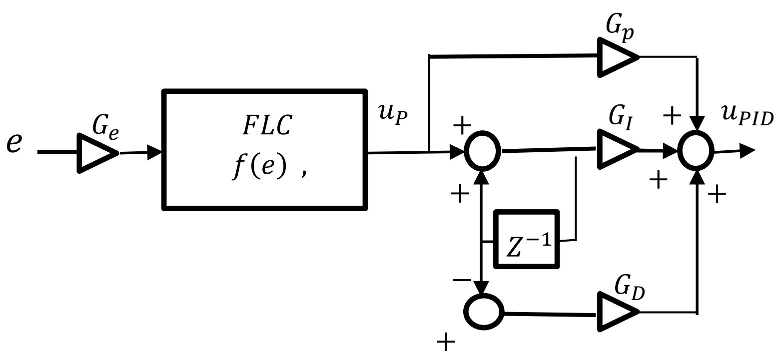


Fig 7 One-input fuzzy PID (coupled rules)

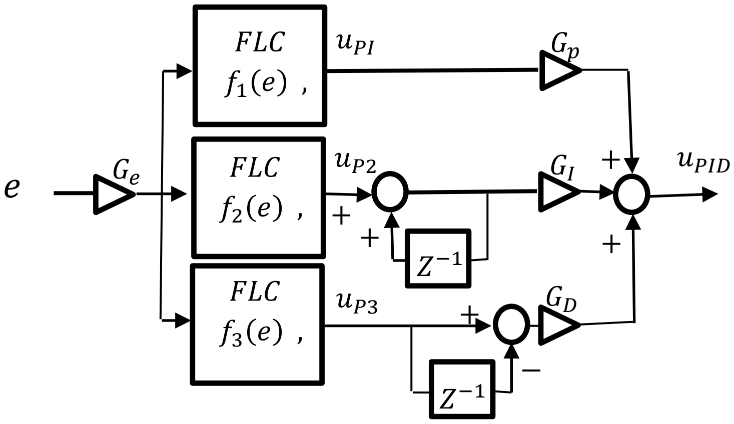


Fig 8 One-input fuzzy PID (decoupled rules)